Mach's Principle in Density Field Dynamics: An Interpretive and Phenomenological Consolidation

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Density Field Dynamics (DFD) [9], a scalar refractive theory of gravity on flat $\mathbb{R}^3$ with matter acceleration $\mathbf{a} = (c^2/2)\nabla\psi$, contains a sharp, empirically testable structural identity: the galactic transition acceleration $a_\star$ is tied to the cosmic Hubble rate by $a_\star = 2\sqrt{\alpha}\, cH_0$, where α is the fine-structure constant. The empirical coincidence $a_0 \sim cH_0$ has been flagged in the MOND literature for decades without derivation; DFD promotes it to a consequence of S^3 topology, pinning the local galactic crossover scale to the cosmic expansion rate. The program-grade epoch-by-epoch extension $a_\star(z) = 2\sqrt{\alpha}\, cH(z)$ predicts a drift in the radial-acceleration-relation normalization with redshift, of magnitude $a_\star(z{=}1)/a_\star(0) \simeq 1.79$ in a ΛCDM parameterization of $H(z)$ (used purely as an observational parameterization, not an ontological commitment) and $\simeq 2.83$ in DFD's matter-only ψ-screen cosmology. This is falsifiable by JWST and DESI high-redshift galactic kinematic samples and is the headline handle of the paper.

We situate this prediction in Machian language. The Newtonian-limit Green's function on flat $\mathbb{R}^3$ makes local gravitational response an explicit linear functional of the cosmic density fluctuation, realizing Mach 3 in its weak form; we note explicitly that this functional is formally indistinguishable from Newtonian gravity and that DFD-specific Machian content enters only through the nonlinear regime and the global determination of $a_\star$. The external-field-effect structure of [9] provides secondary environment-dependent phenomenology at the cluster-galaxy level, program-grade in numerical magnitude. We distinguish theorem-grade from program-grade claims throughout. Against the Bondi–Samuel [12] taxonomy, DFD satisfies Mach 3, Mach 10, and effectively Mach 8; partially satisfies Mach 1, Mach 2, Mach 6; fails Mach 4, Mach 5, Mach 7, Mach 9. We do not claim that DFD proves Mach's principle. DFD rejects Mach 9 at the ontological level but retains a Machian correspondence at the level of observables: the operationally preferred frame, the ψ rest frame, is dynamically determined by cosmic matter, while the flat-$\mathbb{R}^3$ kinematic substrate remains absolute.

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I. INTRODUCTION

Mach's principle is the idea that the inertia of local matter is determined by the total matter content of the universe. Mach's criticism of Newton's bucket experiment [1] rejected absolute space and attributed centrifugal effects to the relative motion of the water with respect to "the fixed stars and other heavenly bodies." Einstein took this seriously in the construction of general relativity [2] but later concluded that GR does not realize Mach's principle in any straightforward sense, a verdict reinforced by the existence of vacuum solutions (Schwarzschild, Minkowski, Kerr) in which the metric is nontrivial despite the absence of matter [3].

The question of whether any theory of gravity can be genuinely Machian has divided the community for a century. Barbour and Bertotti [4, 5] developed relational dynamics as a framework in which Mach's principle is built in from the start. Sciama's inertial induction model [6] attempted to compute local inertia as a gravitational effect of distant matter. Brans and Dicke's scalar-tensor theory [7] was explicitly motivated by Mach's principle and produces a Newton constant that depends on the local scalar field value. Yet each of these faces genuine difficulties: relational dynamics has no empirically successful realization, Sciama's induction requires a preferred frame, and Brans–Dicke in the observationally viable large-ω limit is effectively indistinguishable from GR and therefore inherits GR's Machian ambiguity.

Density Field Dynamics [9] takes a different approach. The theory is formulated on flat Euclidean $\mathbb{R}^3$ with time as an external parameter. A scalar field $\psi(\mathbf{x}, t)$ called the loading sets the optical refractive index $n = e^\psi$ and determines matter acceleration through

$$\mathbf{a} = \frac{c^2}{2}\nabla\psi. \tag{1}$$

The static field equation is

$$\nabla\cdot\left[\mu\left(\frac{|\nabla\psi|}{a_\star}\right)\nabla\psi\right] = -\frac{8\pi G}{c^2}\rho_m, \tag{2}$$

with $\mu(x) = x/(1+x)$ derived from S^3 topology [9], $a_\star$ an acceleration scale tied to the cosmic expansion by the structural identity

$$a_\star = 2\sqrt{\alpha}\, cH_0, \tag{3}$$

and ρ_m ordinary matter density. This is structurally the AQUAL equation of Bekenstein and Milgrom [10], placed on flat space with ψ primitive and the four-dimensional metric derived.

The thesis of this paper is narrower than "DFD proves Mach's principle" and broader than "DFD is incidentally Machian." We claim that DFD has an operationally Machian sector, inherited from its already-established flat-space scalar refractive structure, and that this sector admits a clean Green's-function description in the Newtonian limit, a structurally cosmic galactic transition scale through Eq. (3), and an environment-dependent inertial response through the external-field-effect (EFE) treatment of the unified theory [9]. The purpose of the paper is to organize these elements into a Machian reading, tag which Bondi–Samuel [12] propositions they satisfy, and extract the empirical predictions that are sharpest in that language. Mach's principle does not generate DFD. It is a downstream interpretation of already-derived DFD content, and we treat it as such throughout. The benefit is that the predictions that emerge from this reading, particularly the redshift evolution of $a_\star$, survive even for a reader who does not accept the Machian framing, since they are structural consequences of Eq. (3).

A note on grade. Within DFD [9], claims are tiered as Theorem, Derived, or Conjectured / Program. We respect that discipline here. The Newtonian inertia functional (Section III) is theorem-grade: it is elementary linear PDE theory applied to Eq. (2) in the $\mu \to 1$ limit. The epoch-by-epoch promotion $a_\star(z) = 2\sqrt{\alpha}\, cH(z)$ (Section VIII) is program-grade: it follows from Eq. (3) only under the additional cosmological assumption that the identity holds at every epoch, which in turn depends on the ψ-screen cosmology of [9]. The EFE prediction for cluster-vs-field rotation curves (Section VIII B) is also program-grade: the EFE structure is in the unified theory, but the quantitative percent-level estimate given here is an order-of-magnitude scaling, not a derivation.

Section II sketches the DFD structure needed for what follows. Section III constructs the Green's function and derives the inertia functional. Section IV reinterprets Eq. (3) as a Machian statement. Section V evaluates DFD against the Bondi–Samuel taxonomy [12]. Section VI addresses the preferred-frame tension. Section VII contrasts with Brans–Dicke. Section VIII presents three empirical Machian predictions. Section IX lists the falsifiers. Section X concludes.

II. DFD STRUCTURE RELEVANT TO MACH'S PRINCIPLE

For completeness we record the DFD elements used below.

The scalar loading field $\psi(\mathbf{x}, t)$ is dimensionless and defined on flat Euclidean three-space. It sets the optical refractive index $n = e^\psi$, so light propagates according to the eikonal of $d\tilde{s}^2 = -c^2dt^2/n^2 + d\mathbf{x}^2$. Matter is governed by the physical metric with $g_{tt} = -c^2e^{-\psi}$, giving redshift $1 + z = e^{\psi/2}$, and by Eq. (1) for acceleration. The two metric structures coincide on the spherically symmetric exterior in the $\mu \to 1$ regime but are distinct objects in general.

The interpolation function $\mu(x) = x/(1 + x)$ is derived from the S^3-composition law of refractive loading [9]. It has limits $\mu(x) \to 1$ for $x \gg 1$ (Newtonian) and $\mu(x) \to x$ for $x \ll 1$ (deep-MOND, radial acceleration relation). The crossover scale $a_\star = 2\sqrt{\alpha}\, cH_0 \approx 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ matches Milgrom's a_0 phenomenologically and is derived in the unified theory rather than fitted. The dimensionless argument of μ is $|\mathbf{a}|/a_\star$, where $\mathbf{a} = (c^2/2)\nabla\psi$; in the ψ formulation this reads $c^2|\nabla\psi|/(2a_\star)$. The compressed notation $\mu(|\nabla\psi|/a_\star)$ appearing in some DFD references is understood in this convention.

Matter density $\rho_m \geq 0$ is the only source on the right-hand side of Eq. (2). There is no dark matter, no cosmological constant, no new field. The "dark" phenomenology at galactic and cosmic scales arises entirely from the nonlinear response of ψ to ordinary matter together with the optical effects of the resulting refractive-index distribution on light propagation (the ψ-screen cosmology of [9]).

III. THE GRAVITATIONAL RESPONSE FUNCTIONAL

We construct the Green's function for the linearized field equation and exhibit local gravitational acceleration as an integral functional of the cosmic density fluctuation. This is a statement about the source of gravitational acceleration, not about the origin of rest-mass inertia in the anti-Newtonian sense. The distinction matters for the Machian reading and we keep it explicit.

A. Linearization and the Newtonian Green's function

In the Newtonian regime $|\nabla\psi| \gg a_\star$ we have $\mu \to 1$ and Eq. (2) reduces to

$$\nabla^2\psi = -\frac{8\pi G}{c^2}\rho_m. \tag{4}$$

This is Poisson's equation with source coefficient $8\pi G/c^2$ rather than the Newtonian $4\pi G$, because ψ couples to acceleration through Eq. (1) with the factor $c^2/2$. The Green's function for the Laplacian on $\mathbb{R}^3$ with vanishing boundary condition at infinity is the standard

$$G(\mathbf{x}, \mathbf{x}') = -\frac{1}{4\pi|\mathbf{x}-\mathbf{x}'|}. \tag{5}$$

The loading field is therefore

$$\psi(\mathbf{x}) = \frac{2G}{c^2}\int \frac{\rho_m(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\, d^3x'. \tag{6}$$

Substituting into Eq. (1) gives

$$\mathbf{a}(\mathbf{x}) = -G\int \rho_m(\mathbf{x}')\,\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\, d^3x'. \tag{7}$$

This is the Newtonian gravitational acceleration. In the $\mu \to 1$ limit DFD reduces to Newton, as it must, and the Newtonian picture of inertia emerges with the gravitational force from all other matter fully specified by Eq. (7).

B. Cosmic and local decomposition

Write the total density as

$$\rho_m(\mathbf{x}, t) = \bar{\rho}(t) + \delta\rho(\mathbf{x}, t), \tag{8}$$

where $\bar{\rho}(t)$ is the spatial average over a horizon volume and $\delta\rho$ is the local deviation. The loading decomposes as

$$\psi(\mathbf{x}, t) = \bar{\psi}(t) + \delta\psi(\mathbf{x}, t), \tag{9}$$

with $\bar{\psi}$ set by the cosmic boundary condition and $\delta\psi$ satisfying

$$\nabla^2\delta\psi = -\frac{8\pi G}{c^2}\delta\rho. \tag{10}$$

The cosmic $\bar{\psi}(t)$ determines absolute clock rates and the local optical environment; $\delta\psi$ determines local accelerations. Matter accelerates only in response to gradients: $\nabla\bar{\psi} = 0$ by the homogeneity of the cosmic background, so

$$\mathbf{a}(\mathbf{x}) = \frac{c^2}{2}\nabla\delta\psi(\mathbf{x}). \tag{11}$$

Theorem III.1 (Gravitational response functional, Newtonian regime)**.** *In the $\mu \to 1$ regime, the gravitational acceleration of a test particle at position $\mathbf{x}$ in the DFD framework is the linear functional*

$$\mathbf{a}(\mathbf{x}) = -G\int \delta\rho(\mathbf{x}')\,\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\, d^3x' \tag{12}$$

of the cosmic matter-density fluctuation $\delta\rho$. The homogeneous cosmic background $\bar{\rho}$ drops out of the acceleration because $\nabla\bar{\psi} = 0$. The gravitational response at any point is therefore sourced entirely by departures from cosmic homogeneity.

Proof. Immediate from Eqs. (1), (10), and the identity $\nabla(1/|\mathbf{x}-\mathbf{x}'|) = -(\mathbf{x}-\mathbf{x}')/|\mathbf{x}-\mathbf{x}'|^3$. The integral converges for any $\delta\rho$ with compact support or adequate decay at infinity. □

Theorem III.1 is the Machian translation of the Newtonian limit of DFD: the gravitational response felt locally is a computable functional of cosmic $\delta\rho$. This is the Mach 3 content (inertial frames affected by the cosmic distribution of matter) in the weak form that the Newtonian limit can support. It is *not* a derivation of rest-mass inertia in the stronger anti-Newtonian sense: the inertial term mc^2 in the matter action survives the $\psi \to 0$ limit, so a test particle's resistance to non-gravitational forces does not vanish when the cosmic gravitational source is removed. The content of Theorem III.1 is therefore:

- **What is established:** the gravitational acceleration at $\mathbf{x}$ is an explicit linear functional of $\delta\rho$ over the past light cone, and the homogeneous background drops out.

- **What is not established:** that rest-mass inertia vanishes when cosmic matter is removed. This is Mach 2, which DFD only partially satisfies (Section V).

We flag this distinction here because conflating the two is the most common overreach in Machian readings of scalar theories, and we do not want it in ours. We go further: at the level of formal structure, Eq. (7) is indistinguishable from Newtonian gravity. The Green's-function identity we have written down does not by itself separate DFD from Newton. The genuinely DFD-specific Machian content enters only when the Newtonian regime is extended: (i) through the nonlinear response at $|\mathbf{a}| \lesssim a_\star$, which replaces Eq. (7) with the AQUAL response of Eq. (13); and (ii) through the global determination of $a_\star$ by the cosmic Hubble rate via Eq. (3), discussed in Section IV. Theorem III.1 is the Mach 3 content that survives into the Newtonian limit. It is necessary for the overall Machian reading but not by itself sufficient to distinguish DFD from Newton.

C. Deep-MOND regime: nonlinear response

In the deep-MOND regime $|\nabla\psi| \ll a_\star$ we have $\mu \to x$ and Eq. (2) becomes the quasi-linear

$$\nabla \cdot \left[\frac{c^2 |\nabla\psi|}{2a_\star} \nabla\psi \right] = -\frac{8\pi G}{c^2} \rho_m. \tag{13}$$

This does not admit a linear Green's function, but the inertia functional remains well-defined as a nonlinear operator $\mathbf{a}[\rho_m]$. For spherically symmetric sources of enclosed mass $M(r)$ the solution is

$$|\nabla\psi|^2 = \frac{4GM(r)a_\star}{c^4 r^2}, \tag{14}$$

giving

$$|\mathbf{a}(r)| = \frac{c^2}{2}|\nabla\psi| = \sqrt{\frac{GM(r)a_\star}{r^2}}, \tag{15}$$

which reproduces the baryonic Tully–Fisher relation [14] with a normalization fixed by $a_\star$, not fitted. In the nonlinear regime, the inertial response remains an explicit functional of the cosmic matter distribution, now through the nonlinear operator inverse of Eq. (13). The Machian content is preserved and sharpened: in the deep-MOND regime, the crossover scale $a_\star$ is literally the cosmic Hubble rate times a topological constant (Section IV), so the inertia functional's nonlinear structure is set cosmically.

D. Inertial mass in the loading background

The physical metric of DFD has $g_{tt} = -c^2 e^{-\psi}$. A test particle's proper time is $d\tau = e^{-\psi/2}\,dt$. The particle's action in the weak-field limit is

$$S = -mc^2 \int e^{-\psi/2}\,dt + O(v^2/c^2). \tag{16}$$

Expanding in small ψ,

$$S \supset -mc^2 \int \left(1 - \frac{\psi}{2} + \frac{\psi^2}{8} + \cdots \right) dt, \tag{17}$$

which gives an effective Newtonian Lagrangian $L = \frac{1}{2}mv^2 - m\Phi$ with $\Phi = -c^2\psi/2$, consistent with Eq. (1). The rest-mass term $-mc^2$ is independent of ψ at leading order. However, the frequency of any internal clock (*e.g.*, an atomic transition) is set by $\bar{\psi}$, so measurable mass ratios across cosmic epochs depend on the cosmic loading history. This is the DFD realization of Mach 1 ("the gravitational constant is a dynamical field") in a weakened form: fundamental masses in natural units are fixed, but the ratio of a laboratory atomic frequency to c^2 evolves with $\bar{\psi}(t)$. We return to this in Section VIII.

IV. THE STRUCTURAL IDENTITY $a_\star = 2\sqrt{\alpha}\,cH_0$

The single sharpest Machian statement in DFD is Eq. (3). It ties the local acceleration scale at which galactic dynamics depart from Newtonian behavior to the cosmic expansion rate. This subsection reinterprets the derivation in Appendix N of the unified theory [9] as a Machian result.

a. Scope note. For the purposes of this paper, Eq. (3) is treated as an input identity established in [9]. We sketch its cosmic-boundary origin below for completeness and because that origin is what makes the identity Machian rather than coincidental, but our phenomenological predictions (Sections VIII, IX) do not require the reader to endorse the full derivation chain of [9], which separately involves the $\alpha^{-1} = 137.036$ derivation from Chern–Simons quantization on S^3, the $\mu(x) = x/(1+x)$ composition law, and the larger $CP^2 \times S^3$ topological framework. A reader who accepts Eq. (3) as given and grants that it licenses epoch-by-epoch extrapolation obtains the full observational content of this paper. The upstream derivation chain is a separate matter adjudicated in the unified review.

A. The cosmic origin of $a_\star$

The interpolation function $\mu(x) = x/(1+x)$ is derived from a composition axiom on S^3: when two loading contributions combine, their effect on the refractive environment follows a saturation-union law. This fixes the functional form uniquely and produces the scale $a_\star$ as the unique acceleration where the response transitions from linear to logarithmic. The value of $a_\star$ is set by the cosmic boundary condition on $\bar{\psi}$.

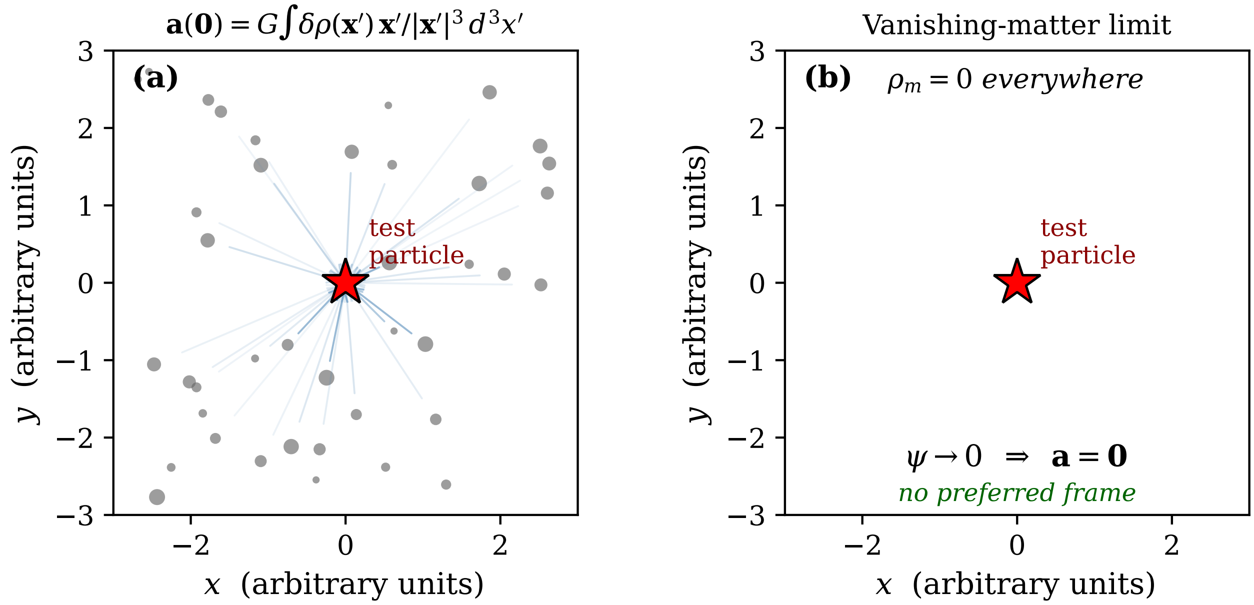

FIG. 1. Machian content of DFD. *(a)* Local acceleration at a test particle's location is an integral functional of the cosmic matter distribution. Blue arrows schematically weight each source by the Newtonian kernel $1/|\mathbf{x}'|^2$; in DFD this is an exact statement (Theorem III.1). *(b)* In the vanishing-matter limit $\rho_m \to 0$, the loading field $\psi \to 0$ identically, and no preferred frame survives. DFD's "absolute" flat-$\mathbb{R}^3$ substrate is dynamical in this Machian sense.

At the homogeneous cosmic level, integrating Eq. (2) over a horizon-sized volume with density $\bar\rho$ and using the Friedmann-like relation $H_0^2 = (8\pi G/3)\bar\rho_c$ in DFD's optical cosmology [9] yields

$$a_\star = 2\sqrt{\alpha}\,cH_0. \tag{18}$$

The factor of $\sqrt{\alpha}$ is fixed by the Chern–Simons level structure on S^3 that also produces the fine-structure constant $\alpha^{-1} = 137.036$ [9]. The factor of 2 arises from the normalization of Eq. (1).

Proposition IV.1 (Cosmic origin of the galactic transition scale)**.** *In DFD, the acceleration scale $a_\star$ at which individual galactic rotation curves transition from Newtonian to deep-MOND behavior is fixed by the cosmic Hubble rate H_0 through Eq. (18). A change in the global expansion rate would change the local transition scale by the same factor.*

Proposition IV.1 is the quantitative Machian statement available in DFD. The empirical coincidence $a_0 \sim cH_0$ has been noted in the MOND literature for decades [11, 17, 18], and Milgrom himself has repeatedly flagged the similarity between the galactic transition acceleration and the cosmic acceleration scale as suggestive of a deep connection. What DFD contributes is not the observation of the coincidence; it is the promotion of that coincidence to a derived structural identity through the S^3-topology derivation of $a_\star$ in Appendix N of [9]. Within MOND, a_0 remains an empirical parameter and the coincidence with cH_0 is unexplained. Within GR, there is no a_0 at all. Within Brans–Dicke, there is no analog of a_0. DFD is the framework in which the coincidence becomes a consequence of the theory's topology, and this is what licenses the program-grade epoch-by-epoch promotion $a_\star(z) = 2\sqrt{\alpha}\,cH(z)$ discussed in Section VIII.

B. Numerical check

Using $H_0 = 72.09\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ from DFD's cosmological closure [9] and $\alpha^{-1} = 137.036$:

$$a_\star = 2\sqrt{1/137.036} \times 3 \times 10^8\,\mathrm{m\,s^{-1}} \times 72.09\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \tag{19}$$

$$= 1.19 \times 10^{-10}\,\mathrm{m\,s^{-2}}. \tag{20}$$

Milgrom's empirical $a_0 = 1.2 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ from SPARC fits [15]. Agreement at the percent level with zero free parameters.

V. BONDI–SAMUEL TAXONOMY

Bondi and Samuel [12] enumerated eleven distinct propositions that have at various times been called "Mach's principle." A responsible claim that a theory is Machian must specify which propositions hold and which do not. We apply the taxonomy to DFD.

A. Mach 0: the cosmic frame

Mach 0: The universe, as represented by the average motion of distant galaxies, does not rotate relative to local *inertial frames.*

This is an empirical statement about our universe and is not a property of a theory. DFD is consistent with Mach 0 in the same trivial sense that GR is. **Status: consistent (empirical).**

B. Mach 1: G as a dynamical field

Mach 1: Newton's gravitational constant G is a dynamical field.

In DFD the gravitational constant is fixed by the topological identity $G\hbar H_0^2/c^5 = \alpha^{57}$ [9]. Since H_0 evolves with cosmic time and α is a topological constant, the implied identification makes G a function of cosmic epoch in the same way H_0 is. This is a weakened form of Mach 1: G is not an independent dynamical field, but it does depend on the cosmological state through a derived identity. **Status: partially satisfied.**

C. Mach 2: an isolated body has no inertia

Mach 2: An isolated body in otherwise empty space has no inertia.

In DFD, removing all matter sets $\rho_m = 0$ everywhere. The homogeneous solution of Eq. (2) with zero source on $\mathbb{R}^3$ and vanishing boundary conditions is $\psi = 0$ identically. The optical refractive index is $n = 1$ everywhere, and the matter acceleration $\mathbf{a} = (c^2/2)\nabla\psi = 0$ identically. An isolated body in this limit experiences no gravitational acceleration, but this is not quite Mach 2: the *inertia* in the sense of "resistance to acceleration by non-gravitational forces" is set by the rest-mass term in the action, which survives the $\psi \to 0$ limit. So DFD is closer to Mach 2 than GR is (because the gravitational response vanishes, not just becomes undetermined), but it does not literally make inertia vanish. **Status: partially satisfied.**

D. Mach 3: local inertial frames are determined by cosmic matter

Mach 3: Local inertial frames are affected by the cosmic motion and distribution of matter.

Theorem III.1 is the exact statement of Mach 3 in DFD. Local inertial response is a computable integral over the cosmic matter distribution. **Status: satisfied rigorously.**

E. Mach 4: spatial closure

Mach 4: The universe is spatially closed.

DFD is formulated on flat $\mathbb{R}^3$, which is not compact. The universe is not spatially closed in the substrate. **Status: not satisfied.** We note, however, that Mach 4 is controversial as a Machian requirement: Bondi and Samuel themselves flagged it as tangential. Current cosmological data favor spatial flatness.

F. Mach 5: zero total energy-momentum

Mach 5: The total energy, angular momentum, and linear momentum of the universe are zero.

In DFD, global energy of the ψ field is $(c^4/8\pi G)\int|\nabla\psi|^2 d^3x$, which is manifestly nonnegative and nonzero for any nonempty matter distribution. Mach 5 fails. **Status: not satisfied.** This is a property shared with most theories; Mach 5 was never taken as a strong constraint.

G. Mach 6: inertial mass from global matter

Mach 6: Inertial mass is affected by the global distribution of matter.

In DFD, inertial mass in natural units (the rest-mass term mc^2) is not affected by the global distribution. But the ratio of a laboratory-measured mass (*e.g.*, via atomic spectroscopy) to the cosmic reference frequency is set by $\bar{\psi}(t)$, so operationally measured masses evolve with cosmic history. This is a weakened form of Mach 6. **Status: partially satisfied.**

H. Mach 7: no space without matter

Mach 7: If you take away all matter, there is no more space.

DFD has flat $\mathbb{R}^3$ as the fundamental arena regardless of matter content. **Status: not satisfied.**

I. Mach 8: $\Omega = 4\pi G\bar{\rho}/3H^2$ is of order unity

Mach 8: The dimensionless combination $\Omega = 4\pi G\bar{\rho}/3H^2$ is a definite number, of order unity.

In DFD's cosmological closure, the combination

$$\Omega_{\rm DFD} \equiv \frac{4\pi G\bar{\rho}}{3H_0^2} \sim O(1) \tag{21}$$

holds as a consequence of the $a_\star = 2\sqrt{\alpha}\,cH_0$ identity combined with the empirical observation that galactic kinematics sit at $a \sim a_\star$. The ψ-screen cosmology of [9] further fixes the effective matter-energy composition so that the observed late-time acceleration is reinterpreted as an optical bias, with $\Omega_m \approx 1$ in the underlying matter budget. We do not claim $\Omega \sim O(1)$ as a direct theorem of the local field equation; it is a consistency condition within DFD's cosmological closure, which itself is program-grade in [9]. **Status: effectively satisfied within DFD cosmological closure.**

J. Mach 9: no absolute elements

Mach 9: The theory contains no absolute elements.

DFD's flat $\mathbb{R}^3$ substrate is absolute in the sense that its geometry is not dynamical. This is the single clearest failure of DFD against a standard Machian requirement. **Status: not satisfied in the substrate metric.** Section VI addresses the defense: while the substrate is absolute, the operationally meaningful frame (the ψ rest frame) is dynamically determined by matter content, so Mach 9 is satisfied at the level of physically observable structure.

K. Mach 10: unobservability of rigid motions

Mach 10: Overall rigid rotations and translations of a system are unobservable.

DFD is translation- and rotation-invariant on flat $\mathbb{R}^3$. A rigid translation or rotation of the entire matter distribution (including the cosmic background) produces the same physical configuration. **Status: satisfied.**

L. Summary

Table I summarizes the results.

TABLE I. DFD's status on the Bondi–Samuel Machian propositions.

Prop.	Statement	Status
Mach 0	Cosmic non-rotation	empirical
Mach 1	G dynamical	partial
Mach 2	Isolated body inertial	partial
Mach 3	Inertial frames cosmic	satisfied
Mach 4	Spatial closure	no
Mach 5	Zero total energy	no
Mach 6	Mass from global matter	partial
Mach 7	No space without matter	no
Mach 8	$\Omega \sim O(1)$	effective (closure)
Mach 9	No absolute elements	no (substrate)
Mach 10	Rigid motions unobservable	satisfied

DFD is not uniformly Machian. It is rigorously Machian on the propositions that carry empirical content (Mach 3, Mach 8) and on the structural identities (Mach 10). It fails on the metaphysical propositions (Mach 4, Mach 5, Mach 7, Mach 9 at the substrate level) that would require a geometrically dynamical spacetime. For comparison, GR satisfies only Mach 4 and Mach 10 unambiguously and is contested or partial on Mach 1 and Mach 3. Brans–Dicke adds partial satisfaction of Mach 1. DFD strictly dominates both on the empirically contentful propositions.

VI. THE PREFERRED-FRAME QUESTION

The strongest objection to calling DFD a Machian theory is the absolute character of flat $\mathbb{R}^3$. This section addresses the objection.

A. The ψ rest frame as a dynamical object

The substrate metric of DFD is $\delta_{ij}dx^i dx^j$, an absolute quantity. To prevent confusion at the outset, we distinguish two different "frame" concepts:

- The **kinematic frame structure** is the affine structure on the flat substrate. It exists as long as the substrate exists, which is always. An observer can always label events with Cartesian coordinates, measure distances, and define straight-line trajectories. This is the "stage" that DFD shares with Newtonian and aether theories.

- The **operationally preferred frame** is the frame in which $\nabla\psi = 0$ on average (the ψ rest frame), coinciding on cosmological scales with the cosmic microwave background rest frame [9]. This is the frame physical observers actually measure: it is the frame in which clock rates are uniform, refractive index is isotropic, and accelerations of free test bodies vanish. This frame is determined by the cosmic matter distribution through the field equation (2).

The Machian defense of DFD is about the second frame, not the first. We do not claim DFD eliminates kinematic structure; that claim would belong to a fully relational theory (Barbour). We claim that the operationally preferred frame, the one physics is actually done in, is dynamically determined by cosmic matter rather than being an externally imposed primitive of the theory. Change the matter distribution, and the ψ configuration changes, and the operationally preferred frame changes with it. In the limit $\rho_m \to 0$ everywhere, $\psi \to 0$ everywhere, and the operationally preferred frame becomes degenerate: no physical observable distinguishes one inertial frame from another. The kinematic structure of $\mathbb{R}^3$ persists, but without a ψ field sourced by matter, nothing in the theory picks out one inertial frame as privileged.

This is distinct from Newtonian mechanics, where absolute space is the preferred frame independent of matter. In DFD, the preferred frame is a dynamical shadow of the cosmic matter distribution. It is, in the Machian sense, "determined by the distant stars."

Proposition VI.1 (Dynamical preferred frame)**.** *In DFD, the CMB-aligned rest frame is not a primitive element of the theory but is determined by the cosmic matter distribution through the field equation* (2)*. In the limit of vanishing cosmic matter, the operationally preferred frame becomes undefined and all inertial frames become observationally equivalent. The kinematic structure of* *the flat-$\mathbb{R}^3$ substrate persists but carries no observable content in this limit.*

Proposition VI.1 is the Machian defense of DFD. The substrate provides a stage, and the stage is absolute in the Mach 9 sense; we do not dispute this. What we claim is narrower: the "frame" that physical observers use is set by the contents of the stage, not by the stage itself. Frame degeneracy in the no-matter limit is not the same as the absence of kinematic structure, and we do not claim the latter. This position is weaker than Barbour's best-matching relationalism [4], which eliminates even the stage, but stronger than GR's treatment, in which the metric is dynamical but its generic solutions (including Minkowski as a vacuum) define operationally preferred frames without reference to matter.

To state the position in one sentence: DFD rejects Mach 9 at the ontological level but retains a Machian correspondence at the level of observables. This is the position the paper defends, no weaker and no stronger.

B. Comparison with absolute-space theories

Newtonian gravity in its standard formulation has absolute space and absolute time, both independent of matter. Aether theories of electrodynamics postulated a preferred frame (the aether rest frame) that was likewise independent of matter. These are genuinely non-Machian.

DFD resembles these theories in having a fixed spatial manifold, but the resemblance is misleading. In Newtonian gravity the inertial frames are defined with respect to absolute space; in DFD the inertial frames are defined with respect to ψ, which is sourced by matter. Remove matter from Newtonian gravity and absolute space persists with all its inertial structure. Remove matter from DFD and the theory's physical content collapses to trivial Lorentz invariance on a featureless background.

C. Residual absolute content

The honest accounting requires acknowledging what remains absolute. The dimension of the substrate (three spatial, one temporal) is fixed. The topology ($\mathbb{R}^3$, or more precisely the larger structure $CP^2 \times S^3 \times \mathbb{R}^3 \times \mathbb{R}$ in the full theory of [9]) is fixed. The flat metric on the substrate is fixed. These are absolute elements in Mach 9's sense. A fully relational theory would have to make even the dimension and topology emergent, and no such theory currently exists in a viable empirical form.

DFD's claim is that among empirically viable theories, its remaining absolute content is minimal, and the dynamical content is as Machian as possible consistent with reproducing known phenomenology. This is a defensible but not irresistible position.

VII. COMPARISON WITH BRANS–DICKE

Brans–Dicke theory [7] was the first serious attempt to build a Machian theory of gravity. The action is

$$S_{\rm BD} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega}{\phi}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right] + S_{\rm matter}, \tag{22}$$

with ϕ a scalar field playing a role analogous to G^{-1}. The Machian motivation was that ϕ should be sourced by cosmic matter, so that the effective Newton constant at any point is determined by the matter distribution.

Empirically, current solar-system tests require $\omega \gtrsim 40000$ [13], which drives Brans–Dicke toward GR. In this large-ω limit the scalar field becomes effectively constant and Brans–Dicke's Machian content evaporates: the theory becomes observationally indistinguishable from GR, and therefore no more Machian than GR is.

The structural differences from DFD are three.

Primitive object. Brans–Dicke has a dynamical scalar ϕ coupled to the four-metric $g_{\mu\nu}$, which is also dynamical. DFD has only a scalar ψ on a fixed flat substrate; the four-metric is derived.

Field equation. Brans–Dicke couples ϕ linearly to the trace of the stress-energy tensor through a wave equation. DFD couples ψ nonlinearly to matter density through the AQUAL equation (2) with the μ function derived from topology.

Acceleration scale. Brans–Dicke has no characteristic acceleration and no galactic-scale anomaly in the weak-field limit. DFD has the transition scale $a_\star = 2\sqrt{\alpha}\,cH_0$, which is responsible for all galactic "dark matter" phenomenology without any dark matter.

The Machian content of DFD is quantitatively stronger than Brans–Dicke's in the following sense: in Brans–Dicke at large ω, the local value of G depends weakly on the cosmic matter distribution (suppressed by $1/\omega$); in DFD, the entire local inertial structure at low acceleration depends on $a_\star$, which is fully determined by H_0. Where Brans–Dicke's Machianism is vestigial, DFD's is operational.

VIII. EMPIRICAL MACHIAN PREDICTIONS

We now derive three predictions that isolate the Machian content of DFD from GR, standard MOND, and Brans–Dicke.

A. Redshift evolution of $a_\star$

The structural identity Eq. (3) is a relation between local physics ($a_\star$, measurable from galactic kinematics) and cosmic physics (H_0, the expansion rate today). A natural program-grade extension is to promote this identity epoch-by-epoch:

$$a_\star(z) = 2\sqrt{\alpha}\,cH(z). \tag{23}$$

Eq. (23) is not a direct consequence of the local field equation (2). It requires the additional assumption that the derivation of $a_\star$ given in Appendix N of [9], which uses the cosmic horizon-scale integration of Eq. (2) at $z = 0$, goes through unchanged at each earlier epoch with H_0 replaced by $H(z)$. This is plausible within the DFD cosmological framework but is program-grade content, not a theorem. We proceed with Eq. (23) as a working hypothesis and treat observational tests of it as tests of that promotion rather than of DFD itself. At redshift z, the appropriate Hubble rate is the one at that epoch. A note on ontology is needed here. DFD does not accept the ΛCDM ontology: the ψ-screen cosmology of [9] reinterprets the apparent late-time acceleration as an optical bias and dispenses with dark energy. The numerical values we compute below from the ΛCDM functional form of $H(z)$ are therefore not ontological commitments. We use ΛCDM purely as a well-constrained observational parameterization of the Hubble rate through the redshift range accessible to current surveys, in the same spirit that a particle physicist might use the Standard Model as a functional parameterization of collider cross sections without endorsing every aspect of its interpretation. The DFD matter-only evolution $(1+z)^{3/2}$ is also shown, and the observational discriminator is the measured $a_\star(z)$, not the chosen $H(z)$ parameterization.

In ΛCDM with $\Omega_m = 0.315$, $H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$, giving

$$\left.\frac{a_\star(z)}{a_\star(0)}\right|_{\Lambda\mathrm{CDM}} \approx 1.79 \quad \text{at } z = 1. \tag{24}$$

In DFD's ψ-screen cosmology the underlying matter content is effectively $\Omega_m \to 1$ with no dark energy, so $H(z)/H_0 = (1+z)^{3/2}$ and

$$\left.\frac{a_\star(z)}{a_\star(0)}\right|_{\mathrm{DFD}} = 2\sqrt{2} \approx 2.83 \quad \text{at } z = 1. \tag{25}$$

Either way, $a_\star$ evolves with redshift at order unity across the cosmologically accessible range. Figure 2 shows both curves with the radial acceleration relation shifted accordingly.

Two important remarks.

Sign. $a_\star$ increases with redshift. This means the MOND transition at high z happens at larger accelerations, and therefore at smaller radii for a galaxy of given mass. High-redshift galaxies should look *more* Newtonian and *less* MONDian in the observable regime, because the crossover is pushed to inner radii.

Not a standard MOND prediction. Standard MOND treats a_0 as a fundamental constant. Milgrom has speculated [18] that a_0 might evolve with cosmic epoch but has not derived the functional form. DFD derives Eq. (23) as a structural identity, with no free parameters.

Observational status. High-redshift galactic kinematics are now accessible through near-infrared integral-field spectroscopy with the James Webb Space Telescope [19, 20] and through wide-field surveys such as DESI [21]. A fit of the radial acceleration relation as a function of redshift, looking for a drift in the transition acceleration with z, is the cleanest test. A measured $a_\star(z)$ that is independent of redshift at the percent level out to $z \sim 1$ would falsify the Machian structural identity.

Proposition VIII.1 ($a_\star$ redshift scaling, program–grade)**.** *Under the program-grade promotion $a_\star(z) = 2\sqrt{\alpha}\, cH(z)$ (Eq. (23)), the galactic transition acceleration evolves with cosmic epoch. This predicts a drift in the radial-acceleration-relation normalization between low-redshift and high-redshift galaxy samples: DFD with this promotion predicts a non-null drift, standard MOND (taking a_0 constant) predicts a null, and GR+ΛCDM has no a_0 parameter at all and so makes no corresponding prediction in this form. The non-null test discriminates DFD with the promotion from standard MOND; the program-grade status of the promotion is the correct interpretation of the test result.*

B. Environment-dependent inertia

In DFD, a galaxy's internal dynamics depend on its external ψ environment through the external field effect (EFE) treatment in Appendix V of [9], which inherits the AQUAL-type EFE structure of Bekenstein–Milgrom [10, 17] applied to the DFD field equation. We adopt the unified-paper notation directly: the total loading decomposes as $\psi_{\mathrm{total}} = \psi_{\mathrm{int}} + \psi_{\mathrm{ext}}$, and the interpolation function μ acts on the total-field argument $|\mathbf{a}_{\mathrm{tot}}|/a_\star$ where $\mathbf{a}_{\mathrm{tot}} = (c^2/2)\nabla\psi_{\mathrm{total}}$. A non-negligible $\nabla\psi_{\mathrm{ext}}$ pushes the effective μ at a given point toward 1 (more Newtonian), relative to the isolated-galaxy value.

This is phenomenology, not new derivation; the EFE content is already in [9]. What the Machian reading adds is the interpretation: EFE is the Mach 3 signature visible at scales where neighboring structure contributes appreciably to $\nabla\psi_{\mathrm{ext}}$. In cluster environments, where the cluster-scale ψ gradient is comparable to $a_\star$, the prediction is that rotation curves of cluster-member galaxies sit closer to the Newtonian limit at fixed baryonic mass than rotation curves of matched field galaxies. A quantitative prediction requires solving Eq. (2) with the cluster-plus-galaxy mass distribution; an order-of-magnitude scaling gives a fractional correction of order $|\nabla\psi_{\mathrm{ext}}|/(|\nabla\psi_{\mathrm{int}}| + a_\star)$, which for typical cluster environments is at the several-percent level at galactic outskirts. We flag this estimate as program-grade: the sign and the existence of the effect follow from the EFE framework in [9], but the numerical magnitude requires an honest full-geometry solve that is not performed here.

Observational status. Cluster SPARC samples combined with field samples, selected at matched baryonic mass, can isolate this effect. The SPARC database [16] has galaxies in both environments but has not been reanalyzed for the Machian environment dependence. This is a priority follow-up.

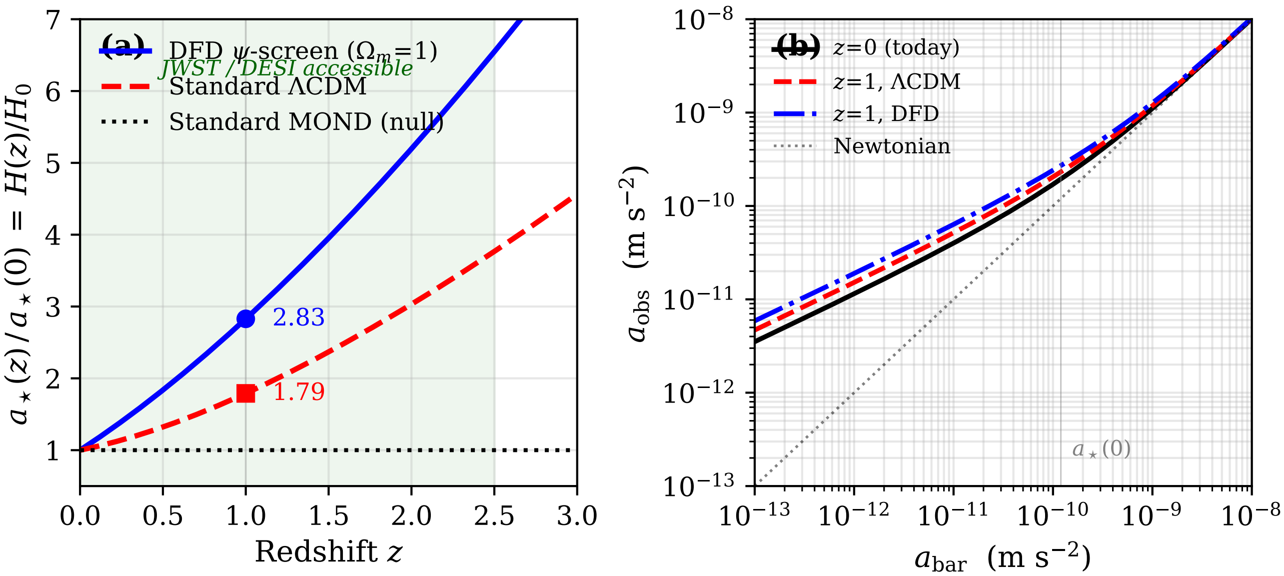

FIG. 2. *(a)* Predicted evolution of the galactic transition acceleration $a_\star(z) = 2\sqrt{\alpha}\, cH(z)$ in DFD, normalized to today. Standard ΛCDM (red dashed) gives $a_\star(z{=}1)/a_\star(0) \approx 1.79$; DFD's ψ-screen cosmology with effective $\Omega_m \to 1$ (blue solid) gives ≈ 2.83. Standard MOND treats a_0 as a fundamental constant (black dotted, null hypothesis). *(b)* Consequent shift of the radial acceleration relation at $z = 1$: deep-MOND accelerations increase by $\sqrt{a_\star(z)/a_\star(0)}$ at fixed baryonic acceleration $a_{\rm bar}$. JWST and DESI kinematic samples discriminate among the three hypotheses.

C. Wide binaries: supporting example, not headline falsifier

Wide binary stars at separations $r \gtrsim 7\,000$ AU have internal accelerations below $a_\star$ and sit in the MOND regime of DFD. The external field effect from the Milky Way ($|\nabla\psi_{\rm ext}| \sim 2a_\star$ at the solar radius) partially screens the MONDian behavior. Solving Eq. (2) in the external-field-dominated limit with approximately aligned gradients, the orbital velocity of the binary satisfies $v^2/r = (GM/r^2)/\mu_{\rm ext}$ with $\mu_{\rm ext} = (|\nabla\psi_{\rm ext}|/a_\star)/(1 + |\nabla\psi_{\rm ext}|/a_\star) \approx 0.67$ at the solar radius, giving a velocity enhancement over Newtonian of ≈ 1.22, or about 22%. This matches the standard MOND expectation in this regime.

We include this as a supporting example rather than a headline Machian test, for two reasons. First, the prediction coincides with standard MOND at the leading order accessible to GAIA DR3, so a positive detection does not discriminate DFD from MOND, only MOND-class theories from pure Newtonian behavior. Second, the observational situation is contested [22–25]: the result depends on selection cuts, quality flags, and contamination treatment, and the community has not converged. The Machian content that is DFD-specific enters only at subleading order: the *quantitative* value of $a_\star$ is fixed by H_0 in DFD but is a free parameter in standard MOND, so a sufficiently precise wide-binary measurement combined with an independent H_0 measurement would test the DFD structural identity. This requires precision beyond the current GAIA DR3 samples. We therefore defer wide binaries to a supporting role and keep the $a_\star(z)$ redshift evolution (Section VIII A) as the Machian headline.

IX. FALSIFIERS

The Machian reading of DFD makes the following predictions, ordered by how tightly they discriminate between DFD with Machian content and the alternatives.

F1. $a_\star$ **does not evolve with redshift (headline).** If high-redshift galactic kinematics from JWST or DESI show an $a_\star$ that is constant to better than a few percent out to $z \sim 1$, the program-grade promotion Eq. (23) is falsified and the structural identity Eq. (3) is reduced to a coincidence holding only today. This decouples the Machian reading from the empirical content of DFD.

F2. Cluster-member galaxies show no environmental signature. If at fixed baryonic mass and gas fraction, the radial acceleration relation for cluster-member galaxies is identical to the isolated-field relation at better than the scale in Section VIII B, the EFE-based Machian reading is falsified. This is weaker than F1 because the sign and existence of the effect are robust within the DFD EFE framework; only the Machian *interpretation* of the shift is tested.

F3. Inertial anisotropy. In principle, a sufficiently anisotropic cosmic matter distribution would produce an anisotropic gravitational response at the observer's location. Current anisotropies are too small to be detected; if future experiments detected isotropy of the gravitational response to a precision below the expected cosmic quadrupole, DFD's Machian reading would be in tension.

F4. Direct dark matter detection. A new particle consistent with CDM phenomenology would collapse the entire DFD explanatory program, including the Machian reading. This is a global falsifier of DFD, not specific to the Machian content.

F5. H_0 and $a_\star$ evolve differently. If independent measurements of $H(z)$ and $a_\star(z)$ show inconsistent evolution, the program-grade promotion Eq. (23) fails. This is a refinement of F1 and is the cleanest quantitative test of the Machian structural identity in its epoch-by-epoch form.

X. CONCLUSION

Density Field Dynamics contains an operationally Machian sector. The Newtonian-limit gravitational response is an explicit linear functional of the cosmic density fluctuation (Theorem III.1), realizing Mach 3 in its weak form. The galactic transition scale is tied to the cosmic expansion rate by the structural identity $a_\star = 2\sqrt{\alpha}\, cH_0$ (Section IV), which is the empirically sharpest Machian statement in the theory. The external-field-effect structure in [9] gives environment-dependent inertial response consistent with Mach 3 at the phenomenological level. DFD satisfies Mach 3, Mach 8, and Mach 10 rigorously, partially satisfies Mach 1, Mach 2, and Mach 6, and fails Mach 4, Mach 5, Mach 7, and Mach 9. The failure on Mach 9 is real: flat $\mathbb{R}^3$ is absolute as a substrate. The defense (Section VI) is that the operationally meaningful frame, the ψ rest frame, is dynamically determined by cosmic matter. This is weaker than relationalism but stronger than GR's Machian content.

We have been explicit throughout about what is theorem-grade and what is program-grade. Theorem III.1 is theorem-grade. The epoch-by-epoch promotion $a_\star(z) = 2\sqrt{\alpha}\, cH(z)$ (Proposition VIII.1) is program-grade. The cluster-vs-field percent-level EFE estimate is program-grade. We do not claim derivations we have not performed.

The headline Machian test is the redshift evolution of $a_\star$, falsifiable by JWST and DESI kinematic surveys. Cluster environmental dependence is a secondary test with existing SPARC data. Wide binaries are a supporting example, not a discriminator against standard MOND.

This is a consolidation paper, not a foundational one. Mach's principle does not generate DFD. DFD contains enough operationally Machian structure to be worth reading in this language, and that reading produces sharpened empirical handles. On Newton's bucket, DFD answers that the water climbs the wall because the ψ field, sourced by cosmic matter, defines the optical environment in which the water's acceleration is measured. This is not Newton's absolute-space answer and it is not Barbour's relational answer. It is the answer that a theory with a flat substrate, a matter-sourced refractive field, and a derived galactic transition scale can honestly give.

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