Two Numerical Relations Linking the Fine-Structure Constant to Gravitational Phenomenology

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Abstract

We highlight two numerical relations connecting the fine-structure constant $\alpha \approx 1/137$ to gravitational phenomenology. First, the MOND acceleration scale satisfies $a_0 = 2\sqrt{\alpha}\, cH_0$ to within the current uncertainty in H_0, where c is the speed of light and H_0 is the Hubble parameter. Second, if atomic clock responses to gravitational potential variations are parameterized as $K_A = k_\alpha S_A^\alpha$, where S_A^α are tabulated α-sensitivity coefficients, then existing clock data are consistent with $k_\alpha = \alpha^2/(2\pi)$ within current $\sim 2\sigma$ uncertainties. These relations involve no free parameters: given α and H_0, both a_0 and k_α are fixed. We present the numerical evidence, offer a vertex-counting heuristic that motivates the appearance of $\sqrt{\alpha}$ and α^2, and identify falsifiable predictions for near-term clock experiments. A multi-month optical clock campaign building on recent cavity-referenced work should be able to confirm or exclude the predicted k_α at $> 10\sigma$ significance.

1 Introduction

The MOND acceleration scale $a_0 \approx 1.2\times10^{-10}$ m/s^2 demarcates the transition between Newtonian and modified gravitational dynamics in galaxies [1, 2]. Its numerical proximity to cH_0—the speed of light times the Hubble parameter—has been noted since MOND's inception [1, 4], but no theoretical framework has explained why these scales should be related.

We show that the relation is more precise than previously recognized:

$$a_0 = 2\sqrt{\alpha}\, cH_0, \tag{1}$$

where $\alpha \approx 1/137$ is the fine-structure constant. This relation is satisfied to within the current "Hubble tension"—the discrepancy between early- and late-universe determinations of H_0. The appearance of α—a purely electromagnetic constant—in a gravitational context is unexpected and, if not coincidental, suggests a coupling between electromagnetism and gravity at cosmological scales.

We further note that if clock sensitivities to gravitational potential follow $K_A = k_\alpha S_A^\alpha$, where $S_A^\alpha \equiv \partial \ln \nu_A/\partial \ln \alpha$ are the relativistic α-sensitivity coefficients tabulated by Dzuba, Flambaum, and collaborators [9, 10, 11], then existing clock comparison data are consistent with

$$k_\alpha = \frac{\alpha^2}{2\pi}. \tag{2}$$

This predicts $k_\alpha \approx 8.5 \times 10^{-6}$, compared to an inferred value of $(-0.4 \pm 0.7) \times 10^{-5}$ from Sr/Cs clock comparisons [16].

Equations (1) and (2) contain no free parameters. Once α and H_0 are specified, a_0 and k_α are determined. The appearance of $\sqrt{\alpha}$ in the MOND relation and α^2 in the clock relation suggests a vertex-counting structure familiar from quantum electrodynamics. Such a structure arises naturally in scalar-tensor frameworks where electromagnetically bound matter couples to a cosmological field [13, 14]. A specific realization—Density Field Dynamics (DFD)—derives both relations from a single Lagrangian [15]; here we focus on the numerical predictions independent of that framework.

2 The Numerical Coincidences

We first establish the numerical relations as empirical facts, independent of any theoretical interpretation.

2.1 Relation I: MOND scale

The observed MOND acceleration is [2, 3]:

$$a_0^{\text{obs}} = (1.20 \pm 0.02) \times 10^{-10} \text{ m/s}^2. \tag{3}$$

The fine-structure constant is [5]:

$$\alpha = 7.2973525693(11) \times 10^{-3} \approx 1/137.036. \tag{4}$$

The Hubble parameter remains subject to the well-known "Hubble tension" [6]:

$$H_0^{\text{Planck}} = 67.4 \pm 0.5 \text{ km/s/Mpc}, \tag{5}$$
$$H_0^{\text{SH0ES}} = 73.0 \pm 1.0 \text{ km/s/Mpc}. \tag{6}$$

From the fine-structure constant:

$$2\sqrt{\alpha} = 0.1708. \tag{7}$$

The cosmological acceleration scale cH_0 depends on which H_0 is used:

$$cH_0^{\text{Planck}} = 6.55 \times 10^{-10} \text{ m/s}^2, \tag{8}$$
$$cH_0^{\text{SH0ES}} = 7.09 \times 10^{-10} \text{ m/s}^2. \tag{9}$$

The predicted MOND scale therefore spans:

$$2\sqrt{\alpha}\, cH_0^{\text{Planck}} = 1.12 \times 10^{-10} \text{ m/s}^2, \tag{10}$$
$$2\sqrt{\alpha}\, cH_0^{\text{SH0ES}} = 1.21 \times 10^{-10} \text{ m/s}^2. \tag{11}$$

The observed value $a_0^{\text{obs}} = 1.20 \times 10^{-10}$ m/s^2 lies squarely within this range. The prediction brackets the measurement:

$$\frac{a_0^{\text{obs}}}{2\sqrt{\alpha}\, cH_0} = \begin{cases} 1.07 & (H_0 = 67.4) \\ 0.99 & (H_0 = 73.0) \end{cases} \tag{12}$$

The agreement is within 7% for Planck and within 1% for SH0ES. Resolving the Hubble tension will sharpen this test; for now, the parameter-free prediction $a_0 = 2\sqrt{\alpha}\, cH_0$ is consistent with observation.

2.2 Relation II: Clock coupling

Local Position Invariance (LPI) requires that atomic frequency ratios be independent of gravitational potential [8]. Violations are parameterized as:

$$\frac{\Delta\nu_A}{\nu_A} = K_A \frac{\Delta\Phi}{c^2}, \tag{13}$$

where Φ is the gravitational potential. Under General Relativity with exact LPI, $K_A = 1$ for all species, so frequency *ratios* are potential-independent.

If α couples to gravity, different atomic species respond proportionally to their α-sensitivity:

$$K_A = k_\alpha \cdot S_A^\alpha, \tag{14}$$

where $S_A^\alpha \equiv \partial \ln \nu_A / \partial \ln \alpha$ are calculated from atomic theory [9, 10, 11]. The differential response between species A and B is:

$$K_A - K_B = k_\alpha (S_A^\alpha - S_B^\alpha). \tag{15}$$

For ^{133}Cs (hyperfine) and ^{87}Sr (optical):

$$S_{\text{Cs}}^\alpha = 2.83, \tag{16}$$
$$S_{\text{Sr}}^\alpha = 0.06, \tag{17}$$
$$\Delta S^\alpha = 2.77. \tag{18}$$

The 2008 Blatt et al. multi-laboratory analysis found [16]:

$$y_{\text{Sr}} = (-1.9 \pm 3.0) \times 10^{-15} \tag{19}$$

for the amplitude of annual variation in Sr/Cs, where Earth's elliptical orbit modulates the solar gravitational potential with amplitude $\Delta\Phi/c^2 = 1.65 \times 10^{-10}$.

This corresponds to:

$$K_{\text{Cs}} - K_{\text{Sr}} = \frac{y_{\text{Sr}}}{\Delta\Phi/c^2} = (-1.2 \pm 1.8) \times 10^{-5}, \tag{20}$$

and thus:

$$k_\alpha = \frac{K_{\text{Cs}} - K_{\text{Sr}}}{\Delta S^\alpha} = (-0.4 \pm 0.7) \times 10^{-5}. \tag{21}$$

The predicted value from Eq. (2) is:

$$k_\alpha^{\text{pred}} = \frac{\alpha^2}{2\pi} = \frac{(7.297 \times 10^{-3})^2}{2\pi} = 8.5 \times 10^{-6}. \tag{22}$$

The difference between prediction and central value is

$$\frac{|k_\alpha^{\rm pred} - k_\alpha^{\rm obs}|}{\sigma_{k_\alpha}} = \frac{|0.85 - (-0.4)|}{0.7} \approx 1.8, \quad (23)$$

i.e. the 2008 result is statistically consistent with the prediction within $\sim 2\sigma$ but does not constitute a detection.

The 2008 error bars were large, precluding detection. However, the central value is in the predicted direction (Sr/Cs smallest at perihelion), and the magnitude is consistent with $k_\alpha \sim \alpha^2$.

3 Vertex-Counting Heuristic

Why might $\sqrt{\alpha}$ appear in the MOND relation and α^2 in the clock relation? We offer a heuristic based on QED vertex counting. A formal derivation within the DFD framework is given in Ref. [15].

In quantum electrodynamics, each interaction vertex contributes a factor of $\sqrt{\alpha}$ to the amplitude. If electromagnetically bound matter couples to a scalar field through QED-like vertices, the coupling strength scales as $(\sqrt{\alpha})^n$ where n is the number of vertices.

3.1 MOND: Two vertices

For the MOND effect—the modification of gravitational dynamics at accelerations below a_0—we consider a two-vertex process:

1. EM-bound matter couples to scalar field ($\sqrt{\alpha}$)
2. Scalar field modifies gravitational response ($\sqrt{\alpha}$)

Combined amplitude: $2 \times \sqrt{\alpha}$.

This gives:

$$a_0 = 2\sqrt{\alpha} \cdot a_\star, \quad (24)$$

where $a_\star = cH_0$ is the cosmological acceleration scale.

3.2 Clock response: Four vertices

For clock response to gravitational potential—requiring coupling between atomic structure, scalar field, and gravitational potential—we consider a four-vertex process:

1. EM-bound matter couples to scalar field ($\sqrt{\alpha}$)
2. Scalar field couples to gravitational potential ($\sqrt{\alpha}$)
3. Gravitational potential couples to scalar field ($\sqrt{\alpha}$)
4. Scalar field modifies atomic transition frequency ($\sqrt{\alpha}$)

Combined: $(\sqrt{\alpha})^4 = \alpha^2$.

Including a standard loop factor of 2π:

$$k_\alpha = \frac{\alpha^2}{2\pi}. \quad (25)$$

We present this as a *heuristic* motivating specific powers of α. The essential point is that the observed numerical relations are consistent with a vertex-counting structure, and this structure yields falsifiable predictions.

4 Universal Clock Prediction

If $K_A = k_\alpha S_A^\alpha$ with $k_\alpha = \alpha^2/(2\pi)$, every atomic clock has a predicted gravitational coupling:

Species	Transition	S_A^α	$K_A^{\rm pred}$ ($\times 10^{-5}$)
^{133}Cs	Hyperfine	2.83	2.40
^{87}Rb	Hyperfine	2.34	1.98
^{1}H	1S-2S	2.00	1.70
^{87}Sr	Optical	0.06	0.05
^{171}Yb$^+$	E2	1.00	0.85
^{171}Yb$^+$	E3	-5.95	-5.04
^{27}Al$^+$	Optical	0.008	0.007
^{199}Hg$^+$	Optical	-2.94	-2.49

Table 1: Predicted gravitational couplings $K_A = k_\alpha S_A^\alpha$ assuming $k_\alpha = \alpha^2/(2\pi) = 8.5 \times 10^{-6}$. Values of S_A^α from Refs. [9, 10, 11, 12].

The prediction is falsifiable: any clock comparison yielding $K_A - K_B \neq k_\alpha(S_A^\alpha - S_B^\alpha)$ would exclude the universal α-coupling hypothesis.

The Cs/Sr channel has $\Delta S^\alpha = 2.77$, among the largest available, amplifying any signal by nearly a factor of 50 compared to channels with $\Delta S^\alpha \sim 0.1$.

5 Comparison with Existing Data

5.1 Blatt et al. (2008)

The three-laboratory Sr clock comparison [16] found:

$$y_{\rm Sr} = (-1.9 \pm 3.0) \times 10^{-15}. \tag{26}$$

Our prediction for $k_\alpha = \alpha^2/(2\pi)$:

$$\begin{aligned} y_{\rm Sr}^{\rm pred} &= -\Delta S^\alpha \cdot k_\alpha \cdot \frac{\Delta\Phi}{c^2} \\ &= -2.77 \times 8.5 \times 10^{-6} \times 1.65 \times 10^{-10} \\ &= -3.9 \times 10^{-15}. \end{aligned} \tag{27}$$

The predicted amplitude (-3.9×10^{-15}) and measured central value (-1.9×10^{-15}) are:

- Same sign (Sr/Cs smallest at perihelion)
- Same order of magnitude
- Statistically consistent within measurement uncertainty: the $y_{\rm Sr}$ amplitudes differ by only 0.7σ, and the corresponding k_α values differ by $\approx 1.8\sigma$

The 2008 measurement could not detect this signal due to large uncertainties, but the data are fully consistent with the prediction.

5.2 Sign convention verification

We explicitly verify the sign agreement. In the convention of Ref. [16]:

- $y_{\rm Sr} < 0$ means $\nu_{\rm Sr}/\nu_{\rm Cs}$ is *smallest* at perihelion.
- Our framework predicts $K_{\rm Cs} > K_{\rm Sr}$ because $S_{\rm Cs}^\alpha > S_{\rm Sr}^\alpha$.
- At perihelion ($\Delta\Phi < 0$), Cs frequency shifts more than Sr, so Sr/Cs decreases.

The signs are consistent. This is a nontrivial check.

6 Prediction for Near-Term Experiments

A multi-month Sr–Si cavity comparison campaign, extending the work of Ref. [17], would cover a substantial fraction of the annual solar potential cycle with precision far exceeding the 2008 measurements. If cross-referenced to Cs standards, such a dataset could decisively test the k_α relation.

6.1 Predicted signal

For $k_\alpha = \alpha^2/(2\pi)$, the expected annual amplitude in Cs/Sr is:

$$|y_{\rm Sr}^{\rm pred}| = 3.9 \times 10^{-15}. \tag{28}$$

Over a six-month baseline spanning perihelion:

$$\Delta\left(\frac{\nu_{\rm Cs}}{\nu_{\rm Sr}}\right) \approx 4 \times 10^{-15}. \tag{29}$$

6.2 Expected significance

Modern optical clock comparisons achieve fractional uncertainties of $\sim 10^{-17}$ at one-day averaging [18, 19]. Over a six-month campaign, systematic-limited precision of $\sim 3 \times 10^{-16}$ is achievable.

If the predicted signal is present:

$$\text{Significance} = \frac{4 \times 10^{-15}}{3 \times 10^{-16}} \approx 13\sigma. \tag{30}$$

This would constitute a definitive detection or exclusion of the specific $k_\alpha = \alpha^2/(2\pi)$ hypothesis.

7 Discussion

7.1 Caveats

We emphasize several limitations:

1. The vertex-counting argument presented here is a heuristic. A complete derivation from the DFD Lagrangian is given in Ref. [15].

2. The 2008 measurement has large uncertainties. While consistent with our prediction, it is also consistent with zero.

3. The factor of 2π in Eq. (2) arises from loop integration in the formal derivation [15].

4. The MOND prediction depends on H_0, which is currently uncertain at the $\sim 8\%$ level due to the Hubble tension [6, 7].

5. Alternative explanations for $a_0 \approx cH_0$ exist [20, 21], though none predict the specific factor of $2\sqrt{\alpha}$.

7.2 If confirmed

If a future campaign measures k_α consistent with $\alpha^2/(2\pi)$, the implications include:

1. **First detection of LPI violation.** This would be the first confirmed departure from the Einstein Equivalence Principle in clock comparisons.

2. **α–gravity coupling.** The fine-structure constant would be directly implicated in gravitational physics.

3. **Parameter-free prediction.** Both a_0 and k_α would be determined by α and H_0 alone.

4. **Unification hint.** The same constant (α) appearing in MOND and clock physics would suggest a common origin, as realized in the DFD framework [15].

7.3 If excluded

If measurements show k_α inconsistent with $\alpha^2/(2\pi)$ at high significance:

1. The universal α-coupling hypothesis would be ruled out.

2. The $a_0 = 2\sqrt{\alpha}\, cH_0$ relation would not extend to clock physics.

3. The numerical coincidence would remain unexplained.

8 Conclusion

We have presented two numerical relations:

$$a_0 = 2\sqrt{\alpha}\, cH_0 \quad \text{(within } H_0 \text{ uncertainty)}, \tag{31}$$

$$k_\alpha = \frac{\alpha^2}{2\pi} \quad \text{(consistent with data at } \sim 2\sigma\text{)}. \tag{32}$$

These relations contain no free parameters. A vertex-counting heuristic motivates the appearance of $\sqrt{\alpha}$ (two vertices) and α^2 (four vertices), connecting MOND phenomenology to atomic clock physics through the fine-structure constant. The formal derivation within the DFD framework is given in Ref. [15].

The prediction $k_\alpha = \alpha^2/(2\pi) \approx 8.5 \times 10^{-6}$ can be tested at $> 10\sigma$ precision by ongoing and planned optical clock campaigns. If confirmed, this would establish a direct link between the fine-structure constant and gravitational phenomenology—a connection uniquely suggested by DFD.

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