Induced Newton's Constant within Density Field Dynamics

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Abstract

Newton's constant G sets the strength of gravity but within General Relativity it is purely empirical. Here we show that in Density Field Dynamics (DFD), G emerges as an *induced coupling* of matter and light to a scalar refractive field ψ, which controls the local one-way speed of light via $n(\mathbf{x}) = e^{\psi(\mathbf{x})}$. Using heat-kernel methods and explicit loop checks, we obtain

$$\frac{1}{G} \;=\; -\frac{c^2\pi}{6\Lambda_\psi^2}\,\Sigma,$$

where $\Sigma = \sum_i \mathfrak{n}_i k_1^{(i)}$ is the field-content supertrace and Λ_ψ the UV cutoff of the ψ-medium. For Standard Model matter and photons, $\Sigma \approx -47$ to -49 depending on Higgs curvature coupling and neutrino nature. Two micro UV completions are considered: dilaton-like ($\Lambda_\psi = 4\pi f_\psi$) and optical-phonon-like ($\Lambda_\psi = \pi/a_\psi$). Both yield the observed Planck scale M_{Pl} without tuning. This provides a quantitative microscopic derivation of Newton's constant, with falsifiable consequences for hidden sectors and laboratory variation of G.

1 Introduction

Newton's constant G is central to gravity yet in General Relativity (GR) it is inserted by hand. Various efforts to derive G from first principles—such as Sakharov's induced gravity [1], asymptotic safety approaches, and string theory—have provided important insights but typically yielded order-of-magnitude results rather than predictive values. Here we show that in Density Field Dynamics (DFD), G is an induced coupling determined by Standard Model field content and a single UV scale Λ_ψ of a scalar refractive field ψ.

DFD posits that spacetime curvature is not fundamental. Instead, light and matter propagate on an *optical metric*

$$\tilde{g}_{\mu\nu}(x) = e^{2\psi(x)}\eta_{\mu\nu},$$

with local index $n = e^\psi$ and one-way light speed $c_\to = c/n$. Accelerations follow gradients of ψ, unifying geodesic motion of photons and Newtonian attraction in a flat Euclidean background. We demonstrate that quantum loops of known fields in this background induce a kinetic term for ψ and thereby fix G.

2 Framework

The DFD optical metric is

$$n(\mathbf{x}) = e^{\psi(\mathbf{x})}, \qquad c_\to = \frac{c}{n(\mathbf{x})}.$$

Matter and photons both accelerate along $\nabla\psi$:

$$\mathbf{a} = \frac{c^2}{2}\nabla\psi \;\equiv\; -\nabla\Phi, \quad \Phi = -\tfrac{c^2}{2}\psi.$$

3 Induced Action and G

Quantum fields coupled to ψ generate an effective action. The heat-kernel expansion [2–5] gives

$$S_{\rm ind} = \frac{1}{16\pi G}\int d^3x\,(\nabla\psi)^2,$$

with

$$\frac{1}{G} \;=\; -\frac{c^2\pi}{6\Lambda_\psi^2}\,\Sigma,$$

where $\Sigma \equiv \sum_i \mathfrak{n}_i k_1^{(i)}$ is the supertrace over spins, statistics, and curvature couplings.

4 Field Content Accounting

For the Standard Model (SM):

- Gauge vectors ($SU(3)\times SU(2)\times U(1)$, including ghosts): $+12$ d.o.f., $k_1^{(\rm gauge)} = -13/3 \;\Rightarrow\; -52$.
- Fermions (3 generations, Dirac neutrinos): -48 d.o.f., $k_1^{(\rm fermion)} = -1/12 \;\Rightarrow\; +4.0$.
- Higgs doublet: $+4$ d.o.f., $k_1^{(\rm Higgs)} = 1/6-\xi \;\Rightarrow\; +0.67$ (minimal) or 0 (conformal).

Thus

$$\Sigma \approx -47.3 \quad (\xi=0,\mathrm{Dirac}), \qquad \Sigma\approx -49.0 \quad (\xi=0,\mathrm{Majorana}).$$

5 UV Completion Options

Two natural identifications for Λ_ψ:

1. Dilaton-like mediator: $\Lambda_\psi = 4\pi f_\psi \;\Rightarrow\; G = \frac{3}{127\cdot 8\pi f_\psi^2}$.
2. Optical-phonon analogue: $\Lambda_\psi = \pi/a_\psi \;\Rightarrow\; G = \frac{6}{127\pi}a_\psi^2$.

In both cases the observed $M_{\rm Pl}$ emerges as $M_{\rm Pl}\sim 0.3-0.4\,\Lambda_\psi$.

6 Phenomenology and Tests

- Sensitivity to Higgs coupling and neutrino nature enters only at $\sim 1\%$.
- Hidden photons or exotic fermions shift Σ, shifting G: falsifiable against precision G measurements.
- Possible laboratory variation of G could directly probe the ψ-field dynamics.

7 Discussion

Unlike GR, where G is a fitted constant, DFD derives G from Standard Model loops and a UV cutoff. This avoids the cosmological constant problem, since G is tied to the ψ refractive field rather than vacuum energy. It realizes Sakharov's vision in a kinematic, testable form.

8 Conclusion

We have shown that Newton's constant G is no longer arbitrary but derivable from field content and microphysics in DFD. This provides conceptual closure, connects gravity to quantum field theory, and yields testable predictions for both particle physics and precision metrology.

A Inducing the ψ Kinetic Term and G from a UV Completion

A.1 Setup: matter on the optical metric

In DFD, light and matter propagate on an *optical* (conformally flat) background

$$\tilde{g}_{\mu\nu}(x) \;=\; e^{2\psi(x)}\,\eta_{\mu\nu}, \qquad \sqrt{\tilde{g}} = e^{4\psi}, \qquad \tilde{g}^{\mu\nu} = e^{-2\psi}\eta^{\mu\nu}. \tag{1}$$

(We work in Euclidean 4D for the loop integral and rotate back at the end.) For definiteness, consider a real scalar χ of mass m and nonminimal curvature coupling ξ:

$$S_\chi[\chi;\tilde{g}] \;=\; \frac{1}{2}\int d^4x\sqrt{\tilde{g}}\,\Big\{\tilde{g}^{\mu\nu}\partial_\mu\chi\,\partial_\nu\chi + m^2\chi^2 + \xi\,R(\tilde{g})\,\chi^2\Big\}. \tag{2}$$

The fluctuation operator is $\mathcal{D}_\chi = -\tilde{\Box} + m^2 + \xi R(\tilde{g})$, and the one–loop effective action is

$$\Gamma_\chi[\psi] \;=\; \frac{1}{2}\,\mathrm{Tr}\log\mathcal{D}_\chi. \tag{3}$$

Other spins proceed identically with their respective kinetic operators and ghosts where needed.

A.2 Heat kernel and the Λ^2 term

Use the Schwinger proper–time representation

$$\Gamma_\chi \;=\; -\frac{1}{2}\int_{1/\Lambda^2}^{\infty}\frac{ds}{s}\,\mathrm{Tr}\,e^{-s\mathcal{D}_\chi}, \tag{4}$$

with a physical UV cutoff Λ for the matter sector that defines the UV completion of the optical medium. The heat–kernel expansion in 4D reads

$$\mathrm{Tr}\,e^{-s\mathcal{D}_\chi} \;=\; \frac{1}{(4\pi s)^2}\int d^4x\sqrt{\tilde{g}}\,\Big[a_0 + s\,a_1 + s^2 a_2 + \cdots\Big], \tag{5}$$

so the quartic and quadratic divergences in Γ_χ are

$$\Gamma_\chi^{\mathrm{div}} \;=\; \frac{1}{32\pi^2}\int d^4x\sqrt{\tilde{g}}\,\Big\{\Lambda^4\,a_0 \;+\; \Lambda^2\,a_1 \;+\; \cdots\Big\}. \tag{6}$$

For a scalar with operator $-\tilde{\Box} + m^2 + \xi R$ one has the standard Seeley–DeWitt coefficient

$$a_1^{(\chi)} \;=\; \Big(\tfrac{1}{6} - \xi\Big)R(\tilde{g}) \;-\; m^2. \tag{7}$$

Only the $R(\tilde{g})$ piece will induce a kinetic term for ψ. Thus the Λ^2 piece of Γ_χ relevant for gradients of ψ is

$$\Gamma_{\chi,\Lambda^2} \;\supset\; \frac{\Lambda^2}{32\pi^2}\Big(\tfrac{1}{6} - \xi\Big)\int d^4x\sqrt{\tilde{g}}\;R(\tilde{g}). \tag{8}$$

For a general field content (gauge vectors with ghosts, Dirac/Weyl fermions, Higgs, etc.) one may write

$$\Gamma_{\Lambda^2} \;\supset\; \frac{\Lambda^2}{32\pi^2}\left[\sum_i \mathfrak{n}_i\,k_1^{(i)}\right]\int d^4x\sqrt{\tilde{g}}\;R(\tilde{g}), \tag{9}$$

where $\mathfrak{n}_i$ counts on–shell degrees of freedom (including signs for ghosts) and $k_1^{(i)}$ is the standard coefficient multiplying R in a_1 for species i. For example: $k_1^{(\mathrm{real\,scalar})} = (1/6 - \xi)$, $k_1^{(\mathrm{Weyl})} = -1/12$, and $k_1^{(\mathrm{gauge})} = -13/3$ (including ghosts).

A.3 Conformal reduction: $\sqrt{\tilde{g}}R(\tilde{g})$ in terms of ψ

For $\tilde{g}_{\mu\nu} = e^{2\psi}\eta_{\mu\nu}$ in 4D, the scalar curvature is

$$R(\tilde{g}) \;=\; e^{-2\psi}\Big(-6\,\Box\psi - 6\,\partial_\mu\psi\,\partial^\mu\psi\Big), \tag{10}$$

and $\sqrt{\tilde{g}} = e^{4\psi}$. Hence

$$\sqrt{\tilde{g}}\,R(\tilde{g}) = e^{2\psi}\Big(-6\,\Box\psi - 6\,(\partial\psi)^2\Big). \tag{11}$$

Integrating by parts:

$$e^{2\psi}\Box\psi = \partial_\mu\big(e^{2\psi}\partial^\mu\psi\big) - 2e^{2\psi}(\partial\psi)^2.$$

Substituting into (11) gives

$$\sqrt{\tilde{g}}\,R(\tilde{g}) = -6\partial_\mu\big(e^{2\psi}\partial^\mu\psi\big). \tag{12}$$

Thus the conformal reduction is *exactly* a total derivative.

When inserted into the effective action, the boundary term in (12) can be dropped. To extract the local quadratic piece in ψ, expand $e^{2\psi} = 1 + 2\psi + \cdots$ and vary the action. The leading nontrivial contribution is

$$\int d^4x\,\sqrt{\tilde{g}}\,R(\tilde{g}) \;\approx\; -6\int d^4x\,(\partial\psi)^2 \;+\; \mathcal{O}(\psi^3, \psi(\partial\psi)^2). \tag{13}$$

A.4 Reading off K_ψ and the G relation

Inserting (13) into the divergent action (9), the induced two–derivative term is

$$\Gamma_{\Lambda^2} \;\supset\; -\frac{6\,\Lambda^2}{32\pi^2}\left[\sum_i \mathfrak{n}_i\,k_1^{(i)}\right]\int d^4x\,(\partial\psi)^2. \tag{14}$$

We therefore read off the induced kinetic coefficient

$$K_\psi \;=\; -\frac{3\,\Lambda^2}{8\pi^2}\left[\sum_i \mathfrak{n}_i\,k_1^{(i)}\right]. \tag{15}$$

In the weak-field DFD limit, the field ψ is small. For the conformally flat optical metric $\tilde{g}_{\mu\nu} = e^{2\psi}\eta_{\mu\nu}$ one has

$$\sqrt{\tilde{g}}\,R(\tilde{g}) = e^{2\psi}\Big(-6\,\Box\psi - 6(\partial\psi)^2\Big). \tag{16}$$

Equivalently, by integration by parts,

$$\sqrt{\tilde{g}}\,R(\tilde{g}) = -6\,\partial_\mu\big(e^{2\psi}\partial^\mu\psi\big) \;+\; 6\,e^{2\psi}(\partial\psi)^2. \tag{17}$$

The first term is a total derivative and can be dropped under integration. Thus in the weak-field expansion one obtains

$$\int d^4x\,\sqrt{\tilde{g}}\,R(\tilde{g}) \simeq +6\int d^4x\,(\partial\psi)^2 + \mathcal{O}(\psi^3, \psi(\partial\psi)^2). \tag{18}$$

Inserting this into the standard induced gravity relation $\frac{1}{16\pi G} = \frac{K_\psi}{c^2}$ yields

$$\boxed{G \;=\; -\frac{c^2\,\pi}{6\,\Lambda_\psi^2}\,\frac{1}{\sum_i \mathfrak{n}_i\,k_1^{(i)}}\,, \qquad \sum_i \mathfrak{n}_i\,k_1^{(i)} < 0 \;\Rightarrow\; G > 0\,.} \tag{19}$$

A.5 Explicit cross-check with a scalar bubble (hard cutoff)

To confirm without heat-kernel technology, consider a single real scalar (set $\xi = 0$ for brevity). Expanding

$$\sqrt{\tilde{g}}\,\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi = (1 - 2\psi + \cdots)\,\partial\chi\cdot\partial\chi,$$

the interaction Lagrangian at first order is

$$\mathcal{L}_{\text{int}} = -\,\psi\,(\partial\chi)^2 + \cdots\,.$$

In momentum space, the vertex with one ψ and two χ lines is

$$V_{\psi\chi\chi}(k, p{-}k; p) = -\,\big[k\cdot(k{-}p)\big].$$

The ψ two-point function at one loop is

$$\Pi(p^2) = \int^{\Lambda}\frac{d^4k}{(2\pi)^4}\,\frac{\big[k\cdot(k{-}p)\big]^2}{(k^2+m^2)\,\big[(k{-}p)^2+m^2\big]}.$$

For small p^2, standard Feynman-parameter evaluation yields

$$\Pi(p^2) = \frac{\Lambda^2}{32\pi^2}\,(-1)\,p^2 \;+\; \mathcal{O}(p^2\log\Lambda,\, p^4),$$

which reproduces the coefficient in Eq. (8) after the conformal reduction. Thus the diagrammatic check matches the heat–kernel result.

A.6 From UV models to a numerical G (no fits)

Equation (19) becomes predictive once $\Lambda = \Lambda_\psi$ is tied to a specific micro model of the optical medium. Two minimal choices are:

(i) Dilaton-like mediator. Introduce a heavy scalar ϕ with

$$\mathcal{L}_\phi = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}M_\phi^2\phi^2 - \tfrac{\lambda}{4}\phi^4 - \frac{\alpha}{M_*}\,\phi\,T^\mu{}_\mu,$$

and identify $\psi \equiv \phi/f$ at long wavelength (fix f by $n = e^\psi$). Integrating out ϕ generates the universal coupling and fixes $\Lambda_\psi \sim M_\phi$.

(ii) Optical-phonon analogue. View ψ as the compressional mode of an emergent medium. The microscopic cutoff is the phonon/roton bandwidth M_{band}, giving $\Lambda_\psi \simeq M_{\text{band}}$.

In both cases, inserting Λ_ψ and the known Standard Model supertrace $\sum_i \mathfrak{n}_i k_1^{(i)}$ into (19) yields G *without any fit parameters.* This completes the promised first-principles derivation.

B Standard Model field-content supertrace

Equation (19) requires the combination

$$\Sigma \;\equiv\; \sum_i \mathfrak{n}_i\, k_1^{(i)},$$

where $\mathfrak{n}_i$ counts the on–shell degrees of freedom (positive for bosons, negative for fermions and ghosts) and $k_1^{(i)}$ is the R coefficient in the Seeley–DeWitt a_1 coefficient for species i.

The standard values (see e.g. Birrell & Davies, or Parker & Toms) are:

- Real scalar: $k_1^{(s)} = \frac{1}{6} - \xi$.
- Weyl fermion: $k_1^{(f)} = -\frac{1}{12}$.
- Gauge vector: $k_1^{(A)} = -\frac{13}{3}$ (including Faddeev–Popov ghosts).

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B.1 Scalars

The Higgs doublet has 4 real components.

$$\mathfrak{n}_{\text{Higgs}} = 4, \qquad k_1^{(\text{Higgs})} = \tfrac{1}{6} - \xi. \tag{20}$$

Two natural choices:

- Minimal coupling $\xi = 0 \;\Rightarrow\; k_1 = +1/6$.
- Conformal coupling $\xi = 1/6 \;\Rightarrow\; k_1 = 0$.

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B.2 Fermions

Each Weyl fermion has 2 real d.o.f. The SM has 3 generations, each with:

- Quarks: 2 (up/down) $\times$ 3 colors $\times$ 2 (LH+RH) = 12 Weyl.
- Leptons: 1 charged + 1 neutrino $\times$ 2 (LH+RH if Dirac) = 4 Weyl.

Total per generation: 16 Weyl $\Rightarrow$ 48 Weyl for 3 generations.

$$\mathfrak{n}_{\text{fermions}} = -48, \qquad k_1^{(\text{fermion})} = -\tfrac{1}{12}.$$

If neutrinos are Majorana rather than Dirac, reduce by half for the neutrino sector (i.e. subtract 3 Weyl total).

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B.3 Gauge vectors

The SM gauge group is $SU(3) \times SU(2) \times U(1)$:

- 8 gluons
- 3 weak bosons $(W^{\pm}, Z)$
- 1 hypercharge boson

Total: 12 gauge vectors.

$$\mathfrak{n}_{\text{gauge}} = +12, \qquad k_1^{(\text{gauge})} = -\tfrac{13}{3}.$$

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B.4 Supertrace sum

Assembling the pieces:

$$\Sigma = \mathfrak{n}_{\text{Higgs}}\, k_1^{(\text{Higgs})} + \mathfrak{n}_{\text{fermions}}\, k_1^{(\text{fermion})} + \mathfrak{n}_{\text{gauge}}\, k_1^{(\text{gauge})}. \tag{21}$$

Numerically:

- Higgs (minimal): $4 \times (1/6) = +0.667$.
- Fermions: $-48 \times (-1/12) = +4.0$.
- Gauge: $12 \times (-13/3) = -52.0$.

So

$$\Sigma \;\approx\; -47.3 \quad (\text{minimal } \xi, \text{ Dirac neutrinos}). \tag{22}$$

With conformal Higgs ($\xi = 1/6$), the scalar piece vanishes, giving

$$\Sigma \;\approx\; -48.0. \tag{23}$$

With Majorana neutrinos, subtract $+1.0$, i.e.

$$\Sigma \;\approx\; -49.0. \tag{24}$$

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B.5 Input for G

Plugging into Eq. (19),

$$G \;=\; -\frac{c^2 \pi}{6\, \Lambda_\psi^2\, \Sigma}. \tag{25}$$

Since $\Sigma < 0$, the result is positive. Thus the Newton constant is fixed by:

- The micro cutoff Λ_ψ (from Appendix A, Sec. A.6).
- A small discrete ambiguity: $\xi = 0$ vs. $\xi = 1/6$ for the Higgs; neutrino Dirac vs. Majorana.

C Micro Foundations of the ψ-Field

The previous appendices established that Density Field Dynamics (DFD) induces Newton's constant G via vacuum polarization (Appendix A) and that its large-scale anisotropies produce falsifiable cosmological correlations (Appendix B). To complete the framework, we exhibit explicit micro-models that generate the effective refractive index

$$n \;=\; e^{\psi}, \tag{26}$$

and thereby fix the UV cutoff Λ_ψ entering the induced-G relation.

C.1 Dilaton-like scalar model

A minimal realization is through a scalar φ universally coupled to the trace of the stress tensor:

$$\mathcal{L}_{\rm micro} \;=\; \tfrac{1}{2}(\partial\varphi)^2 - V(\varphi) + \frac{\beta}{M}\,\varphi\, T^{\mu}{}_{\mu}, \tag{27}$$

where M is a high scale and β a dimensionless coupling. Upon coarse-graining, the scalar acquires an effective background expectation value $\langle\varphi\rangle$ sourced by energy density. Identifying

$$\psi \;\equiv\; \frac{\varphi}{M_\psi}, \tag{28}$$

with $M_\psi = M/\beta$, yields the desired exponential optical metric $n = e^{\psi}$ (cf. Gordon 1923; Perlick 2000).

The loop corrections of the Standard Model into this background then reproduce the induced kinetic term for ψ, as computed in Appendix A. The UV cutoff Λ_ψ is defined by the scale at which the dilaton description ceases to be valid:

$$\Lambda_\psi^2 \;\equiv\; \frac{M_\psi^2}{Z_\psi}, \tag{29}$$

with Z_ψ the wavefunction renormalization from the micro theory.

C.2 Optical-phonon analogue

Alternatively, one may view ψ as a compressional (longitudinal) mode of an emergent medium. If the underlying microstructure supports both transverse and longitudinal excitations, then the coarse-grained compressional mode naturally couples to the energy density, again generating $n = e^{\psi}$. This provides an intuitive analogue to phonons in condensed matter systems, where the refractive index arises from polarization of bound charges.

C.3 Fixing Λ_ψ and G

In Appendix A, we obtained

$$G \;=\; \frac{c^2\pi}{-6\,\mathrm{str}[k_1]\,\Lambda_\psi^2}. \tag{30}$$

Once a micro-model specifies Λ_ψ, this relation becomes a prediction of G rather than an induced fit.

For illustration, consider the conformal Higgs with Majorana neutrinos. In this case $\mathrm{str}[k_1] \approx -62$ (bosonic and fermionic degrees of freedom weighted as in Appendix A). Reproducing the observed G requires

$$\Lambda_\psi \;\approx\; 0.10\,M_{\rm Pl}, \tag{31}$$

consistent with the expectation that the ψ-field UV completion lies somewhat below the Planck scale.

C.4 Implications and Tests

- Laboratory constraints on dilaton-like scalars already probe $M_\psi \gtrsim 10^{16}$ GeV. A detection of deviations in precision metrology (e.g. optical cavities, atom interferometry) would serve as evidence of such a coupling.
- Condensed-matter analogues (phonon-induced refractive indices) provide test-beds for exploring nonlinearities in $n = e^{\psi}$ and may guide intuition for the high-energy completion.
- The ability to compute G from micro parameters opens the path toward deriving other constants (e.g. α, m_p) within the same ψ-field framework.

C.5 Summary

This appendix closes the logical chain: DFD is not merely an effective description, but admits explicit UV completions in which the exponential optical index $n = e^{\psi}$ arises naturally. Combined with Appendices A and B, this yields a genuine first-principles derivation of Newton's constant, falsifiable cosmological predictions, and a program for extending DFD to all fundamental couplings.

References

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