Geometric Cancellation of Cavity–Atom LPI Signals in Density Field Dynamics: A Formal Proof

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February 25, 2026

Abstract

We prove that in Density Field Dynamics (DFD), the tree-level response of electromagnetic cavity resonances and atomic transition frequencies to the scalar field ψ cancel in their ratio, reducing the measurable Local Position Invariance (LPI) violation parameter from $\xi_{\rm LPI} \sim 1$ to $\xi_{\rm LPI} = k_\alpha^{\rm eff} \cdot \Delta S^\alpha \sim 10^{-5}$. Three independent empirical checks confirm this cancellation. The BACON optical clock network (Nature 591, 564, 2021) further constrains the screening regime, requiring that $k_\alpha^{\rm eff}$ be evaluated at the local gravitational environment rather than the source field. This erratum strengthens the theory's consistency with all existing clock data while preserving the ROCIT 13.5σ detection as the primary experimental signature.

1 Statement of the Problem

DFD replaces curved spacetime with a scalar refractive field ψ on flat $\mathbb{R}^3$, with optical metric

$$d\tilde{s}^2 = -\frac{c^2}{n^2}dt^2 + d\mathbf{x}^2, \qquad n = e^\psi. \tag{1}$$

An earlier version of the theory (Sec. 10, DFD v3.1) claimed that the cavity–atom frequency ratio $R = f_{\rm cav}/f_{\rm atom}$ responds to ψ with $\xi_{\rm LPI} \approx$ 1–2, by assigning $K_\gamma = 1$ (photon sector) and $K_{\rm atom} \approx 0$ (atomic sector).

This note proves that the optical metric's constitutive relations *require* both sectors to respond identically at tree level, reducing $\xi_{\rm LPI}$ by a factor $\sim 10^5$.

2 The Constitutive Chain

Step 1: Tamm–Plebanski formalism. The optical metric (1) determines, via the Tamm–Plebanski construction, the vacuum constitutive relations:

$$\varepsilon_{\rm eff} = \varepsilon_0\, e^{+\psi}, \qquad \mu_{\rm eff} = \mu_0\, e^{+\psi}. \tag{2}$$

This is an impedance-matched medium: $Z = \sqrt{\mu_{\rm eff}/\varepsilon_{\rm eff}} = Z_0$. The phase velocity is $v_{\rm ph} = 1/\sqrt{\varepsilon_{\rm eff}\mu_{\rm eff}} = c\, e^{-\psi}$, consistent with $n = e^\psi$.

Step 2: Coulomb potential. Virtual photons propagate on the same optical metric. The static Coulomb potential between charges is:

$$V(r) = \frac{e^2}{4\pi\varepsilon_{\rm eff}\, r} = \frac{e^2}{4\pi\varepsilon_0\, r}\, e^{-\psi}. \tag{3}$$

The fine-structure constant, measured in coordinate frame units, becomes:

$$\alpha(\psi) = \frac{e^2}{4\pi\varepsilon_{\rm eff}\, \hbar c_{\rm local}} = \alpha_0\, \frac{e^{-\psi}}{e^{-\psi}} = \alpha_0. \tag{4}$$

The $e^{-\psi}$ from $\varepsilon_{\rm eff}$ and the $e^{-\psi}$ from $c_{\rm local}$ cancel, so α *is ψ-independent at tree level.*

Step 3: Atomic structure. With α constant, the Bohr radius scales as:

$$a_0(\psi) = \frac{\hbar}{m_e c_{\rm local}\, \alpha} = a_0^{(0)}\, e^{+\psi}. \tag{5}$$

Atoms expand in stronger fields. The Rydberg energy:

$$E_R = \tfrac{1}{2}\alpha^2 m_e c_{\rm local}^2 \propto e^{-2\psi}. \tag{6}$$

For a general transition with relativistic correction ϵ_A:

$$f_{\rm atom} \propto e^{-(2+\epsilon_A)\psi}, \tag{7}$$

where ϵ_A depends on the transition (e.g., $\epsilon_{\rm Sr} = 0.06$).

Step 4: Cavity frequency. A Fabry–Pérot cavity of material spacer length L, mode number m:

$$f_{\rm cav} = \frac{m\, c_{\rm local}}{2L(\psi)}. \tag{8}$$

The spacer is an electromagnetic solid: its lattice constant is set by the Bohr radius, so $L \propto a_0(\psi) \propto e^{+\psi}$. The local light speed is $c_{\rm local} = c\, e^{-\psi}$. Therefore:

$$f_{\rm cav} \propto \frac{e^{-\psi}}{e^{+\psi}} = e^{-2\psi}. \quad (9)$$

Both effects—slower light *and* longer spacer—contribute $e^{-\psi}$ each, compounding to $e^{-2\psi}$.

3 The Cancellation

From Eqs. (7) and (9):

$$R = \frac{f_{\rm cav}}{f_{\rm atom}} \propto \frac{e^{-2\psi}}{e^{-(2+\epsilon_A)\psi}} = e^{+\epsilon_A \psi}. \quad (10)$$

The leading $e^{-2\psi}$ factor—universal gravitational redshift—cancels exactly. The residual geometric variation is:

$$\xi_{\rm geom} = \epsilon_A \approx 0.06 \quad \text{(for Sr/Si cavity)}. \quad (11)$$

Why even $\xi_{\rm geom}$ is unphysical. The residual ϵ_A arises from relativistic corrections to atomic structure that depend on α. But we proved in Eq. (4) that α is ψ-independent at tree level. The $e^{-(2+\epsilon_A)\psi}$ scaling of atomic frequencies is an artifact of expressing frequencies in coordinate time; in proper time (what a local observer measures), $\alpha = \alpha_0$ exactly, and the ratio R is constant.

This is the Weak Equivalence Principle (WEP): in a local freely-falling frame, non-gravitational physics—including α—is position-independent. DFD satisfies WEP at tree level by construction (PPN: $\gamma = \beta = 1$).

4 The Physical Residual

WEP is broken at one loop by Unruh–de Sitter screening of quantum fluctuations. The screened effective coupling is:

$$k_\alpha^{\rm eff}(a) = 2\sqrt{\alpha}\, \mu_{\rm LPI}(a/a_0), \quad (12)$$

where $\mu_{\rm LPI}(y) = (1+y)^{-1/2}$ and a is the local gravitational acceleration.

The measurable LPI violation in a cavity–atom comparison is:

$$\boxed{\xi_{\rm LPI} = k_\alpha^{\rm eff} \cdot (S_A^\alpha - S_{\rm cav}^\alpha),} \quad (13)$$

where $S_A^\alpha \equiv d\ln\nu_A / d\ln\alpha$ is the transition's α-sensitivity and $S_{\rm cav}^\alpha \approx 1$ for an EM-bonded spacer.

5 Three Independent Confirmations

Check 1: Fine-structure splitting. If α varied as $\alpha_0 e^{-\psi}$ (geometric, unscreened), the ratio of two transitions in the same atom with different α-sensitivities would show annual modulation at amplitude $\Delta S^\alpha \times \delta\psi_{\rm annual} \sim 10^{-10}$. Precision spectroscopy constrains such variation to $< 10^{-17}$. The geometric scenario is ruled out by $> 10^7$.

Check 2: PTB Yb^+ E3/E2. The same-ion comparison $|K_{E3} - K_{E2}| < 10^{-8}$ (Lange et al. 2021). Geometric prediction: $|\Delta S^\alpha| \times \delta\psi_{\rm annual} \approx 5.14 \times 1.65 \times 10^{-10} \approx 8.5 \times 10^{-10}$. Ruled out by $\sim 100\times$.

Check 3: BACON network (Beloy et al. 2021). Three species (Al^+, Sr, Yb) compared at 6–8 $\times 10^{-18}$ over 8 months spanning perihelion. Geometric prediction for Yb/Sr: $0.25 \times 1.65 \times 10^{-10} = 4.1 \times 10^{-11}$. Observed stability: $\sim 10^{-17}$. Ruled out by $\sim 10^6$.

All three checks independently confirm the geometric cancellation.

6 Screening Regime Constraint

The BACON data provide a further constraint. With solar-orbit screening ($a = 5.93 \times 10^{-3}$ m/s^2, $k_\alpha^{\rm eff} = 2.4 \times 10^{-5}$), the predicted Yb/Sr annual signal is:

$$\delta R/R = 0.25 \times 2.4 \times 10^{-5} \times 1.65 \times 10^{-10} = 1.0 \times 10^{-15}. \quad (14)$$

The BACON Yb/Sr weighted standard deviation is 1.1×10^{-17}, ruling out this scenario by $\sim 100\times$.

With Earth-surface screening ($a = 9.8$ m/s^2, $k_\alpha^{\rm eff} = 6.0 \times 10^{-7}$):

$$\delta R/R = 0.25 \times 6.0 \times 10^{-7} \times 1.65 \times 10^{-10} = 2.5 \times 10^{-17}. \quad (15)$$

This is comparable to the BACON between-day variability ($\xi_{\rm Yb/Sr} = 10.8 \times 10^{-18}$, $\chi^2_{\rm red} = 6.0$) and therefore *consistent* with the data.

Conclusion: Screening must be evaluated at the *local* gravitational environment, not at the source of the perturbation. Physically, the Unruh–de Sitter mechanism depends on the total local $|\nabla\psi|$, which at Earth's surface is dominated by Earth's own field.

7 Implications

1. **Section 10 erratum:** $\xi_{\rm LPI} \approx$ 1–2 is replaced by $\xi_{\rm LPI} = k_\alpha^{\rm eff}(a_{\rm local}) \cdot \Delta S^\alpha \sim 10^{-7}$ at Earth's

surface.

2. **ROCIT detection preserved:** The 13.5σ ion–neutral modulation uses a different channel (cavity–atom with ionic transition) where $\Delta S^\alpha \sim 6$, giving signals at $\sim 10^{-5}$—unaffected by this revision.

3. **Nuclear clocks become paramount:** With $S^\alpha_{\rm Th} \approx 5900$ (Beeks et al. 2025), the Th-229/Sr annual signal from α-coupling alone is:

$$\delta R/R \approx 6\times10^{-7}\times5900\times1.65\times10^{-10} \approx 5.8\times10^{-13}, \tag{16}$$

detectable at current nuclear clock precision ($\sim 10^{-12}$).

4. **Height tests require space:** The height-separated test needs $\sim 10^{-20}$ precision for $\Delta h =$ 100 m, pushing it to future space missions.

5. **Theory becomes cleaner:** The "why hasn't anyone noticed 10^{-10} drift?" problem disappears. All existing null results are naturally explained.

8 Summary

The optical metric $d\tilde{s}^2 = -c^2 e^{-2\psi} dt^2 + d\mathbf{x}^2$ uniquely determines constitutive relations $\varepsilon = \varepsilon_0 e^\psi$, $\mu = \mu_0 e^\psi$. These modify the Coulomb potential, causing cavity spacers to expand by $e^{+\psi}$ while light slows by $e^{-\psi}$. The compound effect gives $f_{\rm cav} \propto e^{-2\psi}$, identical to the atomic scaling, so the ratio is constant at tree level.

The physical LPI violation is a one-loop quantum correction, screened by the local gravitational environment to $k_\alpha^{\rm eff} \sim 10^{-7}$ at Earth's surface. Three independent empirical checks confirm this picture. The nuclear clock transition in ^{229}Th, with $S^\alpha \approx 5900$, amplifies this residual to $\sim 10^{-13}$—within reach of current experimental programs.

Acknowledgments. The author thanks the BACON collaboration, Jun Ye, and Nils Huntemann for the precision data that constrain these predictions.