

Why Nuclear Clocks: The ^{229}Th Annual Modulation Test of Scalar-Field Gravitational Coupling

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Abstract

Precision atomic clock comparisons constrain pure electromagnetic-sector (α -sector) scalar-field couplings to $|k_\alpha| \lesssim 3.6 \times 10^{-8}$ (95% CL), as demonstrated by the null result of the PTB Yb^+ E3/E2 same-ion comparison [7]. In Density Field Dynamics (DFD), this null is consistent with the framework’s composition-dependent coupling model: same-ion comparisons cancel the composition terms that drive the DFD signal, leaving only the pure α -sector residual—which is indeed negligible.

The ^{229}Th nuclear clock transition accesses a fundamentally different coupling channel: the strong sector, where dimensional transmutation amplifies the scalar-field response by a factor $|\mathcal{R}| \sim 10^3$. The unscreened strong-sector prediction (~ 7 MHz annual half-amplitude) is already excluded by four orders of magnitude by the 220 Hz frequency reproducibility reported by Ooi et al. [2], *independently requiring* strong-sector screening at Earth’s surface. The screened DFD estimate (~ 8 kHz) is likewise excluded by a factor of ~ 30 –50. Since the analogous electromagnetic screening formula overpredicts the PTB bound by $\sim 17\times$, this pattern is consistent: the vertex-coupling prescription systematically overestimates at Earth-surface gravitational acceleration.

The surviving signal window lies at $\lesssim 200$ Hz (constrained by Ooi) down to ~ 26 Hz (the composition-coupling floor from the “family + clock” model). A dedicated orbital-phase reanalysis of existing Th-229/Sr data could detect or constrain signals in this range. General relativity predicts zero. The demonstrated frequency reproducibility of 220 Hz over 7 months in $^{229}\text{Th}:\text{CaF}_2$ [2] places this window within reach, with next-generation nuclear clocks covering it entirely.

1 Introduction

The thorium-229 nuclear isomer has a uniquely low excitation energy of ~ 8.4 eV ($\nu_{\text{Th}} \approx 2,020,407$ GHz), accessible to VUV laser spectroscopy. Recent work has established the absolute transition frequency to kHz precision [1], resolved the five-line nuclear electric quadrupole structure [1], measured the thermal sensitivity at three temperatures with line b shifting by only 5(5) kHz over 79 K [4], identified a zero-shift operating temperature $T_0 = 196(5)$ K [2], and demonstrated sub-kHz frequency reproducibility across independent crystals [2]. The fine-structure constant sensitivity $K_\alpha = 5900 \pm 2300$ has been extracted from the measured quadrupole moment ratio [3].

These developments make ^{229}Th a uniquely powerful probe of scalar-field gravitational theories. But why is a *nuclear* clock needed? After all, atomic clocks already achieve 10^{-18} fractional precision.

The answer lies in a constraint that is simultaneously a clue. The PTB Yb^+ E3/E2 same-ion comparison [7] constrains pure α -sector coupling to $|k_\alpha| \lesssim 3.6 \times 10^{-8}$ (95% CL), while the Ooi et al. frequency reproducibility of 220 Hz over 7 months [2] independently constrains strong-sector coupling to levels far below the unscreened DFD prediction. Together, these results tell us that *screening is operating in both sectors*, and that the surviving signal window for strong-sector coupling lies at the $\lesssim 200$ Hz level—still accessible to precision nuclear clock comparisons, and invisible to atomic clocks.

This paper develops that argument quantitatively, organizing the prediction by coupling channel, citing the PTB bound as the motivation for nuclear clock tests rather than an obstacle, and providing explicit falsification criteria.

2 Coupling Channels and Constraints

DFD predicts that clock transition frequencies couple to the scalar field ψ through multiple channels. The general coupling coefficient for species A is [6]:

$$K_A = k_\alpha \kappa_\alpha^{(A)} + k_s \kappa_s^{(A)} + k_N C_N^{(A)} + k_e C_e^{(A)}, \quad (1)$$

where $\kappa_\alpha^{(A)} = S_A^\alpha$ and $\kappa_s^{(A)} = S_A^{\alpha_s}$ are the α - and α_s -sensitivities, $C_N^{(A)}$ and $C_e^{(A)}$ are nuclear and electronic family charges, and k_α, k_s, k_N, k_e are coupling constants determined by the theory.

For a clock ratio A/B , the annual modulation from Earth’s varying solar potential is:

$$\frac{\delta R}{R} = (K_A - K_B) \cdot \delta\psi_{\text{annual}}, \quad (2)$$

where $\delta\psi_{\text{annual}} = (GM_\odot/c^2 a) \cdot e = 1.65 \times 10^{-10}$.

The key insight is that different comparison types probe different terms in Eq. (1).

2.1 Same-ion comparisons: the α -sector constraint

The PTB Yb⁺ E3/E2 ratio compares two transitions in the *same* ion. Both transitions share identical nuclear composition ($\Delta C_N = 0$), identical electronic structure ($\Delta C_e = 0$), and negligible strong-sector sensitivity ($\Delta\kappa_s \approx 0$: same-ion E3/E2 is dominantly sensitive to α , with any hadronic sensitivity strongly suppressed). Only the α -sensitivity differs: $\Delta\kappa_\alpha = S_{\text{E3}}^\alpha - S_{\text{E2}}^\alpha = -5.95 - 1.0 = -6.95$.

The E3/E2 comparison therefore isolates the α -sector:

$$\Delta K_{\text{E3/E2}} = k_\alpha \cdot (-6.95). \quad (3)$$

Lange et al. [7] report the gravitational coupling parameter $(c^2/\alpha)(d\alpha/d\Phi) = 14(11) \times 10^{-9}$, consistent with zero. In DFD, $\delta\alpha/\alpha = k_\alpha \psi$ with $\psi \approx \Phi/c^2$ in the weak-field limit, so $k_\alpha \equiv (c^2/\alpha)(d\alpha/d\Phi)$ directly. The one-sided 95% CL upper bound is:

$$|k_\alpha| \lesssim |14| + 1.64 \times 11 \approx 3.2 \times 10^{-8}. \quad (4)$$

We round to $\sim 3.6 \times 10^{-8}$ ($|14| + 2\sigma$) for a conservative bound. DFD’s screened prediction $k_\alpha^{\text{eff}}(\oplus) = 5.98 \times 10^{-7}$ exceeds this by a factor of ~ 17 .

DFD interpretation. This bound is *consistent* with DFD’s composition-dependent coupling model. In the “family + clock” framework (Appendix T of [6]), the coupling flows through nuclear and electronic family charges that cancel identically for same-ion comparisons. The E3/E2 null does not constrain the cross-species signal. Same-ion composition cancellation is a generic feature of any composition-dependent scalar theory; we present the E3/E2 consistency as a post-diction compatible with the framework rather than a pre-registered prediction.

What this kills. Any model predicting $K_A = k_\alpha \cdot S_A^\alpha$ with $k_\alpha \gtrsim 4 \times 10^{-8}$ is excluded. This includes DFD’s screened coupling formula $k_\alpha^{\text{eff}}(\oplus) = 5.98 \times 10^{-7}$ (Sec. 9 of [6]), which overpredicts the PTB bound by $\sim 17\times$. The bare vertex-counting value $k_\alpha = \alpha^2/(2\pi) \approx 8.5 \times 10^{-6}$ overpredicts by $\sim 240\times$.

What survives. Couplings through (i) the strong sector ($k_s \neq 0$), or (ii) composition-dependent family charges ($k_N, k_e \neq 0$), or both. These vanish for same-ion comparisons and contribute only to cross-species ratios.

2.2 Cross-species atomic comparisons: composition signal

For different atomic species A and B , $\Delta C_N \neq 0$ and $\Delta C_e \neq 0$ generically. The “family + clock” model (Appendix T of [6]) parameterizes the coupling as:

$$K_i = k_N C_N^{(i)} + k_e C_e^{(i)}, \quad (5)$$

with family charges assigned by chemical group and clock type. The PTB E3/E2 bound forces $k_\alpha \rightarrow 0$, leaving k_N and k_e as the active couplings.

For Cs/Sr, the family model gives $K_{\text{Cs}} - K_{\text{Sr}} \sim 2 \times 10^{-5}$, not yet excluded by existing multi-laboratory clock comparisons (no dedicated annual-modulation analysis has been performed on Cs/Sr data with sufficient phase coverage) [6].

For Th-229/Sr, the composition difference is larger (Th is a lanthanide/actinide, Sr is alkaline earth), giving:

$$K_{\text{Th}} - K_{\text{Sr}} \sim 8 \times 10^{-5}, \quad (6)$$

as derived in Sec. 9.6 and Appendix T of [6]. This

yields an annual modulation:

$$\left(\frac{\delta R}{R}\right)_{\text{family}} \sim 8 \times 10^{-5} \times 1.65 \times 10^{-10} \sim \mathbf{1.3} \times \mathbf{10^{-14}}. \quad (7)$$

This is a *model-dependent estimate* on the Th-229/Sr signal from composition coupling alone, conditional on the family+clock framework being correct. The model has two free parameters (k_N , k_e) with family charges fixed by simple chemical-group rules (not individually fitted), constrained by three independent clock-pair bounds (H/Cs null, Hg⁺/Cs, Dy/Cs), leaving one degree of freedom. The Th-229/Sr value is therefore an extrapolation from this constrained model, not an independent derivation.

2.3 Nuclear clock comparisons: the strong sector

The ²²⁹Th nuclear isomer transition is qualitatively different from any atomic transition because the 8.4 eV energy arises from near-cancellation between Coulomb ($\sim +300$ keV) and nuclear strong-force (~ -300 keV) contributions. This makes the transition exponentially sensitive to α_s through dimensional transmutation [5]:

$$\frac{\delta E_m}{E_m} = S_\alpha \frac{\delta \alpha}{\alpha} + S_q \frac{\delta X_q}{X_q}, \quad (8)$$

where $S_\alpha \approx 10^4$ and $S_q \approx -10^4$ from nuclear structure calculations [5] (denoted K_α and K_q in Flambaum's notation), and $X_q = m_q/\Lambda_{\text{QCD}}$. Variations in α_s enter through $\delta X_q/X_q \approx -(\delta\Lambda_{\text{QCD}}/\Lambda_{\text{QCD}}) \approx -64\delta\alpha_s/\alpha_s$, where the factor of 64 arises from the exponential sensitivity of Λ_{QCD} to α_s via dimensional transmutation.

DFD predicts that each gauge sector couples to ψ with strength proportional to its coupling constant squared [6]:

$$k_i = \frac{\alpha_i^2}{2\pi}. \quad (9)$$

For QCD at the Z -pole scale ($\alpha_s(M_Z) = 0.1180 \pm 0.0009$, PDG 2024):

$$k_s = \frac{\alpha_s^2}{2\pi} \approx 2.2 \times 10^{-3}. \quad (10)$$

(At the confinement scale, $\alpha_s \sim 0.3$ – 0.5 , the bare coupling would be ~ 10 – $100\times$ larger. We use the Z -pole value as a conservative choice where perturbative reasoning is valid; at the confinement scale, both

the coupling and its theoretical uncertainty grow substantially.) This gives $k_s/k_\alpha \approx 260$. Combined with the Λ_{QCD} amplification factor of ~ 64 (from $\delta\Lambda_{\text{QCD}}/\Lambda_{\text{QCD}} = (2\pi/|b_3|\alpha_s^2)\delta\alpha_s/\alpha_s$), the enhancement factor for Th-229 relative to optical atomic clocks is [6]:

$$|\mathcal{R}| \equiv \frac{(\delta\nu/\nu)_{\text{Th}}}{(\delta\nu/\nu)_{\text{optical}}} \approx 1400_{-1000}^{+2600}. \quad (11)$$

Key point. This channel is *invisible* to the PTB E3/E2 comparison. Both E3 and E2 are electronic transitions in Yb⁺, dominantly sensitive to α with any hadronic sensitivity strongly suppressed. The E3/E2 bound constrains k_α only; k_s is unconstrained.

Screening. The unscreened coupling gives the largest signal. In environments with strong gravitational acceleration, DFD predicts screening via the interpolation function $\mu_{\text{LPI}}(a/a_0)$, which suppresses k_s by the same factor as k_α . At Earth's surface ($a = g$):

$$k_s^{\text{eff}}(\oplus) = 2\sqrt{\alpha_s} \mu_{\text{LPI}}(g/a_0) \approx 2.4 \times 10^{-6}. \quad (12)$$

However, the screening prescription for the strong sector has not been independently validated, and the ratio k_s/k_α may differ from the naive α_s^2/α^2 scaling under screening. We therefore present estimates for both screened and unscreened cases.

3 The Prediction

3.1 Strong-sector coupling: existing constraints and surviving window

Unscreened prediction: already excluded. With the bare coupling $k_s = \alpha_s^2/(2\pi) \approx 2.2 \times 10^{-3}$ and $S_{\text{Th}}^{\alpha_s} \sim 10^4$:

$$\left(\frac{\delta R}{R}\right)_{\text{bare}} \sim 2.2 \times 10^{-3} \times 10^4 \times 1.65 \times 10^{-10} \sim 3.6 \times 10^{-9}, \quad (13)$$

corresponding to $\delta\nu_b \sim 7$ MHz. Ooi et al. [2] report 220 Hz reproducibility over 7 months with $\tilde{\chi}^2 = 0.4$ and no systematic drift. Over 7 months the orbital cosine swings by $\gtrsim 1.25$ of its full range, so an unscreened signal would produce $\gtrsim 9$ MHz variation—excluded by ~ 4 orders of magnitude.

This is a *positive result* for the DFD framework: it independently requires strong-sector screening at Earth's surface, consistent with the interpolation function μ_{LPI} predicted from the theory.

Screened prediction: also excluded. With Unruh–de Sitter screening at Earth’s surface:

$$k_s^{\text{eff}}(\oplus) = 2\sqrt{\alpha_s} \mu_{\text{LPI}}(g/a_0) \approx 2.4 \times 10^{-6}, \quad (14)$$

giving:

$$\begin{aligned} \left(\frac{\delta R}{R}\right)_s &= k_s^{\text{eff}}(\oplus) \times S_{\text{Th}}^{\alpha_s} \times \delta\psi_{\text{annual}} \\ &\sim 2.4 \times 10^{-6} \times 10^4 \times 1.65 \times 10^{-10} \\ &\sim 4 \times 10^{-12} \quad (\sim 8 \text{ kHz}). \end{aligned} \quad (15)$$

This is likewise excluded by the Ooi data: 8 kHz \gg 220 Hz reproducibility, by a factor of ~ 36 . This is consistent with the pattern seen in the α -sector, where the screened formula overpredicts the PTB bound by $\sim 17\times$. In both sectors, the vertex-coupling prescription with standard screening overestimates the effective coupling at Earth-surface gravitational acceleration by $\mathcal{O}(10)$ – $\mathcal{O}(100)$.

Surviving signal window. The Ooi data constrain the annual half-amplitude to:

$$\delta\nu_b \lesssim 200\text{--}300 \text{ Hz} \quad (16)$$

(a rough bound; a dedicated orbital-phase fit would sharpen this). The composition-coupling floor from the family model is ~ 26 Hz (Sec. 3.2). The surviving signal window is therefore:

$$26 \text{ Hz} \lesssim \delta\nu_b \lesssim 200 \text{ Hz}, \quad (17)$$

corresponding to $1.3 \times 10^{-14} \lesssim \delta R/R \lesssim 10^{-13}$. This range is below the current per-scan precision of ~ 2 – 4 kHz but accessible to a mature nuclear clock achieving ~ 10 Hz per measurement ($\sim 5 \times 10^{-15}$ fractional).

Open questions. Both k_s^{eff} and $S_{\text{Th}}^{\alpha_s}$ carry order-of-magnitude uncertainty. The sensitivity $S_{\text{Th}}^{\alpha_s} \sim 10^4$ is from Flambaum’s 2006 estimate [5]; modern nuclear structure calculations have not verified this number. A smaller $S_{\text{Th}}^{\alpha_s}$ would reduce the signal proportionally but also weaken the existing Ooi constraint on k_s^{eff} , so the ratio (prediction/constraint) is relatively stable.

Caveat: the EM analog. The vertex-counting formula $k_i = \alpha_i^2/(2\pi)$ has been tested once: for the electromagnetic sector. The screened value $k_\alpha^{\text{eff}}(\oplus) \approx 5 \times 10^{-7}$ exceeds the PTB bound by $\sim 17\times$. For the

strong sector, the screened value exceeds the Ooi constraint by $\sim 36\times$. Both sectors show the same qualitative pattern: the screening prescription reduces the coupling substantially but not enough. Until this discrepancy is understood—whether from additional screening mechanisms, sector-specific corrections, or a fundamentally revised coupling formula—the DFD prediction should be understood as identifying the *channel* and *experiment*, not the precise amplitude.

What makes this channel essential. Despite the amplitude uncertainty, the strong-sector channel is: (i) unconstrained by PTB E3/E2 (which probes only k_α); (ii) uniquely accessed by nuclear clocks (atomic clocks are blind to α_s -variation); (iii) worth testing on model-independent grounds: *any* scalar-field theory coupling to the strong sector will produce an annual modulation in $^{229}\text{Th}/\text{Sr}$ that atomic clocks cannot detect.

3.2 Composition coupling: the model-dependent floor

From the family + clock model (Sec. 2.2):

$$\left(\frac{\delta R}{R}\right)_{\text{family}} \sim 1.3 \times 10^{-14}, \quad (18)$$

corresponding to $\delta\nu_b \sim 26$ Hz. This is below the current reproducibility floor of 220 Hz [2] but above the projected sensitivity of 10^{-15} – 10^{-16} for a mature nuclear clock. This estimate is conditional on the family+clock model; if that model is incorrect, the composition coupling could be smaller or zero.

3.3 α -sector: excluded by PTB

For completeness: a pure α -sector coupling using $k_\alpha^{\text{eff}}(\oplus) = 5.98 \times 10^{-7}$ and $S_{\text{Th}}^\alpha = 5900$ [3] would predict $\delta R/R \approx 5.8 \times 10^{-13}$ (~ 1.2 kHz). This is excluded: $k_\alpha^{\text{eff}} = 5.98 \times 10^{-7}$ exceeds the PTB bound of $\sim 3.6 \times 10^{-8}$ by a factor of ~ 17 . We do not use this channel.

3.4 Signal characteristics

Regardless of which coupling channel dominates, the DFD prediction has these model-independent features:

1. **Frequency:** Exactly $f_{\text{orbit}} = 1/\text{year} = 3.17 \times 10^{-8}$ Hz.

2. **Phase:** Maximum at perihelion (January 3).
3. **Universality:** The hyperfine-averaged (EFG-free) transition frequency, constructed from the five quadrupole lines via [4]

$$\nu_{\text{Th}} = \frac{1}{6}(\nu_{3/2 \rightarrow 1/2} + 2\nu_{5/2 \rightarrow 3/2} + 2\nu_{1/2 \rightarrow 1/2} + \nu_{3/2 \rightarrow 3/2}) \quad (19)$$

should show fractional modulation given by Eqs. (15)–(18).

4. **Crystal independence:** Different $^{229}\text{Th}:\text{CaF}_2$ crystals (and, if available, $^{229}\text{ThF}_4$ thin films [11]) should show identical fractional modulation, since the effect is nuclear.
5. **Sign:** Th-229 frequency increases at perihelion (deeper potential \rightarrow larger $\psi \rightarrow$ nuclear energy shifts upward for $K_{\text{Th}} > 0$).
6. **GR prediction:** Exactly zero. Any nonzero composition-dependent annual modulation in the Th-229/Sr ratio is new physics.

4 Confrontation with Existing Data

4.1 Ooi et al. 2026: existing constraint and reanalysis target

Ooi et al. [2] report line b center frequencies at 195 K across crystals C10 and C13 over 7 months, with weighted mean $\nu_b = 2,020,407,298,701.18$ kHz, standard error 0.22 kHz, and $\tilde{\chi}^2 = 0.4$.

Existing constraint on annual modulation.

The $\tilde{\chi}^2 = 0.4$ indicates the data are *more* consistent than expected from statistical scatter alone—there is no excess variance that could accommodate a multi-kHz annual signal. Over a 7-month window, the orbital cosine swings by $\gtrsim 1.25$ of its full range, making the data sensitive to annual modulation. The 220 Hz reproducibility and absence of drift constrain:

$$\delta\nu_b^{(\text{annual})} \lesssim 200\text{--}300 \text{ Hz} \quad (\text{rough estimate}). \quad (20)$$

A formal orbital-phase fit to the individual measurement dates would sharpen this bound. (Note: the exact phase coverage depends on the measurement schedule; if the 7-month window straddles perihelion–aphelion, the constraint is strongest.)

Implication. This already excludes:

- The unscreened strong-sector prediction (~ 7 MHz) by ~ 4 orders of magnitude;
- The screened strong-sector estimate (~ 8 kHz) by $\sim 30\text{--}40\times$.

The surviving signal window ($\lesssim 200$ Hz) is consistent with the composition-coupling range and with a strong-sector coupling suppressed by the same factor that suppresses the α -sector relative to PTB ($\sim 17\times$).

Thermal systematics. Higgins et al. [4] measured line b at three temperatures: $\nu_b = 2,020,407,298.727(4)$ MHz at 150 K, $298.722(3)$ MHz at 229 K, and $298.784(5)$ MHz at 293 K. Between 150 K and 229 K, line b shifts by only $-5(5)$ kHz over 79 K, with a zero-crossing near $T_0 = 196(5)$ K as reported by Ooi et al. [2]. The second-order coefficient extracted from a quadratic fit to the three points is $b \approx 0.005$ kHz/K². A seasonal cryostat drift of $\Delta T = 0.1$ K at T_0 therefore contributes $\delta\nu_{\text{thermal}} \approx b(\Delta T)^2 \approx 0.05$ Hz—a factor of $\sim 160,000$ below the strong-sector prediction of 8 kHz.

Co-thermometry diagnostic. Line c ($m = -1/2 \rightarrow -1/2$) shifts by ~ 2.3 MHz over 143 K [4], giving a thermal coefficient $\sim 40\times$ larger than line b near T_0 . Measuring lines b and c simultaneously provides a sharp diagnostic: if an apparent annual signal in line b is accompanied by a $40\times$ larger signal in line c at the same phase, the origin is thermal. If line c shows no annual modulation (or modulation at the DFD-predicted fractional level, identical to line b), the origin is gravitational.

Reanalysis protocol. Fit the measurement dates and center frequencies to:

$$\nu_b(t) = \nu_0 + \beta t + A \cos(\omega_{\text{orbit}}(t - t_{\text{per}})). \quad (21)$$

Phase is fixed to perihelion (not fitted). Only A , ν_0 , and β are free. Even a null result constrains the strong-sector coupling.

4.2 PTB Yb⁺ E3/E2

Lange et al. [7] report the gravitational coupling parameter for α : $(c^2/\alpha)(d\alpha/d\Phi) = 14(11) \times 10^{-9}$, consistent with zero. As shown in Sec. 2.1, this constrains $|k_\alpha| \lesssim 3.6 \times 10^{-8}$ and is *consistent* with DFD’s composition-dependent coupling model.

This measurement simultaneously:

1. Excludes pure α -sector coupling models at high significance;
2. Leaves the strong-sector and composition channels unconstrained;
3. Provides the primary motivation for nuclear clock tests.

4.3 Zhang et al. 2024

The ~ 2 -week campaign is too short for annual modulation. Establishes the Sr-referenced baseline and demonstrates the five-line structure from which the EFG-free frequency (Eq. 19) can be constructed.

5 Experimental Strategy

5.1 Dedicated 1-year campaign

$^{229}\text{Th}:\text{CaF}_2$ (C10-type) at $T_0 = 196$ K with co-thermometry via line c . VUV comb referenced to JILA Sr clock, which now achieves instability of 1.5×10^{-18} at 1 s [12]—a factor of $\sim 10^9$ below the strong-sector signal per measurement epoch. The reference clock contributes no meaningful uncertainty. An Al^+ quantum logic clock with 5.5×10^{-19} systematic uncertainty [13] provides an alternative reference species with $S_{\text{Al}}^\alpha = 0.008 \approx 0$, verifying that the signal originates in the Th-229 nuclear transition.

Table 1: Detection significance for signals in the surviving window ($A \sim 200$ Hz upper bound, $A \sim 26$ Hz composition floor) at various per-measurement precisions.

σ_i	N	σ_A	A_{200}/σ_A	A_{26}/σ_A
200 Hz	12	82 Hz	2.4σ	0.3σ
100 Hz	12	41 Hz	4.9σ	0.6σ
50 Hz	24	14 Hz	14σ	1.9σ
10 Hz	24	2.9 Hz	69σ	9σ

If the signal lies near the Ooi upper bound (~ 200 Hz), it becomes detectable at $\sim 5\sigma$ with ~ 100 Hz per-measurement precision and ~ 12 measurements. The composition-coupling floor (~ 26 Hz) requires ~ 10 Hz per-measurement precision, corresponding to fractional uncertainty of $\sim 5 \times 10^{-15}$ —a target for mature nuclear clock technology expected within several years.

5.2 Cross-checks

1. **Hyperfine-averaged frequency:** Construct the EFG-free combination (Eq. 19) from multiple quadrupole lines. This eliminates any crystal-field systematic.
2. **Multi-crystal:** C10, C13, X2 must show identical fractional amplitude.
3. **Temperature proxy:** The line c co-thermometer (thermal sensitivity $\sim 40\times$ that of line b near T_0 [4]) sharply distinguishes thermal from gravitational modulation.
4. **Phase:** Modulation not at perihelion $\pm 30^\circ \Rightarrow$ not gravitational.
5. **Alternative host:** $^{229}\text{ThF}_4$ thin films [11] provide a different crystal environment with two inequivalent Th sites (type 1: $V_{zz} = 310.5 \text{ V}/\text{\AA}^2$, $\eta = 0.437$; type 2: $V_{zz} = 308.9 \text{ V}/\text{\AA}^2$, $\eta = 0.194$). DFD predicts identical fractional modulation of the *unsplit* transition frequency across both sites and both host crystals. Site-dependent modulation beyond the EFG difference would falsify nuclear coupling. (Current ThF_4 linewidths of ~ 10 GHz preclude this test at present.)
6. **Th-229/Al⁺ comparison:** Since $S_{\text{Al}}^\alpha \approx 0$, a future Th-229/Al⁺ comparison would give identical results to Th-229/Sr if the signal is nuclear, providing an independent verification.

6 Relation to Other BSM Searches

Annual modulation in clock ratios is not unique to DFD. Any scalar-field theory coupling to the Standard Model—including ultralight dark matter (ULDM) models and varying- α frameworks [9, 10]—can produce similar signals. What distinguishes DFD:

1. **Channel structure:** DFD’s composition-dependent coupling model is consistent with the PTB E3/E2 null (composition cancellation) while predicting that Th-229/Sr *should* show a signal (strong-sector coupling). Most ULDM models predict universal α -coupling and do not naturally explain the E3/E2 null.

2. **Constrained amplitude:** DFD’s vertex-coupling formula predicts specific coupling constants ($k_s \sim 10^{-3}$, $k_\alpha \lesssim 4 \times 10^{-8}$), with existing data already constraining the effective strong-sector coupling. The surviving signal window (~ 26 – 200 Hz) is a specific, testable range.

3. **Correlated predictions:** The same framework predicts specific amplitudes for Cs/Sr, Yb⁺/Sr, Hg/Sr, and Th-229/Sr, all constrained by a single set of coupling constants. Consistency across channels distinguishes DFD from models with free per-species parameters.

A detection of annual modulation at the predicted phase would constitute evidence for scalar-field coupling generally. Matching the strong-sector amplitude would provide evidence for DFD specifically. A detection at an amplitude consistent with composition coupling but *not* strong-sector coupling would constrain the gauge-sector hierarchy.

7 Falsification Criteria

Sharp conditions:

1. **Null at 10^{-13} precision (dedicated reanalysis):** $|A| < 50$ Hz over 1 year with orbital-phase fit \Rightarrow strong-sector coupling constrained to $|k_s^{\text{eff}} \times S_{\text{Th}}^{\alpha_s}| < 1.5 \times 10^{-4}$, excluding the screened DFD estimate by $>300\times$.
2. **Null at 10^{-14} precision:** $|A| < 0.03$ Hz \Rightarrow composition coupling excluded. This would require the family charges to be effectively zero for Th/Sr.
3. **Wrong phase:** Maximum $> 30^\circ$ from perihelion \Rightarrow solar mechanism ruled out.
4. **Crystal-dependent:** Different specimens show different fractional amplitudes \Rightarrow systematic.
5. **Line-dependent beyond EFG:** Hyperfine-averaged frequency shows different fractional modulation than individual lines by more than expected EFG variation \Rightarrow not nuclear coupling.

What would falsify the overall DFD clock framework: A high-precision null in Th-229/Sr combined with a confirmed null in Cs/Sr, Hg/Sr, and Yb⁺/Sr at 10^{-5} level would eliminate all DFD coupling channels. Any single null constrains one channel but not the theory as a whole.

Table 2: Predictions and constraints by coupling channel. Half-amplitudes quoted. GR predicts zero for all entries.

Channel	$\delta R/R$	$\delta\nu_b$	Status
Strong (bare)	$\sim 4 \times 10^{-9}$	~ 7 MHz	<i>excluded</i>
Strong (screened)	$\sim 4 \times 10^{-12}$	~ 8 kHz	<i>excluded</i>
α -sector	$\sim 6 \times 10^{-13}$	~ 1.2 kHz	<i>excluded</i>
Ooi constraint	$\lesssim 10^{-13}$	$\lesssim 200$ Hz	Data
Composition (k_N, k_e)	$\sim 10^{-14}$	~ 26 Hz	Open
Phase	Perihelion (Jan. 3)		
Frequency	$f_{\text{orbit}} = 1/\text{yr}$		
<i>Existing benchmarks:</i>			
Reproducibility (7-mo)			220 Hz
Per-scan uncertainty			2–4 kHz

8 Summary

Table 2 collects the predictions and constraints. The PTB Yb⁺ E3/E2 null excludes pure α -sector coupling. The Ooi et al. reproducibility data independently exclude the screened strong-sector estimate and *require* screening of the bare strong-sector coupling by ~ 4 orders of magnitude. Both exclusions are consistent with the DFD screening framework systematically overpredicting effective couplings at Earth’s surface by $\mathcal{O}(10)$ – $\mathcal{O}(100)$.

The surviving signal window (~ 26 – 200 Hz) lies between the composition-coupling floor and the Ooi constraint. A dedicated orbital-phase reanalysis of existing data could detect or constrain signals in this range at zero cost. Next-generation nuclear clocks achieving ~ 10 Hz per measurement would cover the window entirely.

Open theoretical issues.

1. The vertex-coupling formula $k_i = \alpha_i^2/(2\pi)$ with standard screening overestimates the effective coupling in both the EM sector ($\sim 17\times$ vs. PTB) and the strong sector ($\sim 36\times$ vs. Ooi). The consistent pattern suggests a common mechanism—possibly additional screening, higher-order corrections, or a revised coupling formula. Understanding this discrepancy is the most important open problem for DFD’s clock predictions.
2. The strong-sector sensitivity $S_{\text{Th}}^{\alpha_s} \sim 10^4$ relies on Flambaum’s 2006 estimate [5]. Modern nuclear

structure calculations are needed.

3. The screening prescription for the strong sector ($k_s^{\text{eff}} = 2\sqrt{\alpha_s} \mu_{\text{LPI}}$) extends the electromagnetic argument to QCD by analogy, not by rigorous derivation.
4. The family charge assignments in Appendix T of [6] are phenomenological. A microscopic derivation from DFD’s postulates would sharpen the composition-coupling prediction.

Practical path.

1. Reanalyze the Ooi et al. [2] dataset for orbital-phase correlation with perihelion-fixed cosine fit. Cost: zero. Potential: sharpen the constraint on strong-sector coupling from the current rough bound ($\lesssim 200$ Hz) to a precise upper limit, or detect a signal in the surviving window.
2. Plan a 1-year Th-229/Sr campaign at ~ 100 Hz per measurement. Uses existing JILA infrastructure with improved averaging. Potential: 5σ on signals near the Ooi bound.
3. Push toward 10^{-15} Th-229 precision for full coverage of the surviving signal window including the composition-channel floor. Requires mature CW VUV or improved comb technology.

The Th-229 nuclear clock is not merely a better clock—it is a qualitatively different probe. The PTB E3/E2 null tells us that atomic clocks have exhausted their sensitivity to scalar-field coupling in the electromagnetic sector. The Ooi frequency reproducibility data independently require strong-sector screening and constrain the surviving signal to $\lesssim 200$ Hz. A dedicated orbital-phase analysis of existing data, followed by a precision campaign at ~ 100 Hz per measurement, would either detect the first strong-sector scalar coupling or set the most stringent hadronic LPI constraints ever achieved. Regardless of whether DFD is correct, a Th-229/Sr annual modulation search is among the most powerful tests of gravitational scalar-field theories achievable with current or near-term technology.

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