Completing the Local Position Invariance Test Suite: A Sector-Resolved Cavity–Atom Frequency Ratio Experiment

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Abstract

Local Position Invariance (LPI) — the universality of gravitational redshift across all physical systems — has been tested for decades using atom–atom, matter–matter, and resonator–resonator comparisons. Yet one critical cross-sector test remains absent: cavity-stabilized optical frequencies (photon sector) compared directly to atomic transitions (matter sector) across a gravitational potential. Here we present the formalism, explicit predictions, and control strategies for this missing experiment, which would complete the LPI test suite. In General Relativity (GR), cavity–atom ratios must remain strictly constant, yielding slope coefficients $\xi^{(M,S)} = 0$. In Density Field Dynamics (DFD), a scalar refractive framework consistent with GR's classic tests but predicting deviations in low-acceleration regimes, evacuated cavities track the refractive index $n = e^{\psi}$ while atomic transitions remain leading-order ψ-insensitive, giving $\xi^{(M,S)} \simeq 1$. This implies a geometry-locked slope of order $\Delta R/R \sim \Delta\Phi/c^2 \approx 1.1 \times 10^{-14}$ per 100 m on Earth. We provide (i) a historical review of LPI tests, (ii) full derivations of the cavity–atom slope observable and its parametrized post-Newtonian (PPN) consistency, (iii) a sectoral decomposition across materials and species, and (iv) an error budget demonstrating feasibility with existing 10^{-16} cavities and 10^{-18} optical clocks. We argue that this cross-sector test provides the final closure of LPI, yielding a binary and decisive discriminator: a null confirms GR and rules out DFD, while a non-null slope falsifies GR's universality.

1 Introduction

The Einstein equivalence principle (EEP) underpins all metric theories of gravity. Its LPI component requires that all systems undergo identical gravitational redshifts, independent of composition or mechanism. The gravitational redshift has been tested in progressively more precise experiments: Pound–Rebka (1960), the 1976 GP-A rocket [1], and modern optical clock comparisons [2, 3]. Each confirmed GR to better than 10^{-6}.

Yet all these comparisons are sector-homogeneous. No experiment has compared cavity-stabilized optical frequencies (tracking photon propagation) against atomic transitions (quantum energy levels) across altitude. This work proposes the missing cross-sector test, completing the LPI suite.

2 Background and Literature Review

2.1 Historical redshift tests

- **Pound–Rebka (1960)**: Mössbauer γ-ray redshift in a tower.
- **GP-A (1976)**: H maser on a suborbital rocket, 7×10^{-5} confirmation.
- **Modern optical clocks**: Yb^+, Sr lattice clocks achieving 10^{-18} stability.

2.2 Sectoral coverage

1. Atom–atom: microwave or optical transition comparisons.
2. Resonator–resonator: cavity or oscillator stability tests.
3. Matter–matter: Mössbauer and nuclear transitions.
4. **Cavity–atom: missing.**

3 Formalism of the Cavity–Atom Test

Define the ratio:

$$\frac{\Delta R^{(M,S)}}{R^{(M,S)}} = \xi^{(M,S)} \frac{\Delta\Phi}{c^2}, \tag{1}$$

where

$$\xi^{(M,S)} = \alpha_w - \alpha_L^{(M)} - \alpha_{\mathrm{at}}^{(S)}.$$

In GR, $\xi^{(M,S)} = 0$. In DFD, $\xi^{(M,S)} = 1$.
For $\Delta h = 100$ m on Earth:

$$\frac{\Delta R}{R} \approx 1.1 \times 10^{-14}.$$

4 PPN Consistency

DFD recovers GR's solar-system predictions by construction. Its effective potential $\Phi = -c^2\psi/2$ yields $\gamma = \beta = 1$, all other PPN parameters vanishing. Thus light deflection, Shapiro delay, and perihelion precession are preserved. Deviations appear only in cross-sector LPI tests, where GR requires null slopes.

5 Sector Decomposition

With two cavity materials and two atomic species:

$$\delta_{\mathrm{tot}} = \alpha_w - \alpha_L^{\mathrm{ULE}} - \alpha_{\mathrm{at}}^{\mathrm{Sr}}, \tag{2}$$
$$\delta_L = \alpha_L^{\mathrm{Si}} - \alpha_L^{\mathrm{ULE}}, \tag{3}$$
$$\delta_{\mathrm{at}} = \alpha_{\mathrm{at}}^{\mathrm{Yb}} - \alpha_{\mathrm{at}}^{\mathrm{Sr}}. \tag{4}$$

Table 1: Mapping of measured slopes to sector parameters.

Measured ratio	Combination	Parameter
ULE/Sr	δ_{tot}	total offset
Si/Sr	$\delta_{\text{tot}} + \delta_L$	cavity diff.
ULE/Yb	$\delta_{\text{tot}} + \delta_{\text{at}}$	atom diff.
Si/Yb	$\delta_{\text{tot}} + \delta_L + \delta_{\text{at}}$	over-determined

Table 2: Illustrative systematic error budget for cavity–atom slopes.

Systematic	Target (frac.)	Control method
Dispersion (dual-λ)	$< 3 \times 10^{-15}$	dual-wavelength probing
Elastic sag	$< 3 \times 10^{-15}$	180° orientation flips
Thermal drift	$< 3 \times 10^{-15}$	environmental stabilization
Polarization/birefringence	$< 3 \times 10^{-15}$	swaps + polarization control
Comb transfer noise	$< 1 \times 10^{-16}$	stabilized links

6 Error Budget and Systematic Controls

7 Feasibility

- Cavities: 10^{-16} fractional stability [4, 5].
- Optical clocks: 10^{-18} stability [6, 2].
- Baseline: 30–100 m suffices for 5σ discrimination.

8 Discussion: Completing the LPI Suite

This test completes the quadrilateral: atom–atom, resonator–resonator, matter–matter, and cavity–atom. Its binary outcome:

- $\Delta R/R = 0$: GR confirmed, DFD falsified.
- $\Delta R/R \sim 10^{-14}$: GR falsified, DFD supported.

Either result is decisive.

9 Conclusion

We have presented the formalism, PPN consistency, predictions, and systematic controls for the final untested LPI sector. This cavity–atom comparison is feasible today, requires no new technology, and provides a definitive discriminator between GR and DFD. Completing the LPI test suite is both achievable and foundational.

References

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