

Density Field Dynamics v5.0: Extended Derivations and Frontier Predictions

Gary Alcock

Density Field Dynamics Program
(Dated: v5.0 — October 2026)

This is the complete extended-derivation companion to the main paper *Density Field Dynamics: A Complete Unified Theory*. It collects the proofs, multi-route derivations, frontier predictions, and high-precision numerical results that support the theory but are too lengthy for the main text: closures and promotions, the new-territory frontier (particle, astrophysical, and cosmological predictions), the CKM/ τ and series IV–VI derivations, the mathematical-foundation and quantum-gravity closures, the multi-route fortifications and sharpenings, and the wave-sector closure (App. WS). It is *not* the statement of the theory; the axioms, theorems, predictions, and falsifiers are in the main paper, whose list of principal corrections is authoritative. The intermediate development record is deliberately omitted. Appendix letters other than AH point to the main paper (App. WS is printed in both volumes); references within this volume are internal.

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Appendix AH: Closures and New Territories

1. Purpose and Scope

This appendix collects the extended results. Each subsection presents the main theorem, the central proof, the principal numerical consequence, and the falsification criterion. Where the proof is constructive, we display the construction explicitly; where it is analytic, we display the central inequality. Where an independent recomputation reached a different conclusion from the derivation, the result is recorded immediately after the proof in a **Status note**, so the reader can weigh the evidence.

2. Closure and Status of Blocking Open Problems

a. Vacuum Energy Feedback Stability Theorem

The Sec. IX.H heuristic “ $\lambda \sim 10^{113}$ violently unstable” is closed in the dimensionless response variable (Sec. AH.6 a); the status note below records an independent recomputation.

Theorem AH.1 (Wilsonian Regulation by Microsector Finiteness). *Let $V_{\text{CW}}^{(1)}$ be the one-loop Coleman–Weinberg correction to the ψ -effective potential, computed against the dynamical microsector spectrum on $\mathbb{CP}^2 \times S^3$ at Toeplitz cutoff $k_{\text{max}} = 60$. Then*

$$V_{\text{CW}}^{(1)} = \frac{1}{64\pi^2} \sum_{n=1}^{60} M_n^4(\psi) \left[\log \frac{M_n^2(\psi)}{\Lambda_{\text{top}}^2} - \frac{3}{2} \right],$$

where $\Lambda_{\text{top}} = \sqrt{\alpha} M_P$ is the largest Toeplitz eigenvalue scale, and the sum is over $\dim \mathcal{H}_{\text{micro}} = 60$ modes.

Proof. The Berezin–Toeplitz quantization of $E = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ over $\mathbb{CP}^2$ at level $k_{\text{max}} = 60$ gives a Hilbert space $\mathcal{H}_{\text{micro}} = H^0(\mathbb{CP}^2, E)$ whose dimension is computed by Hirzebruch–Riemann–Roch: $\dim \mathcal{H}_{\text{micro}} = \chi(\mathbb{CP}^2, E) = 55 + 5 = 60$. This is finite, not regulated infinite. The one-loop effective potential is the standard functional determinant

$$V_{\text{CW}}^{(1)}(\psi) = \frac{1}{2} \text{Tr}_{\mathcal{H}_{\text{micro}}} \log(\hat{D}^2(\psi)/\mu^2),$$

where $\hat{D}$ is the gauge-covariant Dirac operator on E . Because $\mathcal{H}_{\text{micro}}$ is 60-dimensional, the trace is a finite sum over 60 mode masses $M_n^2(\psi)$. Performing the Schwinger proper-time integral for each mode against the standard $\overline{\text{MS}}$ -equivalent scheme (proper-time cutoff Λ_{top}^2) yields the displayed Coleman–Weinberg formula. Because each $M_n^2(\psi) \leq \Lambda_{\text{top}}^2$ by construction (largest Toeplitz eigenvalue), every summand is bounded, and the integral is a finite trace, not a continuum integral to ∞ . $\square$

Lemma AH.2 (One-Loop Per-Mode α -Cancellation). *Under uniform gauge normalization, each of the 57 nonzero modes contributes one factor of α to the Gaussian determinant ratio, as at tree level (Lemma O.5 of*

the main volume); that this survives at one loop is stated, not proved here.

Corollary AH.3 (Vacuum Stability). *$|\delta V_{\text{CW}}^{(1)}/M_P^4| \leq O(\alpha^2)$, and the exponent in $G\hbar H_0^2/c^5 = C\alpha^{57}$ (App. O) survives quantization with corrections bounded by α^2 , given Lemma AH.2; this does not derive measured C .*

Proof. By Theorem AH.1 the one-loop $V_{\text{CW}}^{(1)}$ is bounded by $60 \cdot \Lambda_{\text{top}}^4 \cdot O(1) = 60\alpha^2 M_P^4$, so $|\delta V_{\text{CW}}^{(1)}/M_P^4| \leq O(\alpha^2)$. The per-mode count of Lemma AH.2, as stated, preserves the exponent in $G\hbar H_0^2/c^5 = C\alpha^{57}$ under quantization with relative correction $\delta(\alpha^{57})/\alpha^{57} \leq O(\alpha^2)$; C remains measured. $\square$

Remark AH.4 (Status note). An independent recomputation grades this closure *questionable*. It agrees that the partition-function ratio on $\mathcal{H}_{\text{micro}}$ is finite and scheme-independent, but finds that the 4D Coleman–Weinberg loop integral over external momenta retains a UV-divergent shell beyond the Toeplitz truncation, giving $\delta m_\psi^2 \sim \alpha^2 M_P^2$ rather than the much smaller bound claimed. The discrepancy turns on whether the response variable is ρ_{vac} (master-text reading, instability) or $\delta\alpha/\alpha$ in dimensionless units (Gordon optical reading, stability automatic). Sec. AH.6 a displays the dimensionless bound and traces the 10^{113} of Sec. IX.H to the use of ρ_c in its denominator; both readings are kept there.

In the dimensionless (Gordon optical) reading the “violently unstable” Sec. IX.H warning does not apply: the feedback is stable (Corollary AH.3).

b. Strong-Field Symmetric-Hyperbolic Reformulation

Nonlinear strong-field evolution is locally well-posed wherever $\nabla\psi \neq 0$, for the exact law as for its approximant (main volume, well-posedness section), and the linearised evolution is well-posed on every static background, including points where $\nabla\psi = 0$. What stays open is nonlinear evolution through points where $\nabla\psi = 0$, where the symmetriser A^0 degenerates and both scalar speeds vanish. The scalar characteristic speeds of the exponential response reach $1.017c$ (SOURCE) and $1.066c$ (TOTAL) in the transition regime; every characteristic advances the absolute time t of the preferred foliation, so causality holds and only light-cone containment fails (main volume, well-posedness section).

Theorem AH.5 (15-Component Symmetric-Hyperbolic System). *Let $U \equiv (\psi, \pi_\psi, \phi_i, h_{ij}^{\text{TT}}, k_{ij}^{\text{TT}})$ be the 15-component evolution vector. The DFD coupled trace+TT system can be written*

$$A^0(U) \partial_t U + A^j(U) \partial_j U = S(U),$$

with explicitly symmetric A^0 and symmetric A^j , A^0 positive definite wherever $\nabla\psi \neq 0$. The principal symbol $\sigma(\xi) = (A^0)^{-1} A^j \xi_j$ has real eigenvalues for all spacelike

ξ . *Local well-posedness in H^s for $s > 5/2$ follows by Kato–Friedrichs on any region where A^0 is uniformly positive definite.*

Proof. Write the Lagrangian $\mathcal{L} = \mathcal{L}_\psi + \mathcal{L}_{\text{TT}} + \mathcal{L}_{\text{int}}$. The trace sector contributes the variables (ψ, π_ψ, ϕ_i) where $\pi_\psi = \dot{\psi}$ and $\phi_i = \partial_i \psi$. The TT sector contributes $(h_{ij}^{\text{TT}}, k_{ij}^{\text{TT}})$ with $k_{ij}^{\text{TT}} = \dot{h}_{ij}^{\text{TT}}$. The $O(3)$ block-diagonality of Theorem AH.20 implies no derivative cross-terms between trace and TT, so the Hessian matrix A^0 is block-diagonal, positive definite wherever $\phi \neq 0$ (since $\mu(0) = 0$), with the trace block $\text{diag}(1, c^{-2}, D\mathbf{a}(\phi))$ for (ψ, π_ψ, ϕ_i) , where $D\mathbf{a}$ is the flux Jacobian with eigenvalues μ (twice) and $(x\mu)'$, and $\frac{1}{32\pi G}\delta_{ij}\delta_{kl}$ for each TT pair. The space-derivative matrices A^j are read off from ∂_j in the EL equations and inherit the same block structure. By construction each A^j is symmetric. Compute the eigenvalues of $\sigma(\xi)$ on each block: the trace block gives the scalar-sector characteristic speeds analysed in the main volume’s well-posedness section (real wherever $\phi \neq 0$); TT gives $\pm c|\xi|$ (Theorem AH.20; light and gravitational waves share one metric, App. WS). Kato’s theorem (1975) for quasilinear symmetric-hyperbolic systems with C^1 coefficients gives local existence in H^s for $s > d/2 + 1 = 5/2$ in $d = 3$ space wherever A^0 is uniformly positive definite. $\square$

Theorem AH.6 (Constraint Propagation). *The kinematic trace-sector constraint $C_\psi \equiv \pi_\psi - \dot{\psi}$ and the TT-tracelessness constraint $C_{ij}^{\text{TT}} \equiv \delta^{ij}h_{ij}^{\text{TT}}$ are exactly preserved under the evolution: $\partial_t C_\psi = 0$, $\partial_t C_{ij}^{\text{TT}} = 0$ identically on solution.*

Proof. Differentiate $C_\psi = \pi_\psi - \dot{\psi}$ in time: $\partial_t C_\psi = \dot{\pi}_\psi - \ddot{\psi}$. The EL equation gives $\dot{\pi}_\psi = \nabla_i(\mu\phi^i) - \partial V/\partial\psi = \ddot{\psi}$ on solution, so $\partial_t C_\psi \equiv 0$. For TT-tracelessness: the projection operator $P_{ij,kl}^{\text{TT}}$ commutes with the wave operator $\square_{\text{flat}}$, so if $\delta^{ij}h_{ij} = 0$ at $t = 0$ it remains zero. Algebraically: applying δ^{ij} to the TT evolution equation gives a homogeneous equation for $\delta^{ij}h_{ij}$ with zero source, which by uniqueness of solutions of linear hyperbolic systems is identically zero if it vanishes initially. The numerical implementation verified $\|\text{tr}(h)\|$ stays at machine zero across all evolved steps. $\square$

c. Convergence Kernel Theorem

Theorem AH.7 (Limber-Projected DFD Lensing Kernel). *On the baryon-only apparent-amplitude construction, which is not DFD’s growth carrier (proof, Step 6), the weak-lensing convergence of a source distribution δ_b is*

$$\kappa_{\text{DFD}}(\ell) = \int d\chi \frac{3(H_0^{\text{early}})^2}{2c^2} \frac{\chi g(\chi)}{a(\chi)} \Omega_b \mathcal{R}_{\text{DFD}} \left[k = \frac{\ell + \frac{1}{2}}{\chi}, z; \delta_b \right] \delta_b,$$

with response kernel $\mathcal{R}_{\text{DFD}}(k, z; \delta_b) = Q(k, z; \delta_b)$ and deep-MOND amplification $Q_{\text{DFD}} \approx 502$.

Proof. Step 1. The DFD optical metric $d\tilde{s}^2 = -c^2 dt^2/n^2 + d\mathbf{x}^2$ with $n = e^\psi$ produces the lensing potential $\Phi_{\text{lens}} = -(c^2/2)\delta\psi$ at linear order in $\delta\psi$, identical in

functional form to the GR Newtonian potential. **Step 2.** The Born integral for convergence is standard: $\kappa(\hat{\mathbf{n}}) = \int d\chi W(\chi) \nabla_\perp^2 \Phi_{\text{lens}}/c^2$, where $W(\chi) = (\chi_s - \chi)/\chi_s$ is the lensing efficiency. **Step 3.** On FRW with App. AE Theorem AE.1 ($\bar{a}_{\text{ext}} = 0$), the linearized DFD field equation becomes $k^2 \delta\psi_k = (8\pi G/c^2) \bar{\rho}_b \mathcal{R}_{\text{DFD}} \delta_b$, with $\mathcal{R}_{\text{DFD}} = Q(k, z; \delta_b)$ and $Q \approx 502$ in the deep-MOND regime where $|\nabla\psi| \ll a_*$. **Step 4.** Substitute $\delta\psi_k$ into the lensing-potential integrand: $\nabla_\perp^2 \Phi_{\text{lens}}/c^2 = -(\ell + \frac{1}{2})^2/(\chi^2 c^2) \cdot \Phi_{\text{lens}}$, giving the angular-Fourier convolution. **Step 5.** Limber projection collapses the line-of-sight integral to the displayed form. **Step 6.** Against the spherical-top-hat $R_8 = 8 h^{-1}$ Mpc window the closure takes $\sigma_8 = 0.811$ as its input and solves for Q_{DFD} (App. AE). This is the optical amplification of a baryon-only contrast, the apparent-amplitude construction of App. AE (no dark-matter *halo* entering the lensing kernel). DFD’s physical carrier is the derived χ -matter ($\Omega_\chi h^2 \simeq 0.12$, Λ CDM-like; App. AV), which sources the rest-mass channel on linear scales, and the Rest-Mass Channel confines the baryons’ polarization to bound systems, so this amplification does not act on the R_8 scale and the lensing there follows the matter power that χ and the baryons carry (App. GR of the main volume). $\square$

Three pre-registered falsifiers of this baryon-only construction: direct $\text{Ly}\alpha$ $\sigma_b(R_8, z)$, kSZ amplitude about 0.9 of Λ CDM’s, cross-survey $F_8 = 77.9$ universality at $\pm 10\%$. They test the construction, not DFD’s growth carrier, the cold χ relic with $Q_\chi = 1$ (main volume, App. AE and App. GR).

d. Cluster First-Principles Pipeline

The literature-bounded five-factor cluster budget does not close the cluster shortfall (App. I); clusters need the derived χ . The pipeline below is a fully specified zero-DOF first-principles $\rho_b \rightarrow (\kappa, \sigma, M, M_{\text{HSE}})$ pipeline for the μ -law side.

Theorem AH.8 (Universal μ in clusters, with χ). *The same interpolation function $\mu(x) = x/(1+x)$ that fits SPARC galaxy rotation curves at $a_0 = 2\sqrt{\alpha} c H_0^{\text{loc}}$ also reproduces, without modification, (i) the cluster baryon (gas) dynamics once the derived χ supplies the mass the μ -law leaves (about 0.84 of the cosmic mix inside r_{500} when χ adds its Newtonian pull, and 1.21 ± 0.05 predicted from the derived abundance under Λ CDM-like assembly, an estimate; main volume, App. I), with the Bullet-Cluster collisionless lensing offset carried by χ , and (ii) the multi-scale-averaging effective coefficient $\mu_{\text{eff}} = \mu \cdot (1 + 0.39 f_{\text{sub}})$.*

Proof. (i) Bullet Cluster: χ adds its Newtonian pull through the rest-mass channel, and the μ -law acts on the baryons. During the merger the collisional gas is shock-stripped and lags, while the collisionless χ (and the galaxies) remain at the galaxy centroid and set the lensing (κ) peak, displaced from the gas—reproducing the

observed offset by the same collisionless-mass mechanism as Λ CDM. The μ -law on the baryons alone is insufficient: at the two brightest cluster galaxies the baryonic field is $g_{N,b}/a_0 = 0.21\text{--}0.34$ (boost $\nu = 2.3\text{--}2.7$), and without χ DFD gives only 24–39% of the lensing mass inside 250 kpc; χ at 0.6–0.9 of the cosmic mix fits the lensing map when it adds its Newtonian pull (1.1–1.2 under the floor composition; main volume, App. I). The collisionless carrier is supplied natively by the derived χ field; the universal μ -function governs the baryon/gas dynamics. **(ii) Multi-scale averaging:** substructure with subhalo fraction f_{sub} contributes an additive convex combination $\mu_{\text{eff}} = (1 - f_{\text{sub}})\mu + f_{\text{sub}}\mu_{\text{eff,sub}}$. The constant 0.39 is the angular average over substructure orientations. **Numerics:** an internal Python solver reproduces (i) qualitatively at 64^3 resolution; full 16-cluster public-data run requires AMR and is engineering-pending. $\square$

e. Internal-Manifold Axiom Reduction

The reduction below uses the seven-axiom list of App. AB ((V1), (V2), (V4), (V4'), (V5), (V7), (V10)). It does not take the tangent-bundle gauge partition (3, 2, 1) as an axiom (V*4), nor derive $\dim X = 7$ from it: that partition is topologically impossible on $\mathbb{CP}^2 \times N^3$ (the tangent-obstruction lemma (App. AB, main volume)). And $\pi_3 = \mathbb{Z}$ does not follow from $\dim X = 7$.

Theorem AH.9 (Reduced-Axiom Uniqueness of $\mathbb{CP}^2 \times S^3$). *The internal manifold $X = \mathbb{CP}^2 \times S^3$ is uniquely determined (up to diffeomorphism) by the satisfiable axiom set of App. AB: compactness (V1), minimality of dimension (V2), nondegenerate Kähler-times-frame product structure (V4), parallelizable frame factor (V4'), $\chi_{\text{top}}(M_c) = 3$ (V5), $\pi_1(M_g) = 1$ (V7), and positive internal Ricci curvature (V10).*

Proof. Step 1. Corollary AB ($\dim=7$; main volume): (V10)+Kobayashi force M_c Fano and simply connected with $\dim_{\mathbb{R}} M_c \geq 4$ (the only Fano candidates below complex dimension 2 have $\chi_{\text{top}} \in \{1, 2\} \neq 3$), and (V4')+(V7) force $\dim M_g \geq 3$ (S^1 has $\pi_1 = \mathbb{Z}$; S^2 has $\chi_{\text{top}} = 2 \neq 0$; higher-genus surfaces have $\pi_1 \neq 1$). Minimality (V2), with the witness $\mathbb{CP}^2 \times S^3$ attaining 7, forces $\dim X = 7$ and the split 4 + 3. **Step 2.** M_c is a del Pezzo surface with $\chi_{\text{top}} = 3$; Noether's formula leaves $\mathbb{CP}^2$ as the unique entry (Lemma AB- M_c ; main volume). **Step 3.** M_g is a closed orientable simply connected 3-manifold, hence S^3 by Perelman's proof of the Poincaré conjecture (Lemma AB- M_g ; main volume) — no Lie-group hypothesis is needed. **Step 4.** On the winner, $k_{\text{max}} = \chi(\mathbb{CP}^2, \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}) = 55 + 5 = 60$ by Hirzebruch–Riemann–Roch, and Spin^c -ness, $\pi_3(M_g) = \mathbb{Z}$, and even mapping-torus dimension are derived properties (Corollary AB-properties; main volume). **Conclusion.** $X = \mathbb{CP}^2 \times S^3$ is uniquely determined; four properties ($\dim X = 7$, Spin^c , $\pi_3 = \mathbb{Z}$, even $\dim T_{\mathbb{CP}}$) are theorems

of the axiom list, and $k_{\text{max}} = 60$ is a computed property of the winner. $\square$

3. Statements

a. Padé Identity All-Orders Theorem and Firewall

The [1, 1] Padé identity of the main volume (App. AA) extends to all orders, with an explicit firewall against Yilmaz-type alternatives.

a. Notation. Throughout this subsubsection, $P_{m,m}(u)$ denotes the diagonal $[m, m]$ Padé approximant of e^u , i.e. the unique rational function $N_m(u)/N_m(-u)$ with degree- m numerator and denominator whose Taylor series matches e^u through order u^{2m} . By construction $P_{m,m}(u) \approx e^u$, hence $[P_{m,m}(u)]^2 \approx e^{2u}$.

Theorem AH.10 (Padé Hierarchy with Closed-Form Coefficients). *$[P_{m,m}(u)]^2 = \exp(2u) + O(u^{2m+1})$ with explicit Taylor coefficients of the numerator polynomial*

$$N_m(u) = \sum_{k=0}^{2m} \binom{m}{k} \frac{(2m-k)!}{(2m)!} u^k.$$

The first divergence at $O(u^3)$ has coefficient $\Delta = 3/2 - 4/3 = 1/6$.

Proof. The diagonal Padé approximant has the standard form $P_{m,m}(u) = N_m(u)/N_m(-u)$ with closed-form numerator $N_m(u) = \sum_{k=0}^m \binom{m}{k} \frac{(2m-k)!}{(2m)!} u^k$ (Bender–Orszag). Squaring: $[P_{m,m}(u)]^2 = N_m(u)^2/N_m(-u)^2$. Taylor-expand against $\exp(2u) = \sum_n (2u)^n/n!$. By construction $P_{m,m}(u)$ matches e^u through order u^{2m} , so the squared difference vanishes to order u^{2m} and the leading nonvanishing term is at u^{2m+1} . For $m=1$: $N_1(u) = 1 + u/2$, hence $P_{1,1}(u) = (1 + u/2)/(1 - u/2)$ and $[P_{1,1}(u)]^2 = (1 + u/2)^2/(1 - u/2)^2$. Expand: $(1 + u/2)^2 = 1 + u + u^2/4$; $(1 - u/2)^{-2} = 1 + u + 3u^2/4 + u^3/2 + \dots$. Product: $1 + 2u + 2u^2 + 3u^3/2 + \dots$. Compare $\exp(2u) = 1 + 2u + 2u^2 + 4u^3/3 + 2u^4/3 + \dots$. Agreement through u^2 ($= u^{2m}$ for $m=1$, as the general argument above requires); first deviation at u^3 ($= u^{2m+1}$) with coefficient $\Delta = 3/2 - 4/3 = 1/6$, in agreement with the theorem statement and with the direct series comparison of App. AA (which lists the u^3 coefficients $3/2$ vs. $4/3$ explicitly). The derivation in App. AA tracks the deviation through the difference $\exp(2u) - [P_{m,m}(u)]^2$ to all orders, with leading constant $\Delta = 1/6$, a property of the two functions. $\square$

Theorem AH.11 ($m \rightarrow \infty$ Entire-Function Limit). *$\lim_{m \rightarrow \infty} [P_{m,m}(u)]^2 = \exp(2u)$ uniformly on compact sets in $\mathbb{C}$.*

Proof. Saff and Varga (1977) prove that diagonal Padé approximants of $\exp(u)$ converge uniformly on compact sets in $\mathbb{C}$. Squaring is continuous, so the squared sequence inherits uniform convergence. By Cauchy–Goursat, the limit is the unique entire function matching $\exp(2u)$ at all derivatives at $u=0$. $\square$

Theorem AH.12 (Firewall vs. Yilmaz). *The flat- $\mathbb{R}^3$ DFD geometry has no throat. The DFD field $u_\psi(r) = 2GM/(c^2r) \geq 0$ from the Lyapunov function $W(X) = X - 2\sqrt{X} + 2\ln(1 + \sqrt{X})$ (main volume, App. GR), so no exotic-matter signature appears. Yilmaz solutions arise within GR as wormhole metrics requiring exotic stress-energy.*

Proof. At leading order DFD's exterior solution is the unique solution of the elliptic equation $\nabla^2\psi = 0$ on $\mathbb{R}^3 \setminus \{0\}$ with $\psi \rightarrow 0$ at infinity and $\int \nabla\psi \cdot d\mathbf{S} = -8\pi GM/c^2$. This gives $\psi = 2GM/(c^2r)$, defined and finite on all of $\mathbb{R}^3 \setminus \{0\}$, with no double cover; with the parent strain's σ kept the full exterior is the Schwarzschild metric in DFD's coordinates, sourced by ordinary matter (App. WS). The Lyapunov function $W(X) = X - 2\sqrt{X} + 2\ln(1 + \sqrt{X}) \geq 0$ for all $X \geq 0$ ($W(0) = 0$, $W'(X) = \sqrt{X}/(1 + \sqrt{X}) \geq 0$) confirms positivity of the trapping function. The Yilmaz exponential metric, by contrast, when embedded in 4D GR, requires exotic stress-energy (negative T_{tt} in some patch) and admits a wormhole throat connecting two asymptotically flat regions. The flat- $\mathbb{R}^3$ background of DFD has no second region to connect to; topologically the geometry is $\mathbb{R}^4 \setminus \{r = 0\}$, simply connected, no throat. Detailed energy-condition analysis (App. AA) confirms no NEC violation in DFD. $\square$

b. Microsector Finiteness Theorems

Theorem AH.13 (Microsector Hilbert-Space Finiteness). $\dim \mathcal{H}_{\text{micro}} = \chi(\mathbb{CP}^2, E) = 60$, computed by Hirzebruch–Riemann–Roch on $E = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$.

Proof. The Hirzebruch–Riemann–Roch formula gives $\chi(\mathbb{CP}^2, \mathcal{F}) = \int_{\mathbb{CP}^2} \text{ch}(\mathcal{F}) \cdot \text{Td}(\mathbb{CP}^2)$. For the rank-1 line bundle $\mathcal{O}(d)$: $\text{ch} = e^{dh}$ where h is the hyperplane class, $\text{Td}(\mathbb{CP}^2) = (1 + 3h/2 + h^2)$, giving $\chi(\mathbb{CP}^2, \mathcal{O}(d)) = (d+1)(d+2)/2$. For $d = 9$: $\chi = 55$. The trivial bundle gives $\chi(\mathcal{O}^{\oplus 5}) = 5$. Sum: $\chi = 55 + 5 = 60$. Toeplitz quantization gives a finite-rank Hilbert space, not a regulated infinite one. $\square$

Theorem AH.14 ($k_{\text{max}} = 60$ Index Computation at the Derived Flux). $k_{\text{max}} = 60$ is the Atiyah–Singer index of the canonical Spin^c structure evaluated at the derived $q_1 = 3$ (main volume, App. F, the remark on the flux quantum).

Proof. The isotropy charge $6Y$ fixes $q_1 = 3$: a multiplet of hypercharge Y lives in a homogeneous bundle of slope $3Y$, and $q_1 = 6$ would give the weak doublets an even charge (main volume, App. F, the remark on the flux quantum). The corresponding Spin^c Dirac index on $\mathbb{CP}^2$ is $\text{ind}(D_E) = \chi(\mathbb{CP}^2, E)$ by Atiyah–Singer; with $E = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ (the a_4 Seeley–DeWitt minimization argument for this choice is asserted, not proven; the chain is that of App. F's independence remark, with $q_1 = 3$ derived

and $N_{\text{gen}} = 3$, $n = 5$ inputs), this index is 60 as in Theorem AH.13. The same integer is also reached as $|A_5|$, as $|E_8 \text{ roots}|/4$ and as the shifted level of $SU(2)_{58}$; these routes are stated to give 60, not shown to reduce to one another. $\square$

Theorem AH.15 (Asymptotic Freedom from Frame Stiffness). *The non-Abelian sector β -functions satisfy $\beta_3, \beta_2 < 0$, as in the Standard Model; the reading proposed here, not computed, attributes this to positive Ricci of the projectivised mode-bundle base $P(V_r) = \mathbb{CP}^{n_r-1}$ via frame stiffness $\kappa_r = n_r \kappa_0$.*

Proof. The microsector Berry connection on $P(V_r) = \mathbb{CP}^{n_r-1}$ — the projectivisation of the rank- n_r block $V_r \subset \mathcal{H}_{\text{int}}$ of the internal mode bundle; this projective space is the colour/weak fibre and is *not* the internal factor M_c — has frame stiffness $\kappa_r = n_r \kappa_0$, where κ_0 is the Fubini–Study scalar curvature. Positive Ricci gives a positive-definite quadratic form on gauge fluctuations, contributing $-c_r \log(\Lambda/\mu)$ to the running gauge coupling. For $n_r = 3$ (color $SU(3)$) and $n_r = 2$ (weak $SU(2)$), the contributions are read as exceeding the matter loops (not computed), giving $\beta_3, \beta_2 < 0$. For $n_r = 1$ (hypercharge $U(1)$), matter loops dominate, giving $\beta_1 > 0$ but with a topological cap at E_{UV} where the running freezes (no Landau pole, see Theorem AH.16). $\square$

Theorem AH.16 (No Landau Pole, No Trans-Planckian Regime). $\alpha(E) \rightarrow \alpha_{UV}$ at E_{UV} . *The running stops at the topological cutoff scale because the Hilbert-space dimension caps the flow.*

Proof. The renormalization-group flow involves integrating out modes with $|\mathbf{p}| < E$. Once E exceeds $\Lambda_{\text{top}} = \sqrt{\alpha} M_P$, all 60 microsector modes are in the spectrum and no further modes can be integrated out (since $\dim \mathcal{H}_{\text{micro}} = 60$ is finite). The running freezes at $\alpha(E_{UV}) = \alpha_{UV}$. By Theorem AH.13, this is finite. By the Abelian beta function with $\beta_1 > 0$ but capped flow, no Landau pole exists. $\square$

c. Higgs Hierarchy

Theorem AH.17 (Higgs Vacuum Expectation Value: the Exponent). $v = M_P \alpha^8 \sqrt{2\pi} = 246.09 \text{ GeV}$ (0.05% to PDG), with the exponent 8 derived by dimension count ($8 = \dim(\mathbb{CP}^2 \times S^3) + 1 = 7 + 1$), the prefactor $\sqrt{2\pi}$ fitted (main volume, App. Z), and the loop-counting $(\alpha^2)^4$ and twist-bundle Chern routes recorded as consistency restatements (not independent confirmations, per App. AY).

Proof. Primary route (dimension counting). $\dim X + 1 = 7 + 1 = 8$, where $\dim X = 7$ is Theorem AH.9 ($\dim \mathbb{CP}^2 = 4$ plus $\dim S^3 = 3$) and the +1 is the singlet/radial direction normal to the gauge bundle. This is the *single* derivation that fixes the exponent. **Consistency restatement A (loop counting).** Each gauge-coupling 2-loop factor contributes α^2 ; the

four microsector factors (gauge-Higgs, Higgs-Yukawa, gauge-Yukawa cross, self-Higgs) give $(\alpha^2)^4 = \alpha^8$. **Consistency restatement B (twist-bundle Chern).** The bundle $E = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ has total Chern class $c(E) = (1 + 9h)(1)^5 = 1 + 9h$, with $\int_{\mathbb{C}P^2} c_1(E)^2 = 81$, giving Chern character contribution $\propto 9 + (-1) = 8$ via the Atiyah–Singer correction. **Not independent.** All three agree on 8, but the loop-counting and Chern routes are *consistency restatements*, not independent confirmations: App. AY records the cross-sector lattice identities of this type as tautological (equivalent restatements of a single α -exponent, carrying no information beyond it). The dimension count alone is the derivation. **Numerics.** $v = 1.220890 \times 10^{19} \text{ GeV} \times (1/137.036)^8 \times \sqrt{2\pi} = 246.09 \text{ GeV}$, vs. PDG 246.22 GeV, error 0.05%. $\square$

Remark AH.18 (The Planck-scale boundary). Imposed at M_P , $\lambda_H = 1/8$ runs to $\lambda_H(v) = 0.174$ under two-loop Standard-Model running and gives $m_H = 150 \pm 2 \text{ GeV}$, far above the measured value, so the Planck scale is not the scale at which the conjecture holds. The main volume states $\lambda_H = 1/8$ as a conjecture at the electroweak matching scale (App. Z): imposed at the top-quark mass it gives $m_H = 124.63 \pm 0.15 \text{ GeV}$, 0.40% below the measured $125.13 \pm 0.11 \text{ GeV}$ (PDG 2026; computed), and $\lambda = 1/8$ holds exactly at $182 \pm 4 \text{ GeV}$.

d. $c_T = c$ Structural Theorem

Theorem AH.19 (Parent-Tensor Decomposition). ψ (trace) and h_{ij}^{TT} (TT) are the two $O(3)$ -irreducible components of the same zero-mode metric perturbation on $K = \mathbb{C}P^2 \times S^3$.

Proof. The zero-mode tensor parent on K decomposes under $\text{Iso}(K) \supset SO(3)$ via Lichnerowicz analysis. Koiso (1980) classified scalar zero-modes on Kähler manifolds: only $\chi(\mathbb{C}P^2) = 3$ contributes, of which the trace and squashing are the two components. Higuchi (1987) classified S^3 zero-modes: the trace lifts to S^3 trivially. Combined: K admits a single 4D scalar ψ (trace) and a single TT-spin-2 mode h_{ij} . On the truncated microsector there is no size or shape modulus (main volume, App. O). One-form components have $b_1(K) = 0$ on $\mathbb{C}P^2 \times S^3$ and are excluded. A further harmonic channel is the three-form: by Künneth $b_3(K) = b_0(\mathbb{C}P^2) b_3(S^3) = 1$, the bi-invariant Cartan three-form on $S^3 = SU(2)$, which is *not* a metric mode and so does not affect $c_T = c$ (Theorem AH.20); its 4D reduction on the three-cycle is the pseudoscalar matter field χ (the derived cold dark matter; App. AV). $\square$

Theorem AH.20 ($c_T = c$ from $O(3)$ Block-Diagonality). $O(3)$ block-diagonality of the principal symbol forbids $(\partial\psi)(\partial h^{\text{TT}})$ derivative mixing. Light and gravitational waves share one metric (App. WS).

Proof. The propagator matrix $\mathcal{M}(p)$ of $(\psi, h_{ij}^{\text{TT}})$ in DFD is $O(3)$ -covariant: ψ is a scalar, h^{TT} is spin-2 with helicity

± 2 . By Schur’s lemma, no $O(3)$ -covariant matrix can mix scalar and spin-2 components; the propagator is block-diagonal. On the background the tensor block satisfies $(\partial_t^2 - c^2 \nabla^2) h_{ij}^{\text{TT}} = 0$, giving phase velocity c ; the scalar is an instantaneous constraint that does not radiate (Axiom WS.1). GW170817 is satisfied by construction. $\square$

Block-diagonality forbids the bilinear mixing only; the tensor cone itself is fixed by local relativity at constant ψ_0 and equals the optical cone $c e^{-\psi}$ (main volume, Sec. V A, Remark V.1; App. WS).

$$e. \quad a_0 = 2\sqrt{\alpha} c H_0 \text{ Derivation Chain}$$

Theorem AH.21 (MOND Scale from S^3 Composition). $a_0 = 2\sqrt{\alpha} c H_0$ from the unique microsector composition law on S^3 (Theorem N.14 of App. N).

Proof. Four-step chain following App. N. **Step 1 (Witten partition exponent).** The S^3 partition function with Toeplitz cap at $k_{\max} = 60$ gives a discrete spectrum $\{\lambda_k\}_{k=1}^{60}$ with leading scale $\lambda_1 \propto \alpha^{1/2}$. **Step 2 (composition law).** Lemma N.6 [App. N]: under S^3 saturation-union, the response composes as $\psi_{12} = \psi_1 + \psi_2$ for nonoverlapping sources, fixing the interpolation function μ to the exponential form $1 - e^{-c\psi}$. **Step 3 (weak-response form).** Theorem N.8: with the weak-response level response $k_{\text{eff}} = k_0(1 + s)$ this becomes $\mu(s) = s/(1 + s)$. **Step 4 (variational stationarity).** The variational principle $\delta\Xi/\delta\psi = 0$ at $\Xi_* = 3/2$ gives $a_0 = 2\sqrt{\alpha} c H_0^{\text{loc}} = 1.1966 \times 10^{-10} \text{ m/s}^2$ on bound-matter clocks (main volume, App. ZS) and 1.1208×10^{-10} on the early clock. Against $a_0 = (1.20 \pm 0.02_{\text{stat}} \pm 0.24_{\text{sys}}) \times 10^{-10}$, the bound-matter value lies within the systematic error; the early-clock value is -3.96σ statistical and -0.33σ systematic. $\square$

H_0 caveat. The $C = 1$, $H_0 = 72.09$ clock is excluded as the early clock and is the bound-matter clock of the two-clock postulate (main volume, App. ZS), so bound systems read $a_0 = 1.1966 \times 10^{-10} \text{ m/s}^2$, consistent with SPARC within its systematic error. Its closed form is $a_0 = 2\alpha^{29} \sqrt{c^7/G\hbar}$, the $C = 1$ value of the clock identity, with no measured coefficient; on the early clock, where $G\hbar H_0^2/c^5 = C\alpha^{57}$ with C measured, the same law gives $a_0 = 2\sqrt{C} \alpha^{29} \sqrt{c^7/G\hbar} = 1.1208 \times 10^{-10} \text{ m/s}^2$. Independently, the prefactor $2\sqrt{\alpha} = 0.171$ differs by 7.3% from the empirical Milgrom coincidence $a_0 \simeq c H_0/2\pi$ (0.159): the order $a_0 \sim c H_0$ is robust and mass-independent (turnaround/virialization), whereas the exact coefficient $2\sqrt{\alpha}$ is the DFD-specific claim, resting on the $\Xi_* = 3/2$ stationarity step (App. N). That this step forces $2\sqrt{\alpha}$, rather than merely admitting it within the generic $\mathcal{O}(1)$ spherical-collapse prefactor band (which also contains $1/2\pi$ and $1/6$), is the key item not yet shown.

f. Topological H_0 and Cosmological Constant

Theorem AH.22 (H_0 from Topological Invariant). $G\hbar H_0^2/c^5 = C\alpha^{57}$. The exponent $57 = k_{\max} - N_{\text{gen}}$ is forced by primed-determinant scaling (Thm. AH.23); the dimensionless coefficient C is not derivable from the printed axioms and is measured, $C = 0.87784 \pm 0.00266$ (BAO+ θ_* ; plik-lite 0.8772), and the cross-channel early-clock value is $H_0^{\text{early}} = 67.52 \pm 0.09$ km/s/Mpc; bound-matter clocks read $C = 1$ under the two-clock postulate (main volume, App. ZS).

Proof. By Theorem AH.23 below, $\det'(g\Delta) = g^{k_{\max} - N_{\text{gen}}} \det' \Delta = g^{57} \det' \Delta$ on the dynamical microsector with $g = \alpha$. The dimensionless invariant constructed from $G, \hbar, H_0, c$ with weight zero in mass and length is uniquely $G\hbar H_0^2/c^5$ (other dimensionless products with these constants miss by ≥ 17 decades). Equating to the determinant ratio fixes the exponent: $G\hbar H_0^2/c^5 = C\alpha^{57}$, with the dimensionless C left undetermined by this argument. Solving, $H_0 = \sqrt{C}\alpha^{28.5}/t_P$, where $t_P = \sqrt{G\hbar/c^5}$ is the Planck time. At the measured $C = 0.87784 \pm 0.00266$ (the clock-credit remark, App. AP of the main volume) this gives $H_0^{\text{early}} = 67.54 \pm 0.10$ km/s/Mpc; the adopted cross-channel value is 67.52 ± 0.09 . A $C = 1$ reading returns $H_0 = 72.0902$; that reading is excluded at $\sim 46\sigma$ as the early clock by the CMB-anchored channels; on bound-matter clocks it is the local value $H_0^{\text{loc}} = 72.09$ of the two-clock postulate (main volume, App. ZS). $\square$

Theorem AH.23 (Cosmological Constant via Determinant Scaling). $57 = k_{\max} - N_{\text{gen}} = 60 - 3$ is forced by primed-determinant scaling (Lemma O.1) via per-mode α -cancellation (Lemma O.5).

Proof. Lemma O.1 [App. O]: $\det'(g\Delta) = g^{n_{\text{nz}}} \det' \Delta$ where $n_{\text{nz}} = \dim \ker^\perp \Delta = \dim \mathcal{H}_{\text{micro}} - \dim \ker \Delta$. Lemma O.5 [App. O]: per-mode α -cancellation eliminates eigenvalue dependence; only the count n_{nz} enters. By Theorem AH.13 $\dim \mathcal{H}_{\text{micro}} = 60$, and by chiral generation counting $\dim \ker \Delta = N_{\text{gen}} = 3$. Hence $n_{\text{nz}} = 57$. The current dimensional invariant is $C\alpha^{57} = 1.393 \times 10^{-122}$; the determinant proves the exponent, while the residual coefficient $C = 0.87784 \pm 0.00266$ is measured. The equality $1.586 \times 10^{-122} = \alpha^{57}$ is the $C = 1$ value, excluded as the early clock and held by bound-matter clocks under the two-clock postulate (main volume, App. ZS). $\square$

g. Penrose Superposition Paradox Dissolution

Theorem AH.24 (Penrose Paradox Dissolution). In DFD the optical metric $n = e^\psi$ is sourced by the c-number

expectation $\langle \Psi | \hat{\rho} | \Psi \rangle$, and the μ -monotone elliptic PDE admits a unique weak ψ for any L_{loc}^2 source. There is no manifold branching.

Proof. DFD's field equation $\nabla \cdot [\mu(|\nabla \psi|/a_*) \nabla \psi] = -(8\pi G/c^2)\rho$ is a strictly μ -monotone elliptic PDE (App. U). For a quantum source state $|\Psi\rangle$, the source term is the c-number expectation $\rho \rightarrow \langle \Psi | \hat{\rho} | \Psi \rangle = \sum_i p_i \rho_i$ (probability-weighted sum over branches). By the Browder–Minty theorem applied to the strictly monotone operator, the elliptic equation has a unique weak solution $\psi \in H^1$ for any L_{loc}^2 source. Hence there is a single ψ -field, not a branched superposition of ψ -fields per Penrose-paradox premise (P1). Penrose's premise that “each branch sources its own metric” fails: in DFD the metric is sourced by the *weighted* probability density, just like any classical field sourced by a quantum charge density. The paradox does not arise. $\square$

Theorem AH.25 (Quantum-Operator $\hat{\psi}$). In the hyperbolic relaxation form of its field equation, $\hat{\psi}$ admits canonical commutator $[\hat{\psi}(\mathbf{x}), \hat{\pi}_\psi(\mathbf{y})] = i\hbar \delta^3(\mathbf{x} - \mathbf{y})$ and is a flat-background scalar QFT (dimensionless field with the Planck-scale stiffness $c^4/32\pi G$ in front of its kinetic term, no curvature counterterms). Under Axiom E of the main volume the local sector has no temporal kinetic term, so there $\hat{\psi}$ has no independent canonical momentum and is fixed by its source.

Proof. The DFD scalar action $S_\psi = \int dt d^3x (a_0^2/8\pi G) W(|\nabla \psi|^2/a_*^2)$ of the main volume (no potential term in ψ ; Theorem AH.136, sub-(i)) on flat $\mathbb{R}^{3,1}$ has dimensionless ψ , whose kinetic term in physical units is $(c^4/32\pi G)(\partial\psi)^2 = (\bar{M}_P^2/4)(\partial\psi)^2$: it is not marginal (for a dimensionless field that needs $z = 3$ anisotropic scaling, which DFD does not have), and $\phi = \bar{M}_P \psi / \sqrt{2} = \psi / \sqrt{16\pi G}$ is an ordinary canonical scalar (the decay constant $F = \bar{M}_P / \sqrt{2}$ of the main volume, App. FK). Standard canonical quantization of the relaxation form on a Cauchy slice gives the displayed CCR. Power counting in ϕ : the kinetic term $(\partial\phi)^2$ has dimension 4; a potential term $V(\psi)$, absent in DFD, would also be dimension 4; interactions are non-renormalizable in the standard sense. The Hilbert space is $\mathcal{F}^{\text{DFD}} = \mathcal{F}^\psi \otimes \mathcal{H}_{\text{micro}}$ with $\dim \mathcal{H}_{\text{micro}} = 60$, finite. The two descriptions (flat-scalar QFT and microsector-augmented) agree to $O(\ell_P/L)$ on observables. $\square$

h. Fine-Structure Constant Closed-Form Identity

Theorem AH.26 (α^{-1} Closed-Form).

$$\alpha^{-1} = \frac{\pi^{3/2}}{24} \cdot \text{Tr}(Y^2) \cdot k_{\max} \cdot \frac{k_{\max}+3}{k_{\max}+4} \cdot \left[1 + \frac{7}{80 \cdot 4095}\right] = 137.03599985, \quad \frac{7}{80 \cdot 4095} = \frac{1}{13 k_{\max}^2} \text{ exactly}$$

with residual -0.005 ppm vs. CODATA 2022. The closed form is exact under two stated choices (main volume, the convention-locked closed form for α), and the bracket's coefficient $1/13$ is a stated rational whose provenance is open.

Proof. The prefactor $\pi^{3/2}/24$ is the heat-kernel normalization of the exact construction: the $\zeta(0)$ of the $\mathbb{CP}^2$ charge trace, $-1/24$ per unit charge squared, times the terminating heat trace $(\sqrt{\pi}/4)\Lambda^3$ of the ρ -shifted Casimir on $S^3 = SU(2)$, with the heat-kernel regulator. The factor $k(k+3)/(k+4)$ at $k = k_{\max} = 60$ (Theorem AH.14) combines the exact $\mathbb{CP}^2$ charge-trace ratio $(k+3)/k$ with the Toeplitz dimension $k+4$. The hypercharge trace is $\text{Tr}(Y^2) = 10$, three generations of $10/3$; its identification with $1/e^2$ rather than with the hypercharge coupling is not derived (main volume, the convention-locked closed form for α). The bracket $[1 + 7/(80 \cdot 4095)] = 1 + 1/(13k_{\max}^2)$ has a stated rational coefficient. Numerical evaluation gives $\alpha^{-1} = 137.03599985$, with residual -0.005 ppm vs. CODATA 2022 $137.035999177(21)$; the integers alone give 137.03307 . $\square$

i. Charged Fermion Mass Formula Tightening

Theorem AH.27 (Mass Formula Decomposition). $m_f = A_f \alpha^{n_f} v/\sqrt{2}$, with sector exponents $n_f \in \{0, 1.0, 1.5, 2.5\}$ derived from Cayley-graph hop distance, and prefactors A_f traced to four irreducible blocks: K_d, K_u ($\mathbb{CP}^2$ kernels); Q_d (QCD); D_ℓ (Dirac); G (generation).

Proof. The Yukawa operator factorizes as $Y_f = A_f h_f^{\text{dim } X+1}$ via the Berry-bundle overlap integral on $\mathbb{CP}^2$. The sector exponent n_f counts the hop distance on the Cayley graph from the Higgs vertex to the fermion vertex: $n = 0$ for top and bottom, $n = 1$ for charm and tau, $n = 1.5$ for strange and muon, $n = 2.5$ for up, down and electron (main volume, App. FM). The prefactor A_f decomposes as

$$A_f = K_f \cdot Q_f \cdot D_f \cdot G_f,$$

where each block is fixed by symmetry: $K_d = J_3$, $K_u = I_4$ from $S_3/O(4)$ symmetry on the kernel space, with democratic coupling for K_d (Lemma K.2/K.3); $Q_d = \text{diag}(1, 6/7, 1/42)$ from the QCD anomaly coefficient $b_0 = 7$; $D_\ell = \text{diag}(1, 1, \sqrt{2})$ from Dirac normalization on the lepton sector; $G = \text{diag}(2/3, 1, 1)$ with $G[1, 1] = 2/3$ from primed microsector trace (Theorem K.4). All 9 charged fermion masses are computed at this stage with mean error 1.42% and maximum error below 4%; the full per-fermion table is supplied via the unified Python reproducibility script which evaluates all 34 Standard-Model continuous parameters at 50-digit precision (the running factors of Theorem AH.200 are not Standard-Model running, so no sharper mean error is claimed). *Scope caveat:* the α -power exponent ladder n_f and the equal-exponent ratio structure are derived; the absolute charged-fermion spectrum below the top, however, additionally imports seven data-selected prefactors, one for each species other than the top and the charm (main volume, App. FK and App. FM). The reproducibility script therefore reproduces the 34 values given those selected/fitted inputs; the quoted mean errors (1.42% and 0.61%; the 0.082% of Theorem AH.200 does not stand) measure the fit quality of that structure, not a from-first-principles derivation of all 34 parameters. $\square$

j. Neutrino Sector: Normal Ordering and $\delta_{CP} = -\pi/2$

Theorem AH.28 (Mass Ordering and CP Phase). On the adopted neutrino branch, normal ordering is forced. $\delta_{CP} = -\pi/2$ from the μ - τ reflection, a structural postulate (main volume, App. FK), with the sign set by the Fubini-Study orientation.

Proof. The DFD neutrino mass matrix derived from micro-sector (App. X) yields branch-B exponents $k = \alpha^{-3/11}$, $r = \alpha^{-7/20}$ with absolute scale $m_3 = (14/13)\pi M_P \alpha^{14}$. Both $k, r > 1$, so the mass ratios $m_2/m_1 = k$, $m_3/m_2 = r$ enforce $m_1 < m_2 < m_3$ (normal). Inverted ordering $m_3 < m_1 \approx m_2$ is structurally impossible. The CP phase δ_{CP} is the imaginary part of the PMNS Jarlskog invariant, computed from the complex structure on $\mathbb{CP}^2$ (Fubini-Study Kähler form ω_{FS}). The phase is fixed by the μ - τ (S_2) reflection symmetry $ShS = h^*$ acting on the neutrino mass matrix: this antiunitary symmetry forces $\theta_{23} = \pi/4$ and $|\sin \delta_{CP}| = 1$ simultaneously at all orders, and the Fubini-Study orientation selects the sign, giving $\delta_{CP} = -\pi/2$. Evaluated at the DFD-locked mixing angles ($\sin^2 \theta_{13} = 3\alpha$, $\theta_{23} = \pi/4$, trimaximal $\sin^2 \theta_{12} = 0.3184$, from $|U_{e1}|^2 = 2/3$) with $\sin \delta_{CP} = -1$, the leptonic Jarlskog invariant is $J_{PMNS} = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin \delta_{CP} = -(\sqrt{2}/6)\sqrt{3\alpha}\sqrt{1-9\alpha} = -0.0337$ ($|J_{PMNS}| = 0.03371$, as in the main volume, App. AY); the maximal-angle prefactor $1/8$ is not the physical invariant. This is consistent with current T2K/NOvA hints. Combined predictions: $\Delta m_{21}^2 = 7.48 \times 10^{-5} \text{ eV}^2$, $\Delta m_{31}^2 = 2.51 \times 10^{-3} \text{ eV}^2$, NuFIT 6.0 $\chi^2 = 0.025$, $p = 0.99$. $\square$

4. New Theoretical Territories

a. Topological Pre-Inflation

Theorem AH.29 (Lichnerowicz Rigidity Excludes Slow-Roll Inflaton). On $\mathbb{CP}^2 \times S^3$ the Lichnerowicz Laplacian leaves no slow-roll scalar: on the truncated microsector there is no size or shape modulus, and on a continuous internal space the only scalar moduli are the overall volume and the squashing, whose curvature potential has no slow-roll direction.

Proof. The Lichnerowicz Laplacian on a compact Einstein manifold acts on transverse-traceless symmetric 2-tensors. On $\mathbb{CP}^2$ (Koiso 1980) the only zero modes are conformal Killing forms; squashing of the Kähler–Einstein metric is the unique non-trivial zero mode. On S^3 (Higuchi 1987) the zero modes are the Killing forms; $b_1(S^3) = 0$. Combining via $K = \mathbb{CP}^2 \times S^3$ with $b_1(K) = 0$, the only scalar moduli of a continuous K are its overall volume and the squashing modulus: the shape sits at $\tau_* = 1/\sqrt{3}$, a minimum of the curvature potential for the connection Laplacian, and the volume has no critical point at this order, a runaway rather than a slow roll. The Kähler radius is a modulus of K , not the field ψ , whose homogeneous mode is set by the Friedmann law (main volume). On the truncated microsector none of these moduli exists. No slow-roll-compatible scalar exists. $\square$

Remark AH.30 (Status note on TPI). The Topological Pre-Inflation construction is **refuted as a DFD theorem**. The $\{1, 3, 12, 30, 60\}$ staircase appearing in the construction does not appear in the main volume; the canonical cohomological catalog is $\{3, 7, 8, 13, 19, 31, 49, 57, 60, 108, 137\}$. Lichnerowicz rigidity (above) excludes graviton-mode inflaton candidates but *does not* exclude an external scalar inflaton field added to DFD. Independent recomputation finds n_s indeterminate (closest natural prescription = scale-invariant 1.000, ruled out at $> 8\sigma$), r indeterminate. **TPI is a conjectured extension beyond DFD’s stated scope**, not a derived prediction.

Theorem AH.31 (TPI Predictions, conditional on the construction’s premises). *Conditional on the staircase construction, the predicted observables are: $n_s = 0.964 \pm 0.003$, $A_s = (2.10 \pm 0.08) \times 10^{-9}$, $r = (4.0 \pm 1.5) \times 10^{-3}$, $T_{\text{RH}} \approx 5 \times 10^{12} \text{ GeV}$.*

Proof. The construction posits a discrete sequence of effective dimensions $\{1, 3, 12, 30, 60\}$ each contributing $\Delta N_e \approx 12$ e-folds via vacuum-energy-driven expansion. The spectral tilt is determined by the rate of staircase climbing: at each step, $H \propto \alpha^{-1/2}$ scales, and the slow-roll parameter $\epsilon \sim 1/N_e^2$ gives $n_s = 1 - 2\epsilon - 4\eta = 0.964$ at $N_e = 60$. A_s from the COBE normalization at the staircase’s Hubble rate $H \sim \sqrt{\alpha} M_P$. $r = 16\epsilon \approx 4 \times 10^{-3}$. T_{RH} from $\Gamma_{\text{inf}} = M_P \alpha^6 \Rightarrow T_{\text{RH}} = M_P \alpha^3 \approx 5 \times 10^{12} \text{ GeV}$. Note: this is conditional on the staircase premise, which is challenged by the status note above. Its $r \approx 4 \times 10^{-3}$ also needs a freeze-out at $H \approx 1.6 \times 10^{13} \text{ GeV}$, which erases the χ relic (Theorem AH.40), so the construction is incompatible with DFD’s dark sector. $\square$

b. Baryogenesis from S^3 Topology

Theorem AH.32 (Shared- π_3 Double Duty). $\pi_3(S^3) = \mathbb{Z}$ of the internal winding is the same group as $\pi_3(SU(2)) = \mathbb{Z}$ for sphaleron winding.

Proof. $S^3 = SU(2)$ as a topological space and Lie group. The third homotopy group $\pi_3(S^3) = \mathbb{Z}$ (Hopf invariant) is therefore identical to $\pi_3(SU(2)) = \mathbb{Z}$ (winding number on the $SU(2)$ gauge bundle). The same group $\mathbb{Z}$, on two different three-spheres (main volume, App. F), classifies both S^3 -twists in the DFD microsector (Theorem F.26) and sphaleron transitions in electroweak gauge theory (Manton 1983). $\square$

Thermal leptogenesis does **not** give η_B in DFD: for the minimal DFD lepton sector the Dirac and Majorana neutrino kernels are real and determinant-protected, so the standard thermal-leptogenesis source

$$\varepsilon_i \propto \sum_{j \neq i} \text{Im}[(Y_\nu^\dagger Y_\nu)_{ij}^2] f(M_j^2/M_i^2)$$

vanishes identically: $Y_\nu^\dagger Y_\nu \in \mathbb{R} \Rightarrow \text{Im}[(Y_\nu^\dagger Y_\nu)_{ij}^2] = 0 \Rightarrow \varepsilon_i = 0$, hence $\eta_B^{\text{minimal DFD}} = 0$. This was verified two independent ways: the unflavored sum $\text{Im Tr}[(Y_\nu^\dagger Y_\nu)^2] = 0$ (trace of a Hermitian square), and the resonant regulator $M_i^2 - M_j^2 = 0$ for the exactly-degenerate heavy spectrum of Theorem P.4. *The same real-kernel structure that enters the strong-CP result ($\bar{\theta} = 0$ at tree level, loop-bounded, with $\theta_{\text{bare}} = 0$ fixed by the spectral action given that nothing is added to it for the gauge fields) removes the CP-odd invariant required for leptogenesis* — DFD’s strong-CP virtue and its baryogenesis are in structural tension. A nonzero asymmetry would require two further DFD-SD ingredients, neither presently derived: a geometrically forced leptonic determinant-orthogonal Berry offset (its magnitude, not merely its existence and tree-level, loop-bounded strong-CP safety, which follow given the stated premises) and a forced heavy-neutrino splitting ΔM_R (Theorem P.4 fixes only the determinant/geometric-mean eigenvalue, not the individual masses). *This thermal-leptogenesis no-go is not the final word: the internal axial Berry-holonomy channel of Theorem AH.33 below derives the α^4 power of the asymmetry (its bias current is $B-L$, and with the stated carrier it gives about $(2-3) \times 10^{-9}$, three to five times Planck’s value); the discussion in this note establishes only that the thermal source vanishes.* Given the identified Majorana scale (main volume, App. P), what DFD fixes here: $M_R = 4.744 \times 10^{12} \text{ GeV}$ and the strong-washout efficiency $\kappa_f = 4.87 \times 10^{-3}$ (effective mass $\tilde{m}_1 = m_3 = 50.16 \text{ meV}$, $K = 46.9$; numerically-solved Boltzmann) of the strongest of three washout channels, $K = (2.19, 8.38, 46.9)$ for the degenerate heavy spectrum with real kernels (computed) — these set the scale and washout, but not the CP asymmetry, which is the open quantity.

Rolling-phase (spontaneous) route. A third mechanism, distinct from thermal leptogenesis and from heavy-splitting, was examined: a *time-dependent* determinant-orthogonal Majorana Berry phase $M_R \rightarrow M_R e^{i\Theta_L(\psi)}$, which would generate a spontaneous lepton chemical potential $\mu_L = \frac{1}{2} \dot{\Theta}_L$ on the active-lepton current via field redefinition. This route is attractive because it requires *neither* a nonzero ε_i *nor* a heavy splitting. The mechanism

itself is sound: because N_R is Majorana ($\bar{N}_R \gamma^\mu N_R = 0$), rephasing carries the phase through the Dirac Yukawa onto the nonzero current $j_L^\mu = \bar{L} \gamma^\mu L$, giving $\mu_L = \frac{1}{2} \dot{\Theta}_L$ (standard Majoron field redefinition), and the resulting $B-L$ is reprocessed by sphalerons consistently with proton stability (Thm F.26, Step 2). Define the dimensionless Berry-clock invariant $Q_L \equiv \dot{\Theta}_L / (2H)$. Using the locked inputs ($M_R = 4.744 \times 10^{12}$ GeV, $T_L \simeq 2.0 \times 10^{12}$ GeV, $\kappa_f = 4.87 \times 10^{-3}$, $a_{\text{sph}} = 28/79$, $g_* = 106.75$), the conditional asymmetry is

$$\eta_B \simeq 1.08 \times 10^{-9} (Q_L / \alpha),$$

so the observed η_B would require $Q_L = 0.566 \alpha$. *This does not close baryogenesis*, for three structural reasons established under the no-fitting rule (no fitting to η_B). (i) **The forced phase is static, not rolling.** The geometry forces only the *static* determinant-orthogonal Berry offset of the quark prototype (Postulate Y.10, App. AO): one conjugation-odd coordinate over the *fixed* internal space $\mathbb{CP}^2 \times S^3$, with no ψ -argument. A static Θ_L is a pure rephasing ($\mu_L = 0$), recovering the $\varepsilon_i = 0$ no-go above. (ii) **The ψ -promotion is obstructed.** The FRW flat-direction theorem (Thm AE.1, App. AE) renders the homogeneous cosmological $\dot{\psi}$ non-dynamical—the very drive the mechanism needs. (iii) **The slope is free.** The roll-rate $q_L = \frac{1}{2} d\Theta_L / d\psi$ has no selection rule (strictly weaker than the CKM apex Postulate E.1, which at least admits the convex Euler-projection action); any small rational $\{4/7, 13/23, 30/53, \dots\}$ reproduces 0.566α to $\lesssim 1\%$, the signature of a fit. A more sophisticated dressing—identifying $Q_L = \alpha/2$ from a Spin^c Pfaffian-line “holonomy per generation”—fails for an independent reason: the proposed $\frac{1}{2} = (N_{\text{gen}}/2)/N_{\text{gen}}$ is an N_{gen} -independent algebraic identity (it returns $\frac{1}{2}$ for *any* generation count, so the index computation plays no role), the flux-to-rate normalization $\lambda_\psi = \alpha$ is unforced (no such rule exists in the geometry, and the family “flux” it halves is a degree-0 fiber index over the *fixed* internal manifold, not a degree-2 Berry class over ψ -space), and the resulting $\eta_B = 5.4 \times 10^{-10}$ is excluded by the CMB ($6.13 \times 10^{-10} \pm 0.7\%$) at $\sim 17\sigma$. Moreover, taking the clock-tied $\dot{\psi} \sim H(T_L)$ gives $\eta_B \sim 5 \times 10^{-12}$ for $q_L \sim \alpha$ —two orders *too small*; the closure value tacitly assumes $\dot{\psi}/T$ about $100\times$ larger than H/T . *DFD reduces baryogenesis to one determinant-line Berry-clock invariant Q_L ; the geometry forces it static (no asymmetry), the rolling spacetime- ψ promotion is cosmologically obstructed and the slope is unforced, and the estimate undershoots by two orders. The spacetime- ψ rolling route therefore does not close baryogenesis; the internal axial Berry-holonomy route of Theorem AH.33 below evades all three obstructions and derives the α^4 power of $|\eta_B|$ and its $B-L$ bias; with the stated carrier it gives η_B of about $(2-3) \times 10^{-9}$ when every step is pinned to the print, three to five times Planck’s value, and its sign rests on an adopted axiom (Theorem AH.33).*

We do *not* adopt a baryogenesis branch axiom. Generating $\varepsilon_1 \neq 0$ requires injecting a CP-odd kernel defor-

mation that DFD’s real-kernel/determinant-protection structure *forbids* — the same structure that keeps $\arg \det(M_u M_d) = 0$ — and the resulting η_B would be a fit, not a prediction. That is non-DFD content (a CP source the geometry rules out), not an extension of minimal DFD. The $\varepsilon_1 = 0$ no-go stands.

Internal axial Berry-holonomy channel: the α^4 power. The three obstructions to the rolling-phase route above all attach the clock to the *spacetime* field ψ . Relocating the clock to the *internal* determinant line of the chiral kernel—a connection on the $\mathbb{CP}^2 \times S^3$ fermion bundle, not a $\nabla\psi$ scalar—removes (ii) and (iii) and reduces (i) to a single named insertion. This is *not* the forbidden $\varepsilon_1 \neq 0$ branch axiom (no CP-odd kernel is injected; the real-kernel/ $\bar{\theta} = 0$ protection is untouched); the asymmetry is a CPT-violating chemical-potential bias. We record the resulting theorem.

Theorem AH.33 (Internal axial Berry-holonomy baryogenesis: α^4 power and $B-L$ bias derived). *Let the determinant line of the chiral fermion kernel carry, on the retarded/expanding branch, the flat internal connection*

$$A_5 = Q d \ln A, \quad Q = \frac{\alpha}{\sqrt{\pi}},$$

so that in absolute time its temporal holonomy is a $(B-L)$ chemical potential $\mu_{B-L} = A_{50} = QH$. Then, with the locked seesaw scale $M_R = M_P \alpha^3$ and the conventional freeze-out point $T_L = M_R / \sqrt{2\pi}$ (a convention, not the freeze-out of a computed process; proof, step (4)), the sphaleron-reprocessed entropy-normalized asymmetry is

$$Y_B \equiv \frac{n_B}{s} = a_{\text{sph}} Y_{B-L},$$

$$Y_{B-L} = \frac{15 g_L}{4\pi^2 g_*} \left(\frac{H}{T} \right)_{T_L} Q, \quad Y_B = 0.0292 \alpha^4,$$

and the photon-normalized baryon-to-photon ratio is obtained by the entropy-to-photon conversion $s/n_\gamma = 7.04$,

$$\eta_B^{\text{base}} = \frac{s}{n_\gamma} Y_B = 7.04 a_{\text{sph}} \frac{15 g_L}{4\pi^2 g_*} \left(\frac{H}{T} \right)_{T_L} Q$$

$$= 0.2057 \alpha^4 \simeq 5.83 \times 10^{-10}$$

[base (deposit-free)],

which the per-e-fold transport deposit completes to

$$\eta_B = 0.2057 \alpha^4 (1 + 2\pi\alpha) = 6.1007 \times 10^{-10}$$

(Thm. AH.36). *These are the values at the conventional freeze-out point with the Gaussian carrier $Q = \alpha/\sqrt{\pi}$, which is a premise the print cannot supply (proof, step (1)). The α^4 power of $|\eta_B|$ is derived, and so is the current that carries the bias, $B-L$ (proof, step (3)). The printed coupling fixes the object the transport turns, the determinant line, and its $B-L$ charge, but not its turn per flux step, which is stated: the stated carrier gives each generation a full $B-L$ turn per flux step; with it, on the computed freeze-out, the mechanism gives η_B of about $(2-3) \times 10^{-9}$ pinned to the print, three to five times Planck’s value (proof, step (4)), so the printed transport overshoots. The sign (matter vs. antimatter) is fixed by the expansion*

orientation of Axiom [AH.35](#), a CP-odd datum carried by A_5 on the determinant line — adopted, not derived (the baryon winding itself carries zero information about $\text{sign}(w_0)$).

Proof. (1) The width: its α^1 power is derived, its normalization is not. The clock is the determinant line of the chiral Dirac kernel—a complex determinant line (a complex line bundle), not a real scalar oscillator. DFD’s determinant-scaling lemma (the same that fixes the exponent of H_0 , $M_R = M_P \alpha^3$, and the Higgs/fermion α -tower) assigns one complex Gaussian mode the partition-function ratio

$$\frac{Z_\alpha}{Z_1} = \frac{\int_{\mathbb{C}} e^{-(\lambda/\alpha)|z|^2} d^2z}{\int_{\mathbb{C}} e^{-\lambda|z|^2} d^2z} = \alpha,$$

i.e. scale parameter α (a standalone *real* scalar would give the half-power $\sqrt{\alpha}$; the complex line gives the full α). The two orientation branches average to zero, $\langle q \rangle_{\pm} = 0$; the retarded branch restricts to the oriented half-line $q \geq 0$, giving the folded one-branch mean

$$\begin{aligned} Q &= \langle q \rangle_+ = \int_0^\infty q \frac{2}{\alpha\sqrt{\pi}} e^{-q^2/\alpha^2} dq \\ &= \frac{\alpha}{\sqrt{\pi}} \left(= \sigma\sqrt{2/\pi} \text{ with } \sigma = \alpha/\sqrt{2} \right). \end{aligned}$$

No datum is fitted, but the normalization is not fixed. Reading α as the absolute scale of this Gaussian is a premise, not a result: the Gaussian ratio fixes the α^1 power and not the coordinate in which the folded mean is taken, because it is blind to the normalization of the coordinate it integrates (the flux-period lemma, part (ii), of App. [AV](#)). Nor can a Gaussian carrier supply the bias: a Gaussian vacuum carries no Berry phase, and a single complex Gaussian mode of ratio α has a normally distributed coordinate of size $\sqrt{\alpha}$, not α (computed). The realizable carrier is the determinant line’s holonomy over one transport period, a whole number of turns, $Q = 2\pi\alpha n_w / |c_X|$, with n_w the line’s windings per e -fold and c_X its $B-L$ charge (step (4)).

(2) *The internal holonomy escapes the ψ -clock subtraction.* A_5 is a connection on the internal $\mathbb{CP}^2 \times S^3$ fermion bundle, not a $\nabla\psi$ scalar. The absolute-time segment-identification (App. Q) and the FRW flat-direction theorem (Thm [AE.1](#), App. AE) quantify only over $\nabla\psi$ /screen-flow data and the ψ -action; they leave an internal axial connection untouched. A chemical potential is a *flat* ($F_{A_5} = 0$) temporal connection whose physical content is the thermal/cosmic Wilson line $\oint A_5 = Q \oint d\ln A$, not its curvature; it is therefore not removed by the static-offset reasoning (obstruction (i)) nor by the ψ non-dynamics (obstruction (ii)). Its drive is the Friedmann H at T_L , fixed by the radiation density, which the flat-direction theorems do not zero.

(3) *The CP-odd datum.* A chemical potential for $B-L$ is CP-odd as well as CPT-odd, and the S^3 flux step is CP-even under the CP of App. L of the main volume, so the only CP-odd datum that can drive a net $B-L$ is

the adopted orientation of the transport (Axiom [AH.35](#)), carried by the connection A_5 on the determinant line, which DFD’s antilinear CP lift conjugates, while the flux step only sets one transport period per e -fold; the sign is adopted, not derived. The current is $B-L$, not the axial current J_5 : a current with an axial quark component is QCD-anomalous and, coupled to the flat, never-ending A_5 , would shift $\bar{\theta}$ by $12Q$ per e -fold, at least 10^9 times the neutron-EDM bound even over the 2.3 e -folds in which lepton number is violated; lepton-only currents are P-odd or bias processes that conserve $B-L$; and the charges conserved by all 22 $B-L$ -conserving interactions are exactly $B-L$ and hypercharge (computed). The real-kernel / even-dimensional η -invariant protection (App. L) forbids CP-odd operators of the θ -term type at tree level ($\bar{\theta} = 0$, with $\theta_{\text{bare}} = 0$ fixed by the spectral action given that nothing is added to it for the gauge fields; $\varepsilon_i = 0$) but not a CPT-violating background, so strong CP is preserved and the $\varepsilon_i = 0$ no-go is irrelevant to this channel.

(4) *Magnitude.* With $g_L = 6$, $g_* = 106.75$, $a_{\text{sph}} = 28/79$, $(H/T) = 1.66\sqrt{g_*} T/M_P$, and $T_L = M_R/\sqrt{2\pi} = M_P \alpha^3 / \sqrt{2\pi}$,

$$\begin{aligned} \eta_B^{\text{base}} &= 7.04 \cdot \frac{28}{79} \cdot \frac{15 \cdot 6}{4\pi^2 \cdot 106.75} \\ &\quad \cdot 1.66\sqrt{106.75} \cdot \frac{\alpha^3}{\sqrt{2\pi}} \cdot \frac{\alpha}{\sqrt{\pi}} \\ &= 0.2057 \alpha^4 = 5.83 \times 10^{-10} \\ &\quad [\text{base (deposit-free)}], \\ \eta_B &= \eta_B^{\text{base}} (1 + 2\pi\alpha) = 6.1007 \times 10^{-10} \\ &\quad (\text{deposit factor: Thm. [AH.36](#)}). \end{aligned}$$

The power $\alpha^4 = \alpha^3[M_R] \times \alpha^1[Q]$ is derived; the $O(1)$ coefficient is not. $T_L = M_R/\sqrt{2\pi}$ ($z = M_R/T = \sqrt{2\pi}$) is a convention, not the freeze-out of any computed process. The degenerate heavy spectrum with real kernels has three washout channels, $K = (2.19, 8.38, 46.9)$; computing the relaxation with them, with finite-rate spectators, the sphaleron conversion at the electroweak crossover and a bias switched on at M_R , multiplies the value above by 0.272 (68%: 0.247–0.306; computed, with the plasma inputs estimated). The print has no switch-on: its bias never ends and runs from the start of the hot plasma, at or above T_L , and with the sphaleron rate of the printed $g_2 = 2/3$ the relaxation from that start is computed below. With the Gaussian carrier $Q = \alpha/\sqrt{\pi}$ this would give $\eta_B = 1.66 \times 10^{-10}$, but that carrier is not realizable (part (1)). The printed coupling fixes the object and its charge. The transport couples only through the internal Dirac operator, as the current vertex $A_{5\mu} J_{B-L}^\mu$ inside the fermion term, which acts per particle and contains no topological term. The line it transports is the determinant line of the chiral kernel. The right-handed neutrino lies outside that kernel, since its Majorana mass is not part of the printed D_F (derived from the print, conditional), so $c_X = N_{\text{gen}} \times (-1) = -3$. The coupling acts on $B-L$ alone and requires only whole turns of the line

per transport period, so how far the line turns per flux step is a stated input, and one turn of the whole line per step would satisfy the coupling as well. The stated carrier gives each generation a full $B-L$ turn per step, $n_w = 3$ at one step per e -fold (Axiom AH.35): with the determinant-line convention $c_X = -3$, whose sign sets the sign of the bias, $|c_X|/n_w = 1$ and $Q = 2\pi\alpha$, the per- e -fold deposit itself, counted once. Pinned to the print, with the bias running from the reheating temperature, this gives η_B of about $(2-3) \times 10^{-9}$, three to five times Planck's value, and no draw of the plasma inputs falls below Planck's value; started at M_R the bias gives 1.9×10^{-9} , and started at T_L , the lowest start the print allows, 1.5×10^{-9} (computed). The sphaleron conversion enters once, and the heavy tower of the microsector has annihilated or decayed long before T_L , so it does not dilute the asymmetry (computed). The printed mechanism therefore overshoots. Agreement would need one kernel winding per e -fold, which the stated carrier does not give. The printed Heisenberg–Weyl clock of the three kernel modes (Theorem AH.144) does not supply one. It gives one kernel winding per flux step only if its labels are read as charges, and the choice of window gives 0, 1 or 2; the printed flux step makes every multiplet flow equally, which the clock's zero spectral flow cannot reproduce; its unequal part biases $B-L$ -conserving flavour processes (both CKM links and six off-diagonal Dirac Yukawas), which part (3) above excludes; and once the three washout channels are resolved its value spreads over $4.5-7.3 \times 10^{-10}$, depending on a generation assignment the print does not make (computed). Thermal leptogenesis gives no asymmetry either, by the real-kernel protection above ($\varepsilon_i = 0$). The baryon asymmetry is therefore carried as one calibrated input, ω_b (App. FK, main volume), and the printed transport's first-principles value of η_B , about $(2-3) \times 10^{-9}$, three to five times Planck's value, is recorded. A right-handed neutrino inside the kernel would give no asymmetry at all. $\square$

Theorem AH.34 (Winding-form uniqueness). *Assume the single transport premise (E-transport): the internal determinant line is transported by the cosmological expansion (its microsector modes are comoving). Then the winding form of Theorem AH.33 is forced:*

$$A_5 = Q d \ln A$$

is the unique 1-form consistent with (i) reference invariance, (ii) segment additivity, and (iii) convention invariance of the scale-factor normalization; in particular $A_{50} = QH$ follows rather than being defined.

Proof. (i) *Reference invariance.* The holonomy must vanish on a static background: an explicit dt component would require a preferred time unit and would wind in a static universe — excluded. Hence $A_5 = G(A) dA$ along the expansion flow. (ii) *Segment additivity* is automatic for 1-forms (the same additivity input as the temporal-segment identification of App. Q, main volume). (iii) *Convention invariance.* The normaliza-

tion of the scale factor is a coordinate convention: under $A \rightarrow \lambda A$ the holonomy must be unchanged for all $\lambda > 0$, so $G(\lambda A) \lambda dA = G(A) dA$, forcing $G \propto 1/A$, i.e. $A_5 \propto d \ln A$; any other power $d(A^n)$ fails the scaling test, and the Cauchy functional equation for a multiplicative-to-additive holonomy map gives the same conclusion. The coefficient Q is set in part (1) of the proof of Theorem AH.33: its α^1 power is derived, and its size rests on the stated turn of the determinant line per flux step. $\square$

Status and caveats (no-fitting rule). (a) *Winding form:* the form $A_5 = Q d \ln A$ is not a bare insertion: it is theorem-grade *conditional on the transport premise* (E-transport) of Theorem AH.34 — the weakest residual premise (Postulate Y.10 as written supplies only a *static* determinant-orthogonal offset; that the line is *transported* by the expansion remains an axiom-level input, not a derived statement). The carrier is $d \ln A$, the e -fold count of the matter metric: the split of the scale factor between the arena factor and $\bar{\psi}$ is pure gauge (main volume, App. ZS), so there is no second carrier and no factor β/p . The freeze-out point moves η_B by a factor of order 3, and with the stated carrier the mechanism gives about $(2-3) \times 10^{-9}$, three to five times Planck's value (proof of Theorem AH.33, step (4)). (b) *Sign:* $\text{sign}(\eta_B) = \text{sign}(Q) \text{sign}(H)$; $H > 0$ is C -even and $\text{sign}(Q)$ is an internal-orientation datum logically independent of the external cosmic-time arrow, so the bare action derives the α^4 power of $|\eta_B|$ but does not orient it. The orientation is supplied by Axiom AH.35 (adopted, priced, not derived); all four candidate derivation routes for that orientation are closed by proof (see the axiom's status note below). Subject to (a)–(b), DFD *derives the α^4 power of the baryon asymmetry*, a quantity the Standard Model misses by ~ 8 orders, and its $B-L$ bias; with the stated carrier, its coefficient overshoots Planck's value, giving about $(2-3) \times 10^{-9}$, three to five times Planck's (proof of Theorem AH.33, step (4)). Because $A_5 = Q d \ln A$ is exact ($F_{A_5} = 0$), the step fires on the local e -fold count, so the asymmetry follows the local expansion and carries no isocurvature, $S_B = 0$ exactly for an adiabatic onset of the radiation era, and the baryons follow any adiabatic primordial perturbation (computed); being locally pure gauge, A_5 acts only through $B-L$ -violating processes.

c. *The transport axiom and ω_b*

Axiom AH.35 (E-transport'). *The E-transport advances the S^3 flux by one lattice step per e -fold, oriented with expansion.*

The S^3 flux advanced here is not the sector label j of the χ relic (main volume, App. AV): advancing that label one step per e -fold would raise $\Omega_\chi h^2$ from 0.1193 to 0.126 after one step and to 0.41 by today (computed). This clause is an *adoption*, not a derivation: we adopt it as one priced discrete bit. All four candidate derivation routes (lattice/degree, spectral, Chern–Weil, index) are

closed *by proof*: the unit rate is non-topological, rate-invisible to every operator-path functional, invisible to the flat printed connection (Thm. AH.34's A_5 carries $F_{A_5} = 0$), and sign-blind under every index functional (spectral flow is odd under loop reversal). Beyond its coefficient (Theorem AH.33), the residue of the ω_b chain is therefore *provably* this one axiom-grade sentence; it is minimal by construction, and it is priced: +1 **discrete bit** in the theory's accounting, which reads *derived core + priced discrete bits (including E-transport') + the four continuous calibrations graded in App. FK of the main volume* (the vacuum coefficient R , the amplitude A_s , the coefficient of $\omega_b h^2$, and the astrophysical τ shared with Λ CDM), and the stated two-clock postulate (main volume, App. ZS), which adds no parameter. With the stated carrier, one flux step per e -fold turns each generation once, and the mechanism gives about $(2-3) \times 10^{-9}$, three to five times Planck's value, when every step is pinned to the print (Theorem AH.33, step (4)). The axiom was adopted *after* the conventional-point selection was known, so it is priced as one discrete bit, never “zero assumptions.” The refusal reading, declining the axiom, leaves the deposit exponent w_0 unfixed, so ω_b keeps its derived α^4 power with its coefficient and its deposit factor unfixed; it moves nothing else.

Theorem AH.36 (ω_b at the conventional point under E-transport'). *Under Axiom AH.35 the deposit is quantized with the stated carrier: the unit rate gives $|w_0| = 1$, and the expansion orientation gives $\text{sign}(w_0) = +1$. The per- e -fold deposit $(1 + w_0 2\pi\alpha)$ is independent of the e -fold count N , so with $w_0 = +1$, at the conventional freeze-out point $T_L = M_R/\sqrt{2\pi}$ with the Gaussian carrier $Q = \alpha/\sqrt{\pi}$,*

$$\eta_B = 0.2057 \alpha^4 (1 + 2\pi\alpha) = 6.1007 \times 10^{-10},$$

$$\omega_b h^2 = 0.0222730,$$

against Planck 2018's 0.02237 ± 0.00015 . At full precision the base coefficient is 0.205705378693 , i.e. $\eta_B = 0.205705378693 \alpha^4 (1 + 2\pi\alpha) = 6.1007 \times 10^{-10}$. This is the conventional-point value that the CMB fits use (App. FK, main volume). It is not the mechanism's prediction: the Gaussian carrier is not realizable, and with the stated carrier the mechanism gives η_B of about $(2-3) \times 10^{-9}$ when every step is pinned to the print, three to five times Planck's value (proof of Theorem AH.33, step (4)).

What the axiom does not force. The axiom forces $w_0 = +1$ only. The identification $N = 60$ — the completion reading $w_0 N = k_{\max}$ with the transport-torus fiber index $k_{\max} = 60$ (the fiber of the transported determinant line, not the χ relic's sector label) — is *data-conditional*, not forced: the entropy-anchored e -fold count is $N_{\text{anchor}} = 57.76-59.37$ across the three printed onset conventions (thermal anchor 59.37), so the identification sits 0.63–2.24 e -folds above them. The two values are observationally degenerate in η_B at the 0.05% level (far below the Planck 0.67% width), and η_B and $\omega_b h^2$ above are N -free, but the onset bookkeeping is a named open

item (main volume, open problems).

Grade. $\omega_b h^2$: the α^4 power and the $B-L$ bias current are derived; the coefficient is not. Agreement with Planck holds only at the conventional freeze-out point with the Gaussian carrier, which is not realizable; with the stated carrier, the mechanism gives η_B of about $(2-3) \times 10^{-9}$ when every step is pinned to the print, three to five times Planck's value, and agreement would need one kernel winding per e -fold, where the stated carrier gives three. ω_b is therefore one calibrated input of the cosmological fits, as in Λ CDM, with that value recorded as the mechanism's first-principles result. The chain uses the adopted, priced Axiom AH.35; the axiom is one discrete bit, charged in the knob ledger (App. FK, main volume) and in the trials audit (App. CE row 9, main volume) exactly like every other adopted bit.

d. Transit Times and the Horizon

With the parent strain's σ kept a collapsed object has general relativity's event horizon, at the areal radius $2GM/c^2$ (App. WS): radial transit times and the causal structure outside it are Schwarzschild's, and a signal emitted inside it does not reach large radii, as in general relativity.

e. Quantum-Gravity Amplitude Finiteness

Theorem AH.37 (Finite All-Loop Amplitudes). *The L -loop graviton-graviton amplitude in DFD is a 60^{4L} -term sum of bounded rational functions, bypassing the Goroff–Sagnotti counterterm tower entirely.*

Proof. Each loop integral in DFD's QG sector is a trace over $\mathcal{H}_{\text{micro}}$, finite-dimensional with $\dim = 60$ (Theorem AH.13). For a four-graviton amplitude at L loops, the integration measure is $\prod_{i=1}^L \text{Tr}_{\mathcal{H}_{\text{micro}}}^{(i)}$, a product of L traces, giving 60^L terms per leg insertion. With four external legs, the total is 60^{4L} rational-function summands. Each summand is bounded because mode masses $M_n^2 \leq \Lambda_{\text{top}}^2 = \alpha M_P^2$ are finite. The standard Goroff–Sagnotti R^3 counterterm at 2 loops in pure GR gravity is absent here because the integrand is finite trace, not a divergent loop momentum integral. Tree-level matches GR through $O(s^2/M_P^2)$. $\square$

Remark AH.38 (Status note). The finiteness argument above rests on the identification of each loop integration with a trace over the finite-dimensional microsector, which is asserted at this site rather than derived here; bounded summands alone would not give finiteness without that identification and the finite term count it supplies.

Corollary AH.39 (No LHC Black Holes). *Collisions at LHC energies form no black holes.*

Proof. With the parent strain's σ kept, DFD's gravity where $\mu \rightarrow 1$ is four-dimensional general relativity (App. WS), whose black-hole formation threshold is set by the Planck energy; LHC centre-of-mass energies, $\sim 10^4$ GeV, lie fifteen orders below it, and the truncated microsector has no Kaluza–Klein modes to lower it (App. WS). Predicts null LHC mini-BH searches. $\square$

f. Neutron Stars

With the parent strain's σ kept, static stars obey the TOV equations (main volume, App. AT, Theorem AT.13); for the reference $\Gamma = 2$, $K = 100$ polytrope $M_{\max} = 1.637 M_\odot$, and PSR J0740+6620 ($2.08 \pm 0.07 M_\odot$) is consistent with DFD as it is with GR.

g. Primordial Gravitational Waves

Theorem AH.40 (Tensor-to-scalar ratio in the pre-inflation scenarios). *DFD's primordial tensor ratio is $r \lesssim 10^{-7}$ (main volume, App. SM). Tensor modes and the χ flux coordinate share one cone and one mode equation, so any stage that freezes tensors with power P_t also leaves χ a variance $P_t/(8\alpha^6)$ per e -fold, and the relic's measured abundance caps it. Of the two pre-inflation drivers, (a) $r = 0$ satisfies the cap; (b), an external inflaton with $r = 16\epsilon \approx 4 \times 10^{-3}$ (the value of Theorem AH.31), kicks χ by 2.6 turns per e -fold and erases the relic, so it is excluded on the main volume's branch, which is DFD's cosmology. The Wheeler–DeWitt construction of §AH 12b gives $4\alpha/\pi \approx 9.3 \times 10^{-3}$ on its own premises; that number is the de Sitter dictionary's $16\epsilon_W$, not the B-mode ratio.*

Proof. By Theorem AH.29 no slow-roll-compatible scalar exists on $K = \mathbb{C}P^2 \times S^3$. Tensor modes h_{ij}^{TT} during pre-inflation come from two possible drivers: (a) static initial state with no inflation $\Rightarrow r = 0$; (b) external UV inflaton with reheat at $T_{\text{RH}} \sim 10^{15}$ GeV $\Rightarrow r = 16\epsilon$, and $\epsilon \sim 1/N_e^2$ at $N_e \approx 60$ gives $r \approx 4 \times 10^{-3}$. With $A_s = 2.1 \times 10^{-9}$, driver (b) needs $H = \pi \dot{M}_P \sqrt{A_s r/2} \approx 1.6 \times 10^{13}$ GeV; the massless χ then gains an rms of $H/(2\pi f_\chi) \approx 2.6$ turns per e -fold and is washed to the finite-space ceiling $\Omega_\chi h^2 = 0.139$, excluded at $+16\sigma$. The main volume's cap, $\sigma_\theta^2 < 1.9 \times 10^{-3}$ over the $\simeq 36$ e -folds between today's horizon and χ 's oscillation onset, gives $r \lesssim 10^{-7}$ for any power-law tensor tilt; (a) lies below it. $\square$

h. Connes Spectral-Triple Connection

Proposition AH.41 (DFD/NCG Relationship). *DFD's microsector algebra $M_{60}(\mathbb{C})$ strictly contains Connes' $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$ as a sub-structure carrying the Berezin–Toeplitz tower up to $k_{\max} = 60$.*

Proof. Connes' finite spectral triple $(\mathcal{A}_F, \mathcal{H}_F, D_F)$ has algebra $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$, real dimension $2 + 4 + 18 =$

24. Embed $\mathcal{A}_F$ into $M_{60}(\mathbb{C})$ via the block-diagonal map: $\mathbb{C} \rightarrow e_{1,1}$, $\mathbb{H} \rightarrow M_2(\mathbb{C}) \subset$ block of size 2, $M_3(\mathbb{C}) \rightarrow$ block of size 3. This uses 6 of the 60 dimensions; the remaining 54 encode the Berezin–Toeplitz tower modes that Connes' construction does not natively contain. Hence $\mathcal{A}_F \subset M_{60}(\mathbb{C})$ strictly. $\square$

i. Path Integral Quantization

Theorem AH.42 (Factorized DFD Measure). *The full DFD measure factorizes as a coarse-grained 4D continuum measure times an exact Lebesgue measure on $\mathbb{C}^{60}$.*

Proof. The Hilbert space factorizes as $\mathcal{H}_{\text{DFD}} = \mathcal{F}^\psi \otimes \mathcal{H}_{\text{micro}}$ with $\dim \mathcal{H}_{\text{micro}} = 60$. The path-integral measure is $\mathcal{D}\psi \cdot \mathcal{D}^{60}\zeta$ where ζ are the microsector amplitudes. The microsector measure is the standard Lebesgue measure on $\mathbb{C}^{60}$, finite and non-singular. The continuum $\mathcal{D}\psi$ measure is regulated by the same finite microsector cutoff Λ_{top} , so the entire QFT measure is well-defined modulo the standard 4D continuum issues. No regulator divergence appears at any loop order from the microsector trace. $\square$

Theorem AH.43 (Ghost-Free Spectrum). *Below the spectral cutoff the propagator has a single positive-residue pole at $k^2 = 0$ (above it the Weyl-squared term adds the spin-2 pole of App. WS).*

Proof. Under Axiom WS.1 the scalar ψ is an instantaneous constraint with no propagating pole. For h_{ij}^{TT} : by Theorem AH.20 the propagator is block-diagonal and the tensor block has a positive-residue pole at $k^2 = 0$. $\square$

j. Topology Change of Optical Slices (Optional)

Theorem AH.44 (Topology-Change Admissibility). *DFD admits smooth topology change of its optical slices, the level surfaces of ψ , because its causal ordering is guaranteed by the absolute time t of the fixed flat arena, which every characteristic advances, and the optical metric is constructed, not a fundamental spacetime metric; space itself, the arena $\mathbb{R}^3$, keeps its topology.*

Proof. Geroch's theorem (1967) states that smooth topology change is impossible in 4D Lorentzian manifolds without a closed timelike curve, where causality is the fundamental structure. In DFD, the spatial slice is fixed flat $\mathbb{R}^3$ at the fundamental level; the optical metric $\tilde{g} = -c^2 e^{-2\psi} dt^2 + d\mathbf{x}^2$ is constructed from ψ . Topology change of the optical slices (level surfaces of ψ) is admissible because the fundamental metric is not changing. Geroch's theorem applies to the unchanging fundamental flat $\mathbb{R}^3$, vacuously. $\square$

5. Frontier Territories: Particle and Astrophysical Predictions

Seven experimental fronts follow.

a. Muon and Electron $g - 2$: Structurally Negligible

Theorem AH.45 (DFD Anomalous Magnetic Moment Suppression). *The DFD-specific contribution to the lepton anomalous magnetic moment is bounded by*

$$\delta a_\ell^{\text{DFD}} \leq \frac{1}{\alpha} \left(\frac{m_\ell}{\Lambda_{\text{top}}} \right)^2,$$

where $\Lambda_{\text{top}} = \sqrt{\alpha} M_P$. Numerically, $\delta a_\mu^{\text{DFD}} \leq 1.4 \times 10^{-36}$ and $\delta a_e^{\text{DFD}} \leq 3.3 \times 10^{-41}$, about 26 and 28 orders of magnitude below FNAL and Hanneke sensitivity respectively.

Proof. Heavy-mass decoupling (Appelquist–Carrazone). DFD’s only beyond-QED degrees of freedom are the microsector modes with mass $\sim \Lambda_{\text{top}}$. The leading vertex correction from a heavy mode of mass M to a low-energy observable scales as $(m_\ell/M)^2$ relative to the leading QED contribution $\alpha/(2\pi)$. Hence $\delta a_\ell \sim (\alpha/2\pi)(m_\ell/\Lambda_{\text{top}})^2 \cdot (1/\alpha) = (m_\ell/\Lambda_{\text{top}})^2/(2\pi)$, conservatively bounded by $(m_\ell/\Lambda_{\text{top}})^2/\alpha$. With $\Lambda_{\text{top}} = \sqrt{\alpha} M_P = 1.04 \times 10^{18}$ GeV: $(m_\mu/\Lambda_{\text{top}})^2 = 1.04 \times 10^{-38}$, multiplied by $1/\alpha$: $\delta a_\mu \approx 1.4 \times 10^{-36}$. For electron ($m_e = 5.11 \times 10^{-4}$ GeV): $(m_e/\Lambda_{\text{top}})^2 = 2.4 \times 10^{-43}$, $\delta a_e \approx 3.3 \times 10^{-41}$. $\square$

Corollary AH.46 (FNAL $g_\mu - 2$ Tension is Not a DFD Effect). *The historic ~ 4 – 5σ FNAL tension (at $\Delta a_\mu^{\text{exp-SM}} \approx 245 \times 10^{-11}$ against the 2020 data-driven SM) is 27 orders of magnitude larger than DFD’s bound and therefore cannot originate in DFD. As of the 2025 Muon $g - 2$ Theory Initiative White Paper — with the BMW/lattice HVP consensus and the CMD-3 e^+e^- data shifting the SM prediction upward by $\sim +223 \times 10^{-11}$ — this tension has collapsed to $\sim 0.6\sigma$: the anomaly is now largely removed. This supports DFD’s structural no-new-muon-physics prediction (Theorem AH.45), which forces $\delta a_\mu^{\text{DFD}} \leq 1.4 \times 10^{-36}$ with no free parameter and hence $a_\mu = a_\mu^{\text{SM}}$.*

Proof. Direct comparison against the historic discrepancy: $245 \times 10^{-11} / 1.4 \times 10^{-36} = 1.75 \times 10^{27}$. By Theorem AH.45 the DFD contribution is structurally bounded by the decoupling argument and cannot grow without invalidating microsector finiteness; DFD therefore commits to $a_\mu = a_\mu^{\text{SM}}$ and to any residual tension resolving *outside* DFD (HVP refinement, lattice, or systematics). The 2025 consensus did exactly this — the SM hadronic-vacuum-polarization contribution rose, driving the tension to sub- 1σ — so the structural null is now favored by the data. No DFD-specific central value is introduced; the sole DFD claim is the decoupling upper bound. $\square$

b. QCD Confinement and Glueball Spectrum

Theorem AH.47 (Confinement: area-law form open). *The area-law form is not derived in the proof below. The Wilson loop in DFD-QCD obeys an area law*

$$\langle W(C) \rangle \sim \exp(-\sigma_{\text{DFD}} \text{Area}(C)),$$

$$\sigma_{\text{DFD}}^{1/2} = 440 \pm 25 \text{ MeV}$$

(normalization lattice-calibrated).

Proof. The Wilson loop is computed as a trace over $\mathcal{H}_{\text{micro}}$ (finite-dimensional, $\dim = 60$), $\langle W(C) \rangle = (1/60) \text{Tr}_{\mathcal{H}_{\text{micro}}} [\mathcal{P} \exp(\oint_C iA^{(3)})]$. The string tension is fixed by the spectral spread of the gauge-charge operator on $\mathcal{H}_{\text{micro}}$, computed via Casimir invariants: $\sigma_{\text{DFD}}^{1/2} = (\Lambda_{\text{QCD}}/2\pi) \sqrt{k_{\text{max}}/N_{\text{gen}} \cdot F_{\text{geom}}}$ with $\Lambda_{\text{QCD}} = M_P \alpha^{19/2} = 61.20$ MeV, $k_{\text{max}}/N_{\text{gen}} = 60/3 = 20$, $F_{\text{geom}} \approx 1$ (Fubini–Study volume normalization). Numerical status: the derivation-grade content is the *scaling* $\sigma^{1/2} \propto \Lambda_{\text{QCD}} \sqrt{k_{\text{max}}/N_{\text{gen}}}$; the printed coefficient chain does not close to 440 MeV (it evaluates to ≈ 268 MeV at $N_f = 3$ if one includes an undefined $1/\sqrt{1/N_f}$ factor, ≈ 154 MeV without it — see the status note below), so the absolute normalization is *lattice-calibrated* to 440 ± 25 MeV (main volume, App. AQ: ≈ 465 MeV at $r_0 = 0.5$ fm), not forward-derived. $\square$

Theorem AH.48 (Lightest Glueball 0^{++} Mass). *The DFD target $m_{0^{++}} = 1.69 \pm 0.10$ GeV is tested by the main volume’s App. AQ run (1.71 ± 0.17 GeV) and is consistent with $f_0(1710)$; its normalization is open (status note below).*

Proof. The lightest glueball corresponds to the lowest non-trivial eigenvalue of the gauge-only Laplacian on $\mathbb{CP}^2$ in the singlet 0^{++} representation. The Fubini–Study eigenvalue spectrum on $\mathbb{CP}^2$ has the singlet ladder $\lambda_n = n(n + \dim_{\mathbb{C}} \mathbb{CP}^2)/r_{\text{FS}}^2 = n(n + 2)/r_{\text{FS}}^2$, with r_{FS} the Fubini–Study radius. The lowest non-trivial eigenvalue is at $n = 1$: $\lambda_1 = 3/r_{\text{FS}}^2$. Translating to mass via $m^2 = \lambda_1 \cdot \Lambda_{\text{QCD}, 3\text{-flavor}}^2 \cdot (4\pi)$ ($\overline{\text{MS}}$ matching factor): $m_{0^{++}} = \sqrt{8g_{\text{geom}}} \cdot \Lambda_{\text{QCD}, 3\text{-fl}} with $g_{\text{geom}} = 3/(2\pi)^2$ from the Fubini–Study spectral density. With $\Lambda_{\text{QCD}, 3\text{-fl}} = 210$ MeV: $m_{0^{++}} = \sqrt{8 \cdot 0.076} \cdot 210 \text{ MeV} = 1.69 \text{ GeV}$. Threshold corrections from the η' -anomaly sector add $\delta_{\text{thr}} \approx \pm 0.10$ GeV. $\square$$

a. *Status note (normalization and flavor scheme open).* As printed, the numerical chain above does not close: $\sqrt{8g_{\text{geom}}} \Lambda = 0.78 \times 210 \text{ MeV} \approx 164 \text{ MeV}$, while the intermediate form $m^2 = \lambda_1 \Lambda^2 (4\pi)$ gives $\sqrt{12\pi} \times 210 \text{ MeV} \approx 1.29 \text{ GeV}$ — the two printed coefficient forms are mutually inconsistent and neither reproduces the stated 1.69 GeV. Separately, the input labeled $\Lambda_{\text{QCD}, 3\text{-fl}} = 210$ MeV is numerically the *five-flavor* $\overline{\text{MS}}$ value (cf. $\sqrt{4\pi} \times 61.20 = 217$ MeV in this same chain); the three-flavor value derived from the identical $M_P \alpha^{19/2}$ chain in the τ -sector update is $\Lambda_{\text{QCD}, 3} = 332 \pm 20$ MeV. What holds at

derivation grade is the scaling $m_{0++} \propto \Lambda_{\text{QCD}}$ from the Fubini–Study eigenvalue argument; the overall normalization constant and the flavor-scheme assignment are open items. The ledger value $m_{0++} = 1.69 \pm 0.10$ GeV is retained as the DFD target — consistent with quenched lattice spectroscopy (the Appendix AQ companion run of the main paper gives 1.71 ± 0.17 GeV) — but its normalization from the $\mathbb{CP}^2$ spectral argument is not yet derivation-grade. The same caveat applies to the adjacent string-tension chain (Theorem AH.47): as printed, $\sqrt{20}/(2\pi) \cdot 61.20 \text{ MeV} \cdot \sqrt{4\pi}/\sqrt{1/N_f}$ evaluates to ≈ 268 MeV at $N_f = 3$, not 440 MeV; the scaling structure stands, while the printed normalization factors are open items.

c. Electroweak Phase Transition and Sphaleron Rate

Theorem AH.49 (EWPT Crossover and Critical Temperature). $T_C^{\text{DFD}} = 159.5 \pm 1.5 \text{ GeV}$ (crossover), with the measured $m_H/m_W = 1.56 > 0.9$ forcing crossover.

Proof. DFD’s Higgs sector is identical to SM at the electroweak scale (Theorem AH.17 gives the exponent of $v = 246.09$ GeV, and the conjectured $\lambda_H = 1/8$ gives $m_H \simeq 124.6$ GeV at the top-quark scale (main volume, App. Z); the argument below uses the measured m_H). Hence the finite- T effective potential $V(\phi, T)$ is the SM expression. Lattice SM at $m_H = 125$ GeV (D’Onofrio–Rummukainen 2016) gives crossover at $T_C = 159.5 \pm 1.5$ GeV. The first-order line ends at $m_H \simeq 72$ GeV ($m_H/m_W \simeq 0.9$; Rummukainen, Tsypin, Kajantie, Laine and Shaposhnikov 1998), which the measured $m_H/m_W = 1.56$ exceeds. $\square$

Theorem AH.50 (Sphaleron Rate at Reheating). $\Gamma_{\text{sph}}/V \approx 4.8 \times 10^{46} \text{ GeV}^4$ at $T_{\text{RH}} = 2.4 \times 10^{13} \text{ GeV}$, a reheating temperature that no printed mechanism supplies (main volume, App. Q).

Proof. Above EWPT, $\Gamma_{\text{sph}}/V = \kappa \alpha_W^5 T^4$ with lattice $\kappa = 18 \pm 3$ (D’Onofrio–Rummukainen–Tranberg 2014). The printed $g_2 = 2/3$ runs to $\alpha_W = 1/41.5$ at T_{RH} (computed), so $\Gamma_{\text{sph}}/V = 18 \cdot (1/41.5)^5 \cdot (2.4 \times 10^{13})^4 \approx 4.8 \times 10^{46} \text{ GeV}^4$. The per-particle rate is $(\Gamma_{\text{sph}}/V)/T^3 = 3.5 \times 10^6 \text{ GeV}$, and the $B+L$ relaxation rate carries a multiplicity between $13N_f/4 \simeq 10$, with every Yukawa coupling in equilibrium, and $6N_f = 18$, with none (Burnier, Laine and Shaposhnikov, hep-ph/0511246): $\Gamma_B = (3.4\text{--}6.3) \times 10^7 \text{ GeV}$. Against the radiation-era Hubble rate $H = 1.66\sqrt{g_*} T^2/M_P = 8.1 \times 10^8 \text{ GeV}$ ($g_* = 106.75$), $\Gamma_B/H \approx 0.04\text{--}0.08$ ($\approx 0.14\text{--}0.27$ in the convention $H = T^2/\bar{M}_P$): the sphalerons are out of equilibrium at T_{RH} . Since Γ_B/H grows as $1/T$, they come into equilibrium at $1\text{--}2 \times 10^{12} \text{ GeV}$ ($3.5\text{--}6.4 \times 10^{12} \text{ GeV}$ in the second convention; computed), far above the electroweak crossover, which is all the conversion factor uses. $\square$

Corollary AH.51 (Baryogenesis efficiency: α^4 power and $B\text{--}L$ bias derived). *Thermal leptogenesis gives no*

asymmetry: the minimal real-kernel lepton sector gives $\text{Im}[(Y_\nu^\dagger Y_\nu)_{ij}^2] = 0$, hence $\varepsilon_i = 0$ and the thermal leptogenesis $\eta_B^{\text{minimal DFD}} = 0$ (status note, §AH.4 b). The canonical DFD position is the internal axial Berry-holonomy channel (Theorem AH.33): DFD takes the identified scale $M_R = 4.744 \times 10^{12} \text{ GeV}$ and forces the α^4 power of the asymmetry, and its bias current is $B\text{--}L$. At the conventional freeze-out point with the Gaussian carrier the base value is $|\eta_B^{\text{base}}| \simeq 0.206 \alpha^4 \simeq 5.8 \times 10^{-10}$, completed by the transport deposit to $\eta_B = 0.206 \alpha^4 (1 + 2\pi\alpha) = 6.1007 \times 10^{-10}$ (Thm. AH.36); that carrier is not realizable, and with the stated carrier, pinned to the print, the mechanism gives about $(2\text{--}3) \times 10^{-9}$, three to five times Planck’s value (Theorem AH.33). The sign (matter vs. antimatter) is fixed by the expansion orientation of Axiom AH.35, a CP-odd datum carried by A_5 on the determinant line — adopted, not derived. (The strong-washout efficiency $\kappa_f = 4.87 \times 10^{-3}$ quoted in the status note above belongs to the thermal chain and does not enter the Berry channel. The washout and reprocessing of the Berry channel itself, with three washout channels, finite-rate spectators and the relaxation computed, multiply the conventional-point value by 0.272; proof of Theorem AH.33, step (4).) Baryogenesis is therefore a DFD result whose α^4 power and $B\text{--}L$ bias are derived and whose printed mechanism gives about $(2\text{--}3) \times 10^{-9}$, three to five times Planck’s value, with the sign fixed by an adopted axiom; it is not a closed prediction.

d. Multimessenger Binary Neutron Star Signatures

With the parent strain’s σ kept a binary neutron-star merger is general relativity’s where $\mu \rightarrow 1$ (App. WS; argued for fully nonlinear fields): the post-merger peak frequency and the remnant’s ringdown are GR’s, and there is no echo train.

e. Proton Lifetime

Theorem AH.52 (Proton Stability). *If baryon number is violated only by the electroweak anomaly, $\Gamma(p \rightarrow e^+ \pi^0) = 0$ and $\Gamma(p \rightarrow \bar{\nu} K^+) = 0$ exactly.*

Proof. Sphaleron transitions change B and L together, by $\Delta B = \Delta L = \pm N_{\text{gen}} = \pm 3$ (from the chiral anomaly with three SM generations). Standard $p \rightarrow e^+ \pi^0$ has $\Delta B = -1$, not a multiple of 3, hence forbidden. Similarly $p \rightarrow \bar{\nu} K^+$ is forbidden. The minimal allowed processes change baryon number by three units, $\Delta B = \Delta L = \pm 3$ (main volume, App. F). $\square$

Lemma AH.53 (Allowed-Channel Lifetime). *The $\Delta B = \pm 3$ rate is suppressed by $\exp(-S_{\text{inst}}/\hbar) = \exp(-4\pi/\alpha_W) \approx \exp(-373)$.*

Proof. Electroweak instanton action is $S_{\text{inst}} = 4\pi/\alpha_W \approx 4\pi \cdot 30 \approx 373$ at the EW scale. At $T = 0$, the tunneling

rate is $\Gamma \sim m_p \cdot \exp(-S_{\text{inst}})$. Hence $\tau = \exp(373)/m_p \approx 10^{130}$ yr. Even with the $\sim 10^{35}$ nucleons of Hyper-K and 10^{10} years, expected events $\sim 10^{35} \cdot 10^{10}/10^{130} = 10^{-85}$ – unobservable. $\square$

Corollary AH.54 (Hyper-K, DUNE, JUNO Null Prediction). *If baryon number is violated only by the electroweak anomaly, DFD predicts zero signal at every projected sensitivity in standard channels.*

Proof. Hyper-K targets $\tau_p > 10^{35}$ yr on $p \rightarrow e^+ \pi^0$ and $p \rightarrow \bar{\nu} K^+$. By Theorem AH.52 both are forbidden. By Lemma above, the allowed multi-baryon channel has $\tau > 10^{120}$ yr. Therefore expected events at any current or planned proton-decay experiment is exactly zero from DFD. A single positive event in any standard channel falsifies the selection rule of Theorem AH.52. $\square$

f. Holographic Entropy Bounds

Theorem AH.55 (Bulk Information Density). $\rho_{\text{info}} = \log_2(\dim \mathcal{H}_{\text{micro}})/\ell_P^3 = \log_2(60)/\ell_P^3 \approx 5.91$ bits per Planck volume.

Proof. Each Planck-volume cell carries $\mathcal{H}_{\text{micro}}$ as its quantum state space. Maximum information storable in a cell of Hilbert dimension d is $\log_2 d$ bits (standard quantum information). With $d = 60$: $\log_2 60 = 5.91$ bits per cell. Density: $5.91/\ell_P^3$ bits/volume. Note this is finite because $\dim \mathcal{H}_{\text{micro}} < \infty$, in contrast with continuum QFT where the per-volume capacity formally diverges. $\square$

Corollary AH.56 (Λ -Topology Reciprocity). $S_{\text{ds}} \cdot \Lambda/M_P^4 = O(1)$.

Proof. By Theorem AH.23 and the clock dictionary, the observed dark-energy density is $\Lambda/M_P^4 = \Omega_\Lambda(3/8\pi)C\alpha^{57}$ (main volume, App. AU). By Theorem AH.22, $H_0^2 = Cc^5\alpha^{57}/(G\hbar)$. The de Sitter entropy from Gibbons–Hawking is therefore $S_{\text{ds}} = (\pi c^5)/(\hbar G H_0^2) = \pi/(C\alpha^{57})$. The clock coefficient C cancels in the product: $S_{\text{ds}} \cdot \Lambda/M_P^4 = 3\Omega_\Lambda/8 = O(1)$. $\square$

g. Ultra-High-Energy Cosmic Rays and Photon Dispersion

Theorem AH.57 (DFD GZK Threshold = SM). $E_{\text{th}}^{\text{DFD}} = 1.07 \times 10^{20}$ eV, identical to SM at the 10^{-14} level.

Proof. The GZK threshold satisfies $E_p \cdot \varepsilon_{\text{CMB}} = m_\pi(m_p + m_\pi/2)$ at the Δ^+ resonance. Pion mass m_π and proton mass m_p in DFD descend from the same spectral construction as α^{-1} and the fermion mass ladder (Theorem AH.27); their numerical values match PDG to $\leq 1\%$. The CMB photon distribution is unchanged by DFD (the cosmological photon sector is not modified). Hence $E_{\text{th}}^{\text{DFD}}$ equals SM. Any structural correction is at the Padé-protected $O((E/E_{\text{Pl}})^6)$ order, giving relative shift $(10^{20} \text{ eV}/10^{28} \text{ eV})^6 = 10^{-48}$ – negligible. $\square$

Theorem AH.58 (No Lorentz-Violating Photon Dispersion). *The DFD LIV parameter $\xi_{\text{LIV}} \equiv 0$ identically.*

Proof. DFD’s optical metric is conformally flat on equipotential slices ($\psi = \text{constant}$): $\tilde{g} = -c^2 e^{-2\psi} dt^2 + d\mathbf{x}^2$ has zero Weyl curvature in the spatial sector. The dispersion relation on such a slice is $\omega^2 = c^2 |\mathbf{k}|^2$ exactly, with no $\xi(p^2/M_{\text{Pl}})^n$ correction (those would require non-conformal slice metric). Off-slice variations of ψ produce only conformal-factor changes, which preserve the light-cone structure. By Theorem AH.20, no derivative coupling between ψ and h^{TT} exists; in particular, no quartic LIV operator. Hence $\xi_{\text{LIV}} = 0$ structurally. Pierre Auger bounds $|\xi| < 10^{-21}$ are consistent by structural identity. Future tightening cannot “test DFD harder” on LIV – the prediction is rigid zero. $\square$

6. Standard Model Lagrangian, BSM Constraints, and Cosmology

The Topological Pre-Inflation construction is kept as a conjectured extension beyond DFD’s stated scope, not a DFD prediction (Sec. AH 6b).

a. Vacuum Stability

An independent recomputation graded the vacuum-stability theorem of Sec. AH 2a questionable; the disagreement is a units artifact, not a physics conflict.

Theorem AH.59 (Vacuum Stability in the Dimensionless Variable). *In the dimensionless ratio $\delta V/M_P^4$, one-loop vacuum feedback satisfies $|\delta V_{\text{CW}}^{(1)}/M_P^4| \leq O(\alpha^2)$. The Sec. IX.H “ $\lambda \sim 10^{113}$ violently unstable” warning is a presentation defect: it implicitly uses ρ_c (a cosmological output) as if it were the ψ effective-mass input. The Planck-scale quantity in that denominator is the Newtonian gradient stiffness $c^4/(32\pi G) \sim M_P^2$ of the ψ kinetic term, not ρ_c ; ψ itself carries no potential term (Theorem AH.136, sub-(i)).*

Proof. Theorems AH.1 and AH.3 establish $|\delta V_{\text{CW}}^{(1)}/M_P^4| \leq O(\alpha^2)$ in the dimensionless trace over $\mathcal{H}_{\text{micro}}$. The 10^{113} of the main text is obtained by inserting $\rho_c \sim H_0^2 M_P^2/(8\pi G)$ into the ψ self-energy denominator, but ρ_c is itself an output of the exponent identity $G\hbar H_0^2/c^5 = C\alpha^{57}$ (Theorem AH.23; C measured), not a fundamental input. The correct denominator is the Planck-scale Newtonian gradient stiffness of the ψ kinetic term, $c^4/(32\pi G) \sim M_P^2$ (“ ψ effective mass” is a mislabel: ψ has no potential term, Theorem AH.136, sub-(i)). Substituting M_P^2 instead of ρ_c removes the 10^{113} enhancement and recovers the dimensionless α^2 bound. $\square$

Remark AH.60 (Verification Reconciliation). One computation works in the dimensionless variable (correct), the other in the dimensional ρ_{vac} variable (also correct

for that variable); they agree on the underlying integrals. The presentation defect is the carrying of the dimensional variable, with ρ_c in the denominator, into Sec. IX.H; that section states the dimensionless-variable bound beside the estimate, and this appendix keeps both readings.

b. Topological Pre-Inflation Status

The TPI construction (Theorems AH.29–AH.31) is not derivable from DFD’s stated structure (five independent checks; Remark AH.109), and Theorem AH.136 excludes a slow-roll-compatible native inflaton. The TPI material is kept as a *conjectured extension beyond DFD’s stated scope*; it is not a DFD prediction.

Remark AH.61 (Status note: the TPI staircase). The integers $\{1, 12, 30\}$ appearing in the TPI staircase do not match DFD’s canonical cohomological catalog $\{3, 7, 8, 13, 19, 31, 49, 57, 60, 108, 137\}$. The Lichnerowicz rigidity argument excludes graviton-mode inflatons but does not exclude an external scalar inflaton coupled to DFD; therefore the “no slow-roll” premise is narrower than the stated theorem. Independent recomputation of n_s, A_s, r without the construction’s internal staircase parameters does not converge to Planck values. The verification record is in Remark AH.109.

c. CKM Per-Channel Agreement

The magnitude integers are suggestive of cohomological origin, and the CP-even apex is fixed at $\bar{\rho} = \frac{43}{2}\alpha = 21.5\alpha$ by the Euler-projection postulate.

Remark AH.62 (Per-channel CKM check). Three independent recomputations against PDG 2024 ($\lambda = 0.22501 \pm 0.00068$, $A = 0.826^{+0.016}_{-0.015}$, $\bar{\rho} = 0.1591 \pm 0.0094$, $\bar{\eta} = 0.3523^{+0.0073}_{-0.0071}$) give per-channel deviations λ at $\sim 0.5\%$, $\bar{\eta}$ at $\sim 1.5\%$, A at $\sim 4.6\%$, and $\bar{\rho}$ at $\sim 1.4\%$ for the Euler-projected apex $\bar{\rho} = \frac{43}{2}\alpha$ (the raw assignment $\bar{\rho} = 19\alpha$ gives a $\sim 13\%$ deviation). The pattern $(31, 108, \frac{43}{2}, 49) \times \alpha$ has no continuous fitted parameter; its integers are selected (App. FK of the main volume) and the apex is fixed by the Euler-projection postulate (App. AO).

d. Full Standard Model Lagrangian from Microsector

Theorem AH.63 (SM Lagrangian Derivation). *Every coefficient of $\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\theta} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}} + \mathcal{L}_{\nu_R}$ is given a structural source on $\mathbb{CP}^2 \times S^3$ at $k_{\text{max}} = 60$ or is counted among the inputs of App. FK of the main volume (the fitted Higgs prefactor, the Higgs quartic, the imported η_q and the discrete selections); the absolute charged-fermion Yukawas additionally carry data-selected labels, graded separately*

in App. FM of the main volume, and the statement here is not a derivation of that absolute spectrum.

Proof. Gauge sector. The $(3, 2, 1)$ partition + singlet on the bundle $E = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ carries the gauge group $SU(3) \times SU(2) \times U(1)$ as a dictionary entry: the electroweak factor is the isotropy group of $\mathbb{CP}^2$ under one stated identification, and colour is a stated input (main volume, App. F; Theorem AH.9). Couplings: $\sin^2 \theta_W = 3/13$ (App. Z), and $\alpha_s(M_Z) = 0.1187$ from the stated strong scale $\sqrt{4\pi} M_P \alpha^{19/2}$ (main volume, open problems).

Fermion sector. Three generations are a stated input identified with $\chi_{\text{top}}(\mathbb{CP}^2) = 3$, and the representations of a family are inputs, as in the Standard Model; their hypercharge twists use the derived flux quantum $q_1 = 3$ (main volume, App. F).

Higgs sector. $v = M_P \alpha^8 \sqrt{2\pi} = 246.09$ GeV, with the exponent derived and the prefactor fitted (Theorem AH.17); the conjectured $\lambda_H = 1/8$ from dimension counting (main volume, App. Z).

Yukawa sector. $y_f = A_f \alpha^{n_f} / \sqrt{2}$ via the Berry-bundle overlap integral on $\mathbb{CP}^2$ (Theorem AH.27). Each prefactor A_f is a product of four blocks (kernel, QCD, Dirac, generation); for seven of the nine charged fermions the absolute prefactor is data-selected (main volume, App. FM).

θ -term. $\theta_{\text{QCD}} = 0$ at tree level, loop-bounded, with $\theta_{\text{bare}} = 0$ fixed by the spectral action given that nothing is added to it for the gauge fields (Theorem AH.66 below).

Gauge-fixing + ghost. Standard BRST quantization on the 60-dim Hilbert space; finite-dim ghost sector arises from the same Toeplitz truncation.

Right-handed neutrinos. $M_R = M_P \alpha^3 = 4.744 \times 10^{12}$ GeV (App. P; machine-verified to 40 digits).

Anomaly cancellation. Gauge anomalies $\sum_i Y_i^3 = 0$, $\sum_i Y_i T_i^a T_i^b = 0$, $\sum_i Y_i = 0$ vanish for the Standard Model content of one family, which is a stated input; the flux quantum does not force them.

Custodial $SU(2)$. $\rho = m_W^2 / (m_Z^2 \cos^2 \theta_W) = 1$ at tree level by the Higgs being a doublet under $SU(2)_L \otimes SU(2)_R$; structurally derived from the $(3, 2, 1)$ partition. $\square$

e. Beyond-Standard-Model Constraints

DFD’s microsector finiteness imposes sharp BSM exclusions.

Theorem AH.64 (BSM Forbidden List). *The following BSM scenarios are forbidden at theorem level, items (iv)–(vi) given the adopted gauge group (its electroweak factor rests on one stated identification and colour is an input; main volume, App. F): (i) supersymmetry, (ii) QCD axion/Peccei–Quinn, (iii) magnetic monopoles, (iv) dark photon $U(1)'$, (v) leptiquarks, (vi) Z' bosons, (vii) cosmic strings, (viii) domain walls, (ix) composite Higgs, (x) extra macroscopic spatial dimensions, (xi) Lorentz-violating photon dispersion. There is no postulated dark*

matter and no thermal WIMP candidate; the cold dark matter is instead the derived χ -matter field (App. AV), the harmonic b_3 three-form on the internal $S^3 = SU(2)$ — a topology-forced geometric mode, not an added particle and not a Peccei–Quinn axion (item ii). DFD adds no free dark-sector parameter: χ supplies the cosmological clustering scaffolding while the μ -law ($\mu(x)$, a_0 ; Theorem AH.21) covers galactic-scale phenomenology; inside galaxies the two compose by the adopted floor law with a stated three-dimensional filling rule (main volume, App. AV).

Proof. Each scenario fails a specific DFD axiom or theorem.

(i) **SUSY.** Adding even one superpartner forces $k_{\max} \geq 65$ to accommodate the additional internal modes: the closed Spin^c index $\chi(\mathbb{CP}^2, E) = \chi(\mathcal{O}(9)) + 5\chi(\mathcal{O}) = 55 + 5 = 60$ is already saturated (equivalently, $60 = 3$ geometric zero modes + 57 nonzero KK modes), so minimal padding admits no further internal mode (Theorem AH.9). A hypercharge-charged superpartner additionally violates the hypercharge-twist integrality condition (the $q_1 Y + \frac{3}{2} \in \frac{1}{2}\mathbb{Z}$ requirement on $L^{q_1 Y}$, App. F); a would-be gauge-singlet ($Y = 0$) superpartner direction, to which the twist integrality has no content, is instead excluded by geometric rigidity — the Koiso/Lichnerowicz zero-mode classification with $b_1(K) = 0$ leaves only the trace ψ , the truncated microsector carries no size or shape modulus, and the vacuum moduli space has dimension 0 and cardinality 1 (Theorem AH.179).

(ii) **Axion.** Theorem AH.66 below: $\theta_{\text{QCD}} = 0$ at tree level ($\theta_{\text{bare}} = 0$ fixed by the spectral action given that nothing is added to it for the gauge fields), loop-bounded below 10^{-16} . Peccei–Quinn relaxation is unnecessary, and a new Peccei–Quinn axion — an added $U(1)_{\text{PQ}}$ field introduced to solve strong-CP — would correspond to a 60th-mode-violating internal direction and is forbidden. This does *not* forbid the χ -matter field of App. AV: χ is the pre-existing harmonic b_3 three-form on S^3 (a bosonic geometric zero-mode, not one of the 60 chiral Spin^c modes), it is not introduced to solve strong-CP, and it carries no Peccei–Quinn charge. It is categorically distinct from the forbidden PQ axion.

(iii) **Monopoles.** $\pi_2(S^3) = 0$ and $\pi_2(\mathbb{CP}^2) = \mathbb{Z}$ structure does not admit topological monopole solutions in DFD’s (3,2,1) gauge embedding because the second homotopy group is exhausted by the Kähler class.

(iv) **Dark photon.** The gauge group is $SU(3) \times SU(2) \times U(1)$ from (3,2,1)+singlet, adopted rather than forced: its electroweak factor rests on one stated identification and colour is an input (main volume, App. F). Adding $U(1)'$ would require a sixth chiral multiplet, ruled out by minimal-padding theorem (Theorem AH.9).

(v) **Leptoquarks.** Mediating quark-lepton transitions requires gauge bosons charged under both $SU(3)$ and $SU(2)_L$. No such bosons exist in E .

(vi) Z' . Same argument as (iv).

(vii) **Cosmic strings.** The electroweak vacuum manifold $(SU(2) \times U(1)_Y)/U(1)_{\text{em}} \simeq S^3$ has $\pi_1 = 0$. No

topologically stable string solutions.

(viii) **Domain walls.** The ψ -vacuum is unique by the strict monotonicity of $\mu(x) = x/(1+x)$ on $[0, \infty)$.

(ix) **Composite Higgs.** The Higgs is the unique $(1, 2, +1/2)$ zero mode of the Spin^c Dirac operator on $\mathbb{CP}^2$ in E (App. H). Compositeness would require a strongly-interacting subsector, absent from $\mathcal{L}_{\text{SM}}$.

(x) **Extra macroscopic dimensions.** The 4D space-time is fixed flat $\mathbb{R}^{3,1}$; the additional 7 internal dimensions are bundle directions of $\mathbb{CP}^2 \times S^3$ at fixed compact size, not large macroscopic dimensions.

(xi) **LIV.** Theorem AH.58: $\xi_{\text{LIV}} \equiv 0$. $\square$

Corollary AH.65 (Sharpest BSM Null Predictions). *The most experimentally-decisive null predictions are:*

- (a) *Detection of any squark/gluino/neutralino at LHC, FCC-hh, or any future collider $\Rightarrow$ DFD falsified.*
- (b) *Observation of $p \rightarrow e^+ \pi^0$ or $p \rightarrow \bar{\nu} K^+$ at any rate at Hyper-K, DUNE, or JUNO $\Rightarrow$ DFD falsified.*
- (c) *Detection of QCD axion at any haloscope (ADMX, IAXO, ABRACADABRA, CASPER, MADMAX) across the KSVZ/DFSZ window $\Rightarrow$ DFD falsified.*
- (d) *Detection of a millicharged particle — any charge εe , any mass, any production channel (accelerator, cosmic-ray, or terrestrial accumulation) $\Rightarrow$ DFD falsified.*

Proof. Each item is a direct consequence of Theorem AH.64. (a) SUSY’s superpartners would force $k_{\max} \geq 65$, exceeding the saturated minimal-padding index $\chi(\mathbb{CP}^2, E) = 60$ (Theorem AH.14 fixes $k_{\max} = 60$); hypercharge-charged superpartners additionally violate the hypercharge-twist integrality condition (App. F), and any gauge-singlet superpartner direction is excluded by the single-vacuum geometric rigidity (Theorem AH.179). (b) Standard proton-decay channels are forbidden by Theorem AH.52; a positive observation in any standard channel violates its selection rule. (c) The QCD axion’s existence requires the Peccei–Quinn relaxation mechanism, which is unnecessary in DFD because $\bar{\theta} = 0$ at tree level, with $\theta_{\text{bare}} = 0$ fixed by the spectral action given that nothing is added to it for the gauge fields, and loop-bounded by Theorem AH.66; a haloscope detection in the KSVZ/DFSZ window would imply a non-vanishing axion-photon coupling that DFD’s spectral-pairing closure forbids. (d) follows from item (iv) of Theorem AH.64: a kinetic-mixing millicharge requires the forbidden dark-photon $U(1)'$, and a bare millicharge is a new charged internal mode excluded by the saturated index $\chi(\mathbb{CP}^2, E) = 60$; accumulator-augmented shielded-Cavendish tests (Berlin, Bogorad, Graham, and Ramani 2026; arXiv:2510.25825, 2510.25834) reach the irreducible cosmic-ray-produced population within years, so the null is dated as well as one-sided. Each of (a)–(d) corresponds to a single theorem failing. $\square$

f. *Strong-CP Closure: Tree Level, Loop-Bounded*

Theorem AH.66 ($\theta_{\text{QCD}} = 0$ at Tree Level, Loop-Bounded). *With $\theta_{\text{bare}} = 0$ fixed by the spectral action given that nothing is added to it for the gauge fields, the strong-CP angle $\bar{\theta} = 0$ at tree level, with loop corrections at the Standard-Model level ($\lesssim 10^{-16}$; App. AO). The neutron electric dipole moment from this $\bar{\theta}$ sector is therefore at most $\sim 2 \times 10^{-32}$ e·cm ($d_n \simeq 2.4 \times 10^{-16} \bar{\theta}$ e·cm), comparable to the CKM background; the residual EDM is set by the CKM phase alone and is dominated by the long-distance chiral pion-loop, $d_n^{\text{DFD}} \sim (0.9\text{--}5.6) \times 10^{-32}$ e·cm (Corollary below; App. AT neutron-EDM ledger).*

Proof. The CP mapping torus on $X = \mathbb{C}P^2 \times S^3$ is $T_{CP} = (X \times [0, 1]) / (x, 0) \sim (CP(x), 1)$, with $\dim T_{CP} = 7 + 1 = 8$ (even). On any closed even-dimensional oriented manifold carrying a $\mathbb{Z}_2$ -graded Clifford-module bundle (here $W = S \otimes E \oplus \bar{S} \otimes \bar{E}$ with the antilinear CP lift; T_{CP} itself admits no Spin^c structure, App. L), the chirality operator Γ anticommutes with the Dirac operator: $\{\Gamma, D_{T_{CP}}\} = 0$. Therefore the spectrum is $\pm$ -paired: every eigenvalue λ has matching $-\lambda$. The Atiyah–Patodi–Singer η -invariant

$$\eta(D_{T_{CP}}) = \lim_{s \rightarrow 0^+} \sum_{\lambda} \text{sgn}(\lambda) |\lambda|^{-s}$$

vanishes term-by-term by spectral pairing for $\text{Re } s > 8$, where the series converges absolutely by Weyl’s law, and hence identically after continuation to $s = 0$. The Dai–Freed factor entering the partition function is

$$A_{CP} = \exp(i\pi\eta(D_{T_{CP}})/2) = 1$$

exactly, hence $\bar{\theta}_{CP} = 0$. Combined with $\arg \det(M_u M_d) = 0$ at tree level for det-orthogonal mass matrices positive on the branch of Postulate Y.10 (App. AO) and the bare angle $\theta_{\text{bare}} = 0$, fixed by the spectral action given that nothing is added to it for the gauge fields (main volume, App. L), this enforces $\bar{\theta} = 0$ at tree level. Loop-by-loop ledger: at tree level, $\theta_{\text{tree}} = 0$; the shift of $\arg \det(M_u M_d)$ vanishes identically through Yukawa degree eleven for Hermitian tree-level mass matrices with the Standard-Model chiral couplings; from the degree-thirteen Jarlskog structure onward (a multi-loop effect), corrections are possible and are at the Standard-Model level, $\lesssim 10^{-16}$ (App. AO). On the product background the Dirac operator has no zero modes, so the anomaly phase is trivial for either a determinant or a Pfaffian fermion measure. $\square$

Corollary AH.67 (Neutron and Electron EDM Predictions). *$d_n^{\text{DFD}} \sim (0.9\text{--}5.6) \times 10^{-32}$ e·cm (the CKM long-distance ledger of the “Neutron-EDM ledger and falsifier” corollary in App. AT of the main paper; ~ 6 orders below the current PSI bound 1.8×10^{-26} e·cm), and $d_e^{\text{DFD}} \lesssim 10^{-38}$ e·cm (CKM/SM floor, $\gtrsim 8$ orders below ACME-III/JILA $\sim 10^{-30}$ e·cm).*

Proof. With $\bar{\theta} = 0$ at tree level from Theorem AH.66, the CP sources are the CKM phase and the loop-induced

$\bar{\theta} \lesssim 10^{-16}$. The dominant neutron-EDM contribution is the long-distance chiral charged-pion loop (Khriplovich–Zhitnitsky; Seng, Phys. Rev. C **91**, 025502 (2015)), $d_n^{\text{SM,LD}} \sim (1\text{--}6) \times 10^{-32}$ e·cm, rescaled by the DFD Jarlskog ratio $J_{\text{DFD}}/J_{\text{SM}} = 0.94$ to $d_n^{\text{DFD}} \sim (0.9\text{--}5.6) \times 10^{-32}$ e·cm. The short-distance three-loop quark/chromo-electric channel ($d_n^{\text{SD}} \sim 10^{-34}$ e·cm) is *subdominant* by 2–3 orders and is **not** the only contribution; the residual $\bar{\theta}$ -channel is $\lesssim 2 \times 10^{-32}$ e·cm for the upper-end induced $\bar{\theta} \sim 10^{-16}$, comparable to the CKM background. (A short-distance-only estimate, $\sim 10^{-37}$ e·cm, omits the dominant long-distance channel; the value here is set by the App. AT neutron-EDM corollary.) The electron EDM sits at the tiny CKM/SM floor, $d_e \lesssim 10^{-38}$ e·cm; the leptogenesis Majorana channel is *inert*, because DFD’s leptonic CP-odd invariant $\text{Im}[(Y_\nu^\dagger Y_\nu)^2] = 0$ (the same real-kernel structure that gives the $\varepsilon_1 = 0$ baryogenesis no-go), leaving only the CKM contribution. $\square$

g. *Precision QED Beyond $g - 2$*

Other precision-QED channels (Lamb shift, positronium, muonic-H, helium HFS) receive $\delta \sim (m_\ell/\Lambda_{\text{top}})^2 \sim 10^{-43}\text{--}10^{-38}$ DFD corrections, structurally invisible.

h. *Lepton Flavor Violation: Structurally Negligible*

Theorem AH.68 (LFV Branching Ratios in DFD). *With $M_R = M_P \alpha^3 = 4.744 \times 10^{12}$ GeV and DFD’s PMNS matrix structure (order-of-magnitude bounds; the see-saw rates scale as $|\Theta|^4 \sim (m_\nu/M_R)^2$): $\text{BR}(\mu \rightarrow e\gamma) \lesssim 8 \times 10^{-51}$, $\text{BR}(\mu \rightarrow eee) \lesssim 2 \times 10^{-55}$, $\text{BR}(\tau \rightarrow \mu\gamma) \lesssim 3 \times 10^{-48}$, $\text{BR}(\tau \rightarrow e\gamma) \lesssim 7 \times 10^{-50}$, $\text{CR}(\mu\text{--}e, \text{Ti}) \lesssim 3 \times 10^{-51}$.*

Proof. Standard see-saw with heavy Majorana neutrinos at M_R gives effective dimension-6 operators $\mathcal{L}_{\text{LFV}} \supset c_{\text{dipole}} (m_\ell/M_R^2) \bar{\ell}_L \sigma^{\mu\nu} \ell_R F_{\mu\nu} + \dots$, with the heavy-neutrino estimate $\text{BR}(\ell_i \rightarrow \ell_j \gamma) \sim (3\alpha/32\pi) |\Theta|^4$ times a PMNS overlap factor, $|\Theta|^2 \sim m_\nu/M_R \sim 10^{-23}$. With DFD’s PMNS values and neutrino masses (Theorem AH.28): PMNS overlaps $\sim 10^{-1}$. The various channels differ by PMNS overlap factors that scale predictably. $\square$

Corollary AH.69 (All Planned LFV Experiments Are Null Predictions). *MEG-II ($\sim 6 \times 10^{-14}$ for $\mu \rightarrow e\gamma$), Mu3e (10^{-16} for $\mu \rightarrow eee$), Belle II (10^{-9} for τ channels), Mu2e/COMET (10^{-17} for $\mu\text{--}e$ conversion) all sit ≥ 31 orders above DFD’s predictions. DFD predicts null at every planned LFV facility. A positive signal at any of them at level $> 10^{-18}$ falsifies the see-saw scale $M_R = M_P \alpha^3$.*

Proof. Direct comparison of DFD predictions (Theorem AH.68) with experimental thresholds; gap is ≥ 31 orders in every channel. $\square$

i. *Galaxy Formation in DFD: the High- z Halo Abundance*

On the adopted carrier ($Q_\chi = 1$) DFD’s halo abundance at $z = 6$ – 15 is Planck Λ CDM’s, lower by 3–19% at 10^{10} – $10^{11} M_\odot$ because σ_8 and the small-scale power are lower (Sheth–Tormen, computed). DFD’s halo growth therefore produces no excess of early galaxies. The luminous galaxies at $z \simeq 14$ need a star-formation efficiency of 0.13–0.25 of their halo baryons, about 4% more than in Λ CDM, and a confirmed population at $z \simeq 14$ with $n \geq 10^{-6} \text{ Mpc}^{-3}$ and $M_* > 4 \times 10^9 M_\odot$ would exceed the baryons of every DFD halo. The deep-MOND apparent-amplitude branch ($Q \simeq 502$) gives a 10–30 $\times$ excess, but it is not DFD’s growth carrier, and its excess is not a prediction.

If the epoch law holds on the bound-matter clock, compact galaxies enter the MOND regime early: a $10^{10} M_\odot$ galaxy within 1 kpc has $\nu = 1.6$ at $z = 6$ and 2.2 at $z = 14$ (computed), which deepens its potential and shortens its free-fall time by $\nu^{-1/2}$. Whether this raises the star-formation efficiency needs hydrodynamics; each 30% gain in efficiency raises the counts of bright galaxies at $z = 8$ – 14 by a factor 2–2.5. Kinematics of compact galaxies at $z \geq 6$ decide it: the epoch law predicts $M_{\text{dyn}}/M_{\text{bar}} \geq 1.5$ inside a kiloparsec from the boost alone.

j. *Type Ia Supernovae and the Hubble Diagram Plateau*

Theorem AH.70 (Retired CMB-anchored Type-Ia bridge forecast). *The former $+0.59 \pm 0.05 \text{ mag}$ forecast was conditional on the CMB-anchored bridge of Theorem AH.85. That bridge is retired by the exact conformal-invariance and homogeneous-mode results of App. ZS; this number is not a current DFD prediction. Inhomogeneous distance effects remain an open numerical program.*

k. *Big Bang Nucleosynthesis: Standard Abundances*

Theorem AH.71 (DFD-BBN Equivalence to SM-BBN). *DFD predicts the standard primordial abundances: $D/H = 2.55 \times 10^{-5}$ (vs. obs $2.547 \pm 0.025 \times 10^{-5}$), $Y_p = 0.2479$ (obs 0.2453 ± 0.0034 , 0.7σ), ${}^7\text{Li}/H \simeq 4.7 \times 10^{-10}$ (obs $1.6 \pm 0.3 \times 10^{-10}$). $N_{\text{eff}} = 3.044$ (obs 2.99 ± 0.17): the species count is $N_\nu = 3$ with no dark radiation (see proof), the count itself being the generation count, a discrete input of the main volume (App. F), the residual $+0.044$ being the standard SM thermal correction adopted as input. DFD does not solve the lithium-7 puzzle.*

Proof. At BBN ($T \approx 1 \text{ MeV}$), DFD’s optical metric on FRW background reduces to the standard FRW ($\bar{a}_{\text{ext}} = 0$ by Theorem AE.1). The ratio of every nuclear and weak rate to the Hubble rate is unchanged from Λ CDM at $T \sim \text{MeV}$ scales (main volume, App. ZS: gravity enters as $\lambda^2 G$ and the early rates run faster by λ). Baryon density $\Omega_b h^2$ is fixed by $\eta_B = 6.1 \times 10^{-10}$, whose α^4 power

is derived and whose coefficient is open (Thm. AH.33, §AH.4c); BBN here uses the observed value, so the numerical BBN inputs are the standard ones. Effective neutrino species: the integer count is the generation count, $N_\nu = 3$, a discrete input of the main volume that coincides with $\chi_{\text{top}}(\mathbb{CP}^2) = 3$, with *zero* extra relativistic degrees of freedom. The absence of dark radiation is triple-locked — Berezin–Toeplitz/Spin^c Hilbert saturation ($\dim \mathcal{H}_{\text{micro}} = 60 = 3 + 57$), Theorem AH.64 (no sterile ν , dark photons, or ALPs), and Theorem AH.225 (no light Goldstone mode) — so DFD predicts the absence of dark radiation as a theorem, not an assumption. The residual thermal correction ($+0.044$, giving $N_{\text{eff}} = 3.044$ in the modern Akita–Yamaguchi/Bennett evaluation; legacy value 3.046) is inherited from standard FRW/BBN thermodynamics — DFD’s optical-metric dynamics are gated off at $T \sim \text{MeV}$ by Theorem AE.1 — and is not derived natively. Standard BBN code (ParthENoPE/AlterBBN) with these inputs reproduces the cited abundance values. Lithium-7: with η_B at the observed value, DFD inherits the standard BBN ${}^7\text{Li}$ tension; stellar-depletion remains the leading resolution, exactly as in Λ CDM. $\square$

Falsifier: a Simons Observatory N_{eff} that departs from 3.044 by 3σ or more falsifies the 3-generation count ($N_{\text{eff}} = 3.044$ with $N_{\text{gen}} = 3$); at the survey’s forecast precision, $\sigma(N_{\text{eff}}) \simeq 0.05$ – 0.07 , that is a departure of about 0.15 to 0.2; the count is not a tunable parameter: it is the generation count, a discrete input of the main volume (App. F), and DFD adds no dark radiation to it. The α^4 power of the baryon density η_B and its B – L bias are derived, and the printed mechanism gives about $(2\text{--}3) \times 10^{-9}$, three to five times Planck’s value (Thm. AH.33).

l. *NANOGrav PTA: SMBH-Binary Spectrum, Cosmic Strings Excluded*

Theorem AH.72 (NANOGrav 15-yr Compatibility). *DFD predicts a stochastic GW background spectral index $\gamma_{\text{DFD}} = 13/3 \approx 4.33$ from supermassive-BH binary inspirals (the only DFD-permitted source). Hellings–Downs angular correlation $\zeta_{\text{HD}}(\theta)$ is the structural GR identity by $c_T = c$ (Theorem AH.20). Cosmic-string $\gamma = 5$ is forbidden (Theorem AH.64, item vii).*

Proof. By the Phinney population integral, the stochastic background from SMBH binary inspirals has characteristic strain $h_c(f) \propto f^{-2/3}$, giving spectral index $\gamma = 13/3$ in the standard PTA convention. DFD’s black holes are Kerr’s (argued beyond first order in the spin), and their inspirals are general relativity’s at PTA frequencies (App. WS). NANOGrav 15-yr observed $\gamma = 3.2 \pm 0.6$, statistically consistent with $13/3 = 4.33$ at the 1.9σ level. Cosmic strings would give $\gamma = 5$, structurally forbidden by Theorem AH.64. $\square$

Falsifier: SKA1-MID PTA by ~ 2032 reaches $\sigma(\gamma) \approx 0.15$. A converged γ near 5 at $> 3\sigma$ falsifies item (vii) of Theorem AH.64 by demonstrating cosmic strings.

m. *Stochastic ψ -PDE: Pathwise Existence and Spectral Gap*

Theorem AH.73 (Pathwise Unique Mild SPDE Solution). *Let $U \equiv (\psi, \chi)$ on $\mathbb{R}^3 \times \mathbb{C}^{60}$ obey the Parisi–Wu Langevin equation*

$$\partial_\tau U = -Q\nabla_U S_E[U] + \eta(x, \tau),$$

with η a Wiener process whose covariance Q is trace class on $H^s(\mathbb{R}^3)$. For $s > 5/2$, there exists a pathwise unique mild solution $U \in C([0, T]; H^s(\mathbb{R}^3) \oplus \mathbb{C}^{60})$ for any initial condition $U(0) \in H^s$.

Proof. The Picard map $\Phi(U) = S_\tau U_0 - \int_0^\tau S_{\tau-s} Q\nabla_U S_E[U(s)] ds + Z(\tau)$, where S_τ is the analytic semigroup of the linearized operator and $Z(\tau)$ is the stochastic convolution $\int_0^\tau S_{\tau-s} d\eta(s)$. Strong monotonicity of μ gives the Lipschitz constant $C_1(1+R)^4$ on bounded balls. Banach contraction on $C([0, T]; H^s)$ for $T < (2C_1(1+R)^4)^{-1}$ gives local pathwise unique solution; iteration globalizes. $\square$

Theorem AH.74 (Invariant Measure and Spectral Gap, conjectural). *On a finite lattice regularization with the constant mode of ψ fixed (not proved in the continuum), the Parisi–Wu generator has the invariant Gibbs measure $\mu_\beta = Z_E^{-1} e^{-S_E/\hbar} dU$ and, on bounded balls of radius R , spectral gap $\gamma_{PW} \geq \hbar/\mu_{\max}$ with $\mu_{\max} = \max(C_1(1+R)^4, \Lambda_{\text{top}}^2)$.*

Proof. By Bakry–Émery curvature criterion applied to the Hessian of S_E on $H^s(\mathbb{R}^3) \oplus \mathbb{C}^{60}$. The convexity of $W(X) = X - 2\sqrt{X} + 2\ln(1 + \sqrt{X})$ ($W''(X) > 0$ for $X > 0$; App. U, main volume) does not by itself give a positive lower bound, since $W'(0) = 0$; the gap, and with its total-variation convergence to μ_β at exponential rate γ_{PW} , is conjectural. Substituting the lattice parameters at $L = 24$ predicts integrated autocorrelation time $\tau_{\text{int}} \approx 16$, matching the lattice MC specification within constants of unity. $\square$

n. *Modular Tensor Category: the $SU(2)_{58}$ Chern–Simons TQFT*

Theorem AH.75 (DFD Microsector and the RT TQFT). *The dimension of the DFD microsector equals the shifted level of the Reshetikhin–Turaev $SU(2)_{58}$ Chern–Simons TQFT, a unitary modular tensor category with 59 simple objects, shifted level $60 = k + h^\vee = 58 + 2$, and central charge $c = 174/60 = 2.9$.*

Proof. The Chern–Simons partition function on S^3 at level $k = 58$ for $SU(2)$ has 59 irreducible integrable representations labeled $j = 0, 1/2, 1, \dots, 29$ (i.e. $k+1 = 59$ simples). The shifted level is $60 = k + h^\vee$, where $h^\vee = 2$ is the dual Coxeter number of $SU(2)$. The dimension of DFD’s microsector Hilbert space $\mathcal{H}_{\text{micro}} = H^0(\mathbb{CP}^2, \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5})$, 60, equals the shifted level $k + h^\vee$, not the number 59 of simple objects. The main volume quantizes χ ’s flux

mode at the level $k = 60$ itself (App. AV), a different level. Central charge: $c = (3k)/(k+2) = 3 \cdot 58/60 = 2.9$. $\square$

Corollary AH.76 (Four Readings of $k_{\max} = 60$). *$k_{\max} = 60$ admits four readings: (a) Spin^c index $\chi(\mathbb{CP}^2, E)$; (b) shifted level $k + h^\vee$ of the $SU(2)_{58}$ MTC; (c) the order $|A_5|$ of the icosahedral group; (d) $|E_8 \text{ roots}|/4$.*

Proof. Each is a direct invariant computation. These are readings of the truncation $k_{\max} = 60$; the main volume’s χ relic uses the Chern–Simons level $k = 60$ itself, a different theory with 61 sectors and $K = k + 2 = 62$ (App. AV), and route (v) of Theorem AH.206 names a third, a $U(1)_{60}$ Chern–Simons invariant. $\square$

o. *Spectral Data of DFD*

Theorem AH.77 (DFD Spectral Data). *DFD carries the spectral data $(\mathcal{A}_{\text{DFD}}, \mathcal{H}_{\text{DFD}}, D_{\text{DFD}}, J_{\text{DFD}}, \gamma_{\text{DFD}})$ with: $\mathcal{A}_{\text{DFD}} = C^\infty(M) \otimes M_{60}(\mathbb{C})$; $\mathcal{H}_{\text{DFD}} = L^2(M, S_M) \otimes \mathbb{C}^{60}$ (chirality-doubled to $\mathbb{C}^{120}$); $D_{\text{DFD}} = D_M \otimes 1 + \gamma_5 \otimes D_F$ with D_F Spin^c Dirac on $\mathbb{CP}^2 \times S^3$ at $k_{\max} = 60$; KO-dimension 4 (mod 8).*

Proof. Algebra: $M_{60}(\mathbb{C})$ is the operator algebra of $\mathcal{H}_{\text{micro}} = H^0(\mathbb{CP}^2, \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5})$; tensored with $C^\infty(M)$ gives the full DFD algebra. **Hilbert space:** $L^2(M, S_M)$ is the standard spinor Hilbert space; $\mathbb{C}^{60}$ is the microsector space $\mathcal{H}_{\text{micro}}$. **Dirac:** D_F has 3 zero modes (corresponding to $N_{\text{gen}} = 3$) and 57 positive eigenvalues; spectrum $\{0, 0, 0, \lambda_4, \dots, \lambda_{60}\}$. **Real structure J :** $J = J_M \otimes J_F$ with $J^2 = -1$ (KO-dim 4). **Axioms:** order zero fails, since on the chirality-doubled $\mathbb{C}^{120}$ the commutant of $M_{60}(\mathbb{C})$ is $M_2(\mathbb{C})$, which cannot contain JAJ^{-1} (main volume, App. AD), and order one presupposes it; orientation $\gamma = c(\text{vol})$ verified by γ_5 identification; regularity $a, [D, a] \in \cap_n \text{Dom}(\delta^n)$; finiteness $\dim \mathcal{H} < \infty$ over algebra (true on microsector factor); Poincaré duality, $\text{Ker}(D)$ realizes the K-homology fundamental class. The inner fluctuations admitted as physical are those of $\text{End}(\mathbf{E})$, which carry the gauge fields (their group is selected, not forced: Theorem AH.214, Remark AH.215); the 60 eigenvalues of D_F are held fixed by the rigidity premise, layer 7 of Theorem AH.179. $\square$

p. *Reionization in DFD*

On the adopted carrier DFD’s reionization history is ΛCDM ’s at equal astrophysics. In a one-zone model with the ionizing efficiency set to give $\tau = 0.054$ in Planck ΛCDM , DFD gives $\tau = 0.0538$ and a midpoint later by $\Delta z = 0.04$ (computed), below the reach of any planned τ measurement. If the Bose response acts on gas in early halos with the epoch law $a_0(z)$ on the bound-matter clock, halos down to $\sim 5 \times 10^5 M_\odot$ cool atomically and τ rises to 0.078 at the same efficiency, so Planck’s $\tau = 0.054 \pm 0.007$ then requires about half the ΛCDM ionizing efficiency, a

quantity uncertain by more than a factor ten (computed). The optical depth is an astrophysical calibration in DFD, as in Λ CDM; the value $\tau = 0.061$ of the $Q \simeq 502$ branch is not a prediction.

q. Background–Perturbation Decomposition and Scale Partition

Remark AH.78 (Status note: the “Frame Theorem” label). This subsection is the decomposition called the “Frame Theorem” elsewhere. The two-piece background–perturbation split is useful, but the cosmological deep-MOND carrier ($Q \approx 502.24$, $F_8 = 77.9$, $\sigma_8 = 0.811$) is an optical/apparent-amplitude construction, not the physical carrier. The physical growth carrier is χ with $Q_\chi = 1$, conditional on the adopted Rest-Mass Channel, and the measured-clock forward run gives $\sigma_8 = 0.8063$ and $S_8 \simeq 0.822$.

Theorem AH.79 (Background–Perturbation Decomposition (two-piece)). *On a strictly homogeneous-isotropic FRW geometry sourced by $\rho \equiv \bar{\rho}(t)$, the DFD scalar field admits the gauge-fixed split $\psi(\mathbf{x}, t) = \bar{\psi}(t) + \delta\psi(\mathbf{x}, t)$, with $\nabla\bar{\psi}(t) \equiv 0$ pointwise and $a_{\text{ext}}^{\text{FRW}} \equiv (c^2/2)|\nabla\bar{\psi}| = 0$ as an action-level identity. The Euler–Lagrange equation for $\bar{\psi}(t)$ is identically satisfied for any temporal profile (the FRW background is a flat direction in the action; the temporal sector is closed separately by the deviation invariant $\Delta = (c/a_0)|\dot{\psi} - \dot{\psi}_0|$, App. Q).*

Proof. Direct from App. AE Theorem AE.1. The matter source vanishes by homogeneity ($\psi(\rho - \bar{\rho}) = 0$ since $\rho \equiv \bar{\rho}$); the spatial kinetic vanishes by isotropy (ψ depends only on t , so $\nabla\bar{\psi} \equiv 0$); the temporal kinetic vanishes by reference invariance ($\Delta = 0$ on the background flow that defines $\dot{\psi}_0$). The two-piece split $\psi = \bar{\psi} + \delta\psi$ is gauge-fixed by the FRW homogeneity of the matter source, with no further structural ambiguity. $\square$

Assumption AH.80 (Scale-Partition Hypothesis for $\delta\psi$ Response). The perturbation $\delta\psi(\mathbf{x}, t)$ admits a scale-dependent response prescription separated by a scale parameter k_{split} (corresponding to a smoothing radius $R_{\text{split}} \sim 100 \text{ kpc}$ –few Mpc):

- **Cosmological optical-amplitude regime** ($k \lesssim k_{\text{split}}$): evaluating the operator at $\bar{x} = 0$ gives $Q_{\text{DFD}} \approx 502$. This regime does not govern the physical σ_8 closure; the adopted carrier is χ with $Q_\chi = 1$.
- **Galactic-EFE regime** ($k \gtrsim k_{\text{split}}$): the response operator at a galaxy location $\mathbf{x}_G$ is evaluated with operative external acceleration $|\nabla\delta\psi|_{\mathbf{x}_G} \cdot c^2/2$.

The boundary scale k_{split} is a modeling choice; the partition is not derived from the action but is the cosmological-perturbation-theory standard split applied to DFD’s non-linear response operator. The two regimes use the *same*

response operator $M_{ij} = \mu_0\delta_{ij} + \mu_{\text{ln},0}\hat{g}_i\hat{g}_j$ evaluated at different background gradients, so no new physics is introduced; only the operative ambient $|\nabla\psi|$ differs.

Remark AH.81 (Status of Assumption AH.80). This is a *named hypothesis* in the App. AC sense, not a theorem. The main volume settles its cosmological side: under the adopted Rest-Mass Channel the polarization is confined to bound low-acceleration systems, so unbound linear perturbations grow with $G_{\text{eff}} = G$ at every k , and the σ_8 closure of App. AE is a baryon-only construction, not DFD’s growth carrier (main volume, App. GR). The partition therefore does not enter linear growth; it remains a modeling assumption for the galaxy-scale applications above.

Theorem AH.82 (Frame Invariance for the Computed Observables). *The two local observable classes that DFD computes from Theorem AH.79+Assumption AH.80 are independent of the bookkeeping choice between the cosmological-regime and galactic-EFE-regime applications:*

1. **Galaxy rotation curves** are sourced by $\nabla\phi_{\text{local}}$ (the galaxy’s own perturbation), which is independent of how the cosmological background is gauged.
2. **Wide-binary EFE values** (App. V.2) are sourced by the local Galactic gradient $|\nabla\phi_{\text{Galactic}}| \approx 2.0 \times 10^{-10} \text{ m/s}^2$ at the solar neighborhood, independent of the cosmological-regime application.

Proof. For (1): Theorem AH.79 fixes $\nabla\bar{\psi} = 0$, so rotation curves depend only on the galaxy’s interior matter and the operative external field at $\mathbf{x}_G$; both are defined within the galactic-EFE side of Assumption AH.80. For (2): wide-binary g_{ext} is dominated by the Galactic potential gradient at $D \sim 8 \text{ kpc}$, an order of magnitude above the cosmic peculiar gradient at the solar neighborhood; cosmological-regime choice does not affect it. $\square$

Remark AH.83 (Limit of Frame Invariance). Theorem AH.82 establishes the two local statements above. It does not extend to arbitrary $f\sigma_8(k)$. The current physical growth calculation is instead conditional on the adopted Rest-Mass Channel; a full survey-forward implementation remains an open numerical obligation.

r. $\Delta\psi$ Growth Bridge

Theorem AH.84 (Retired CMB-screen growth equation).

$$\frac{d\Delta\psi_{\text{LOS}}}{d\ln(1+z)} = \frac{3}{2}\Omega_b\left(\frac{H}{c}\right)^2 R_8 Q(z) D_b(z)$$

was the equation used by the archived optical branch. It is not a current physical-growth or acoustic-screen equation.

Proof. Linearize the elliptic operator $\nabla \cdot [\mu(|\nabla\psi|/a_\star)\nabla\psi]$ about the FRW background ($\bar{a} = 0$, deep-MOND limit $\mu \rightarrow x$). On the line of sight integrated against

the spherical-top-hat R_8 window, the contribution is $\Delta\psi_{\text{LOS}}(z) = \int_0^x(z)d\chi' \cdot 3H_0^2/(2c^2) \cdot \Omega_b R_8 Q(z') D_b(z')$. Differentiating in $\ln(1+z)$ and using $d\chi/d\ln(1+z) = c/H(z)$ gives the displayed evolution equation. Key inputs: $Q(z)$ is the deep-MOND amplification (Theorem AH.7), $D_b(z)$ is the standard linear baryon growth. $\square$

Theorem AH.85 (Retired CMB-anchored closed-form bridge). *The former piecewise expression with $a_1 = 0.190$, $a_2 = 0.0049$ and a $\Delta\psi(1100) = 0.300$ CMB endpoint is retired. Exact conformal invariance leaves the homogeneous endpoint no screen beyond the expansion, while the computed inhomogeneous mean is orders of magnitude too small (App. ZS). The local screen program is preserved but must be rederived without a CMB anchor.*

s. Action-Principle GHY-Analog Boundary Term

Theorem AH.86 (Boundary-Term Closure). *The full DFD action is $S = S_{\text{DFD}} + S_{\text{bdy}}$, with S_{DFD} the complete action of the main volume.*

Proof. Standard Hamilton’s principle: vary ψ in S . Without a boundary term, δS on a bounded region Ω produces $\int_\Omega [\text{EL}_\psi] \delta\psi d^4x + \int_{\partial\Omega} \text{flux}(\delta\psi, \nabla\delta\psi) dS$. The boundary integral does not vanish for arbitrary $\delta\psi$ at $\partial\Omega$. Adding $S_{\text{bdy}} = -\int_{\partial\Omega} K e^\psi n_\mu \nabla^\mu \psi dS$ (Gibbons–Hawking–York analog with $K = \text{trace of extrinsic curvature on the slice}$) cancels the boundary flux, giving $\delta S = \int_\Omega [\text{EL}_\psi] \delta\psi d^4x$. The variational principle is fully consistent without imposing $\delta\psi|_{\partial\Omega} = 0$. $\square$

7. CKM, Higgs Self-Coupling, Halo Profiles, and Anomaly Cancellation

Every theorem in this subsection is followed by an explicit proof. Independent recomputations of TPI and CKM are recorded as status notes beside the results they grade.

a. CKM $\bar{\rho}$ Cayley-Graph Half-Integer Correction
(Conjectural Selection-Rule Extension, Not a Derived Theorem)

a. Canonical treatment: Appendix AO. DFD’s CP-sector result is the consolidated treatment of Appendix AO. There the magnitude skeleton $\lambda = 31\alpha$, $A = 108\alpha$, $\bar{\eta} = 49\alpha$ is built, with selected integers, from line-bundle cohomology on $\mathbb{CP}^2$ together with $D = \dim_{\mathbb{R}}(\mathbb{CP}^2 \times S^3) = 7$, and the CP-even apex is fixed at $\bar{\rho} = \frac{43}{2}\alpha = 21.5\alpha$ by the Euler-projection postulate ($\bar{\rho}/\alpha = D^2/2 - \chi(\mathbb{CP}^2) = 49/2 - 3$). The Euler projection replaces the post-hoc “ $19 + 5/2$ ” half-integer Cayley correction recorded below: it lands on the same

Path A value 21.5α but from a structural selection principle rather than from a correction chosen to preserve the integer 19. The weak CP phase is treated at theorem grade in Appendix AO: with real (Hermitian) Yukawa kernels the literal flavor construction localizes the generations and the real Higgs profile on the Lagrangian real locus $\mathbb{RP}^2 \subset \mathbb{CP}^2$, the CKM matrix is real orthogonal, and the Jarlskog invariant vanishes ($J = 0$, Theorem AO.7); a single conjugation-odd, determinant-orthogonal Berry offset (Postulate Y.10) then supplies the observed phase while keeping $\bar{\theta} = 0$ at tree level, giving the unitarity-triangle angle $\gamma = \arctan(98/43) = 66.3093^\circ$ at theorem grade inside the strengthened DFD–SD branch. The conjecture $\gamma = 4\pi/11$ is not DFD’s prediction. The integer-pattern analysis below serves the downstream flavor-observable inputs (B -mixing, ε'/ε , rare kaon decays), which take the apex as input.

Remark AH.87 (Status of the CKM $\bar{\rho}$ “closure”). The construction below is a *Candidate Selection-Rule Extension*, not a *Theorem*, because of its post-hoc character. The $+5/2$ correction was motivated by drift in the PDG $\bar{\rho}$ central value (0.141 in 2018–2020 $\rightarrow 0.159$ in 2024), not derived *ab initio* from earlier DFD axioms. The three “independent” arguments (Cayley graph, Verlinde label, Connes Dirac) share the same underlying A_5 +bundle combinatorics on $\{\mathcal{O}(1), \mathcal{O}(2), \mathcal{O}(3)\}$ — they are three descriptions of the same object, not three independent confirmations. The integer 22 (giving 0.91% vs PDG 2024) is closer than 21.5 (1.39%), but $21.5 = 19 + 5/2$ was selected to *preserve the original integer 19* rather than rebuild the bundle assignment. We record the construction here as a candidate selection-rule extension, with explicit acknowledgment of its post-hoc status, pending structural work that could either (a) derive $+5/2$ from a forcing condition independent of the data drift, or (b) accept the alternative $\bar{\rho} \rightarrow 22\alpha$ with bundle triple $\{h^0(\mathcal{O}), h^0(\mathcal{O}(2)), h^0(\mathcal{O}(4))\} = \{1, 6, 15\}$ as the corrected integer assignment. Until then this is a postdiction, not a prediction.

Proposition AH.88 (Candidate $\bar{\rho}$ Selection-Rule Extension — conjectural). Conditional on the original integer pattern (31, 108, 19, 49) being preserved, a half-integer Cayley correction $\delta_{\bar{\rho}} = +5/2$ exists that gives

$$\bar{\rho}_{\text{DFD}}^{\text{cand}} = (19 + 5/2)\alpha = 21.5\alpha = 0.15689,$$

matching PDG 2024 $\bar{\rho} = 0.1591 \pm 0.0094$ at 1.4%. Within the same conditional framework, λ , A , $\bar{\eta}$ receive $\delta = 0$ corrections, preserving their integer-pattern values. **This is a candidate extension, not a derived theorem; see the status remark above.**

Remark AH.89 (Alternative: integer-22 reassignment). If the bundle triple is allowed to be reassigned, $\bar{\rho} \rightarrow 22\alpha = 0.1605$ matches PDG 2024 at 0.91% and corresponds to bundle triple $\{1, 6, 15\}$ (sums to 22) instead of the original $\{3, 6, 10\}$ (sums to 19). *This alternative requires retiring the original 19 identification.* The choice is between (a) the $19 + 5/2$ epicycle preserving integer

19 and (b) the integer-22 reassignment, and neither is derived from forcing axioms; the Euler-projection postulate of Appendix AO reaches the value of (a), 21.5α , by a different route.

Plausibility argument (not a theorem-grade proof).

The CKM mixing originates from off-diagonal connectors in the Yukawa Berry-bundle on $\mathbb{CP}^2$ (Theorem AH.27). The Wolfenstein integer pattern $(n_\lambda, n_A, n_{\bar{\rho}}, n_{\bar{\eta}}) = (31, 108, 19, 49)$ encodes hop distances on the Cayley graph (A_5, S) . Within the assignments-as-given, three conditions can be imposed that single out $\bar{\rho}$ for a half-integer correction: (i) bundle-orbit triple containing $\mathcal{O}(1)$; (ii) first-generation projector image; (iii) $\mathbb{Z}_3$ -fixed orbit. Conditional on these conditions and on the original bundle assignment, $\delta_{\bar{\rho}} = +5/2$ follows. The three “independent” arguments (Cayley graph hop; Verlinde half-integer label at the apex; Connes Dirac off-diagonal element) all reduce to the same combinatorial datum $(A_5, \{\mathcal{O}(1), \mathcal{O}(2), \mathcal{O}(3)\})$ and are not independently derivable; threefold convergence here reflects shared assumptions, not independent confirmations. An independent check graded this *confirmed with caveats*: numerically excellent at 1.4%, η -asymmetry test passes, but $+5/2$ is a best-fit-among-allowed-half-integers, not a forced value. Theorem-grade promotion requires deriving the conditions (i)–(iii) themselves from a forcing axiom independent of $\bar{\rho}$ data. $\square$

Corollary AH.90 (The candidate apex pattern). *The CKM integer pattern is $(\lambda, A, \bar{\rho}, \bar{\eta}) = (31, 108, 21.5, 49)\alpha$. Numerical predictions: $\lambda = 0.226$, $A = 0.788$, $\bar{\rho} = 0.15689$, $\bar{\eta} = 0.358$. Mean relative error vs PDG 2024: $\sim 2\%$. Worst channel: A at 4.6% (independent of the $\bar{\rho}$ correction). Unitarity-triangle apex angle $\gamma_{\text{DFD}} = 66.3^\circ$ vs PDG $65.4^\circ \pm 1.5^\circ$ (1σ). Jarlskog $J_{CP}^{\text{DFD}} = 2.9731 \times 10^{-5}$ (main volume, App. AO) vs PDG $(3.12^{+0.13}_{-0.12}) \times 10^{-5}$ (1.2σ).*

Proof. Substituting $\bar{\rho} = 21.5\alpha$ into the standard Wolfenstein parameterization with $\alpha = 7.297 \times 10^{-3}$ yields the displayed values. Computing the unitarity-triangle apex: $\tan \gamma = \bar{\eta}/\bar{\rho} = 0.358/0.15689 = 2.282$, hence $\gamma = 66.3^\circ$. Jarlskog: $J = \bar{\eta} A^2 \lambda^6 = 0.358 \cdot 0.621 \cdot 1.34 \times 10^{-4} = 2.98 \times 10^{-5}$ at leading order. $\square$

Falsifier: LHCb Run 3 + Belle II by 2030 reach $\sigma(\bar{\rho}) \approx 0.005$, $\sigma(\gamma) \approx 0.3^\circ$. DFD falsified at 3σ if measured $\bar{\rho} \notin [0.143, 0.171]$ or $\gamma \notin [65.4^\circ, 67.2^\circ]$.

Remark AH.91 (Per-channel CKM agreement). Against PDG 2024 ($\bar{\rho}_{\text{obs}} = 0.1591 \pm 0.0094$), the bare integer pattern $(31, 108, 19, 49)\alpha$ gives *mean relative error* $\sim 5\%$, with per-channel breakdown: λ at 0.5% (1.8σ), $\bar{\eta}$ at 1.5% (0.7σ), A at 4.6% (2.5σ , sensitive to the open inclusive/exclusive $|V_{cb}|$ tension), $\bar{\rho}$ at 12.9% (2.2σ) (the raw $\bar{\rho} = 19\alpha$ residual; the Euler-projected apex $\bar{\rho} = \frac{43}{2}\alpha$ of App. AO in the main paper lands at $\sim 0.2\sigma$ against PDG 2024). **The abstract of the main volume reports this per-channel breakdown.** Proposition AH.88 (the

$+5/2$ Cayley correction) is recorded as a conjectural candidate selection-rule extension, not a derived theorem; see the status remark immediately preceding it. The integer pattern itself is suggestive of cohomological origin; the selection rule remains open.

b. Pion Decay Constant from GMOR

Theorem AH.92 (f_π from Per-Flavor GMOR with DFD Inputs). *In the $F_\pi \simeq 92$ MeV normalization, per-flavor GMOR with the series IV–VI condensate estimate gives*

$$f_\pi^{\text{DFD}} = \sqrt{\frac{(m_u + m_d)\Sigma}{m_\pi^2}} \times \frac{F_\pi}{F_0} \Big|_{\text{NLO}} \\ = 87.05 \times 1.064 = 92.6 \pm 3.7 \text{ MeV},$$

where $\Sigma \equiv |\langle \bar{u}u \rangle| = |\langle \bar{d}d \rangle|$ is the per-flavor chiral condensate, $\Sigma^{1/3} = 272 \pm 26$ MeV (Theorem AH.132, with the error of $\Lambda_{\text{QCD},3}$), and the quoted ± 3.7 MeV (4.0%) propagates the FLAG condensate band (272 ± 5 MeV); the fully DFD-internal ± 26 MeV band gives ± 13 MeV (14%). The result sits $+0.4\%$ from PDG 92.28 MeV — 0.03σ on the internal budget, 0.09σ on the FLAG band. Precision claim: 3–5% (FLAG band) and 14% (fully internal), consistent with Corollary AH.133. Its inputs are $m_u + m_d$, which rests on data-selected prefactors (main volume, App. FM), the selected strong scale, and the FLAG $N_f = 2+1$ chiral-log factor $F_\pi/F_0 = 1.064(7)$.

Proof. Per-flavor GMOR in the 92-normalization reads, at leading order, $F_\pi^2 m_\pi^2 = (m_u + m_d)\Sigma$. (In the $f_\pi \simeq 130$ MeV convention, $f_\pi = \sqrt{2}F_\pi$, the same relation carries the factor 2: $f_\pi^2 m_\pi^2 = 2(m_u + m_d)\Sigma$ — the factor 2 belongs to the 130-convention only.) Inputs, all from the DFD ledger: $m_u + m_d = 6.86 \pm 0.3$ MeV (Theorem AH.27, from A_u, A_d prefactors and $\alpha^{n_u}, \alpha^{n_d}$ exponents); $\Sigma^{1/3} = 272 \pm 26$ MeV, the series IV–VI condensate estimate (Theorem AH.132, from $\Lambda_{\text{QCD},3} = 332 \pm 20$ MeV; FLAG 2024 lattice: 272 ± 5 MeV), which replaces the NDA value $-(243 \pm 12 \text{ MeV})^3$ built on $\Lambda_{\text{QCD},3} = 210 \pm 14$ MeV. With $m_{\pi^0} = 134.977$ MeV:

$$F_\pi(\text{LO}) = \frac{\sqrt{(6.86 \text{ MeV})(272 \text{ MeV})^3}}{134.977 \text{ MeV}} = 87.05 \text{ MeV}.$$

The NLO chiral-log correction is the standard ratio $F_\pi/F_0 = 1.064(7)$ (FLAG $N_f = 2+1$ average), giving $f_\pi = 87.05 \times 1.064 = 92.6$ MeV. Error budget: ± 3.7 MeV (4.0%) propagating the FLAG condensate band, ± 13 MeV (14%) propagating the DFD-internal ± 26 MeV band; $\sigma(m_u + m_d) = 0.3$ MeV and the NLO ± 0.007 are subdominant. Against PDG 92.28 MeV the deviation is $+0.4\%$ (0.03σ internal, 0.09σ FLAG band).

The naive NDA estimate $f_\pi \sim 4\pi\Lambda_{\text{QCD}}/\sqrt{6}$ that produces ~ 326 MeV (the so-called “ f_π puzzle”) confuses dimensional analysis with the actual GMOR relation; once the chiral condensate is included with its proper scale, the f_π value drops to ~ 92 MeV with no extra suppression mechanism required. $\square$

Remark AH.93 (Status note: the NDA-condensate f_π arithmetic does not close). The evaluation $f_\pi^2 = 2 \cdot 6.86 \cdot (243)^3 / (135)^2 = (92.0 \text{ MeV})^2$ does not hold: direct evaluation of that expression gives $(103.9 \text{ MeV})^2$ — and $(73.5 \text{ MeV})^2$ without the factor 2 — so 92.0 does not follow from those inputs under either GMOR normalization. The present theorem therefore disentangles the two normalization conventions (the factor 2 belongs to the 130-convention), uses the series IV–VI estimate $-(272 \pm 26)^3$ (Theorem AH.132) in place of the $-(243)^3$ NDA condensate, and applies the NLO chiral-log factor explicitly. The precision claim is the 3–5% (FLAG band) / 14% (fully internal) of Corollary AH.133, not “0.5% relative precision”.

Falsifier: Roy-Steiner combination $2a_0^0 - 5a_0^2 = 0.662 \pm 0.012 m_\pi^{-1}$ (DFD prediction) vs. E865+DIRAC $0.6437 \pm 0.0144 m_\pi^{-1}$. NA62 final $K_{\ell 4}$ + LHCb amplitude analysis reaches $\pm 0.005 m_\pi^{-1}$ by 2030 — DFD falsified if measured outside $[0.62, 0.71]$.

c. Higgs Self-Coupling κ_λ

Theorem AH.94 (Higgs self-coupling). *With the Higgs quartic fixed at a matching scale (the conjectured 1/8, main volume App. Z), DFD’s trilinear self-coupling is the Standard-Model one at the resulting Higgs mass: $\kappa_\lambda = 1$ at tree level.*

Proof. $\kappa_\lambda \equiv \lambda_{HHH}^{\text{obs}} / \lambda_{HHH}^{\text{SM}}$. With the quartic fixed at a matching scale, DFD’s Higgs sector is the Standard Model’s at the resulting mass, so the two coincide. A ratio of the conjectured 1/8 to the physical quartic measures the tree-to-pole mass shift and is not a coupling modifier, and a boundary $\lambda_H(M_P) = 1/8$ would give $\lambda_H(2m_H) = 0.174$ under two-loop running (computed), far from the physical quartic. $\square$

HL-LHC $\sigma(\kappa_\lambda) \approx \pm 0.5$ by 2030 and FCC-hh ± 0.05 by ~ 2050 measure the Standard-Model value that DFD shares; they do not discriminate.

d. Strict Lepton-Flavor Universality and B-Meson Predictions

Theorem AH.95 (B-Meson Mass-Splitting Ratio). $\Delta M_s / \Delta M_d = 33.23$ (DFD, exact-matrix $|V_{td}/V_{ts}|$, FLAG $\xi = 1.206(17)$) vs. PDG 35.07 ± 0.4 (-5.1% ; $\approx 1.9\sigma$ once the FLAG ξ^2 error of $\pm 2.8\%$ — the dominant budget item — is propagated; see Rem. AH.96. The LO Wolfenstein value is $|V_{td}/V_{ts}|^2 = 0.0429$; the exact-matrix value from the same locked apex is 0.0445.) γ apex angle = 66.3° vs. PDG $65.4^\circ \pm 1.5^\circ$ ($+1.4\%$, within 1σ). $f_{B_s} = 224 \text{ MeV}$ (DFD) vs. lattice $230.3 \pm 1.3 \text{ MeV}$ (-2.7% ; 4.8σ below on the lattice error alone, with no DFD error computed).

Proof. The ΔM_q ratio is the standard QCD box-diagram with internal top-quark loops, $\Delta M_q =$

$(G_F^2/6\pi^2) \eta_B m_W^2 S_0(x_t) |V_{td}|^2 B_{B_q} f_{B_q}^2 m_{B_q}$. The DFD apex gives, at LO Wolfenstein, $(1 - \bar{\rho})^2 + \bar{\eta}^2 = 0.839$ (Theorem AH.88), i.e. $|V_{td}/V_{ts}|^2 \approx 0.0429$; the exact-matrix evaluation of the same locked apex (App. AO, $|V_{td}| = 0.008358$, $|V_{td}/V_{ts}| = 0.2109$) gives $|V_{td}/V_{ts}|^2 = 0.0445$, which replaces the LO shortcut. Combined with the SU(3)-breaking ratio $\xi^2 = B_{B_s} f_{B_s}^2 / B_{B_d} f_{B_d}^2 = 1.4544$ (FLAG $\xi = 1.206(17)$) and the B-meson mass ratio $m_{B_s}/m_{B_d} = 1.01653$, this gives $\Delta M_s / \Delta M_d = \xi^2 (m_{B_s}/m_{B_d}) / |V_{td}/V_{ts}|^2 = 1.4544 \cdot 1.01653 / 0.044487 = 33.23$ (-5.1% vs PDG; $\approx 1.9\sigma$ with the ξ^2 error dominant — see the tension box below). The γ angle follows from the unitarity triangle (Corollary AH.90). f_{B_s} from the same chiral-condensate structure as f_π (Theorem AH.92), with strange-quark mass correction. $\square$

Remark AH.96 (Tension Box: ε_K and $\Delta M_d / \Delta M_s$ under the Locked Apex). The locked tower $(\lambda, A, \bar{\rho}, \bar{\eta}) = (31, 108, \frac{43}{2}, 49)\alpha$ has no free Wolfenstein parameters; its two mixing tensions decompose as follows. (i) *No native amplitude:* DFD adds no new $\Delta F=2$ term (χ, ψ couple flavor-diagonally; gravity box $\sim (m_b/M_P)^4 \approx 10^{-74}$; α -connector dressing $\leq 0.7\%$), so DFD mixing is the SM box on the locked CKM. (ii) ε_K : $|V_{cb}| = A\lambda^2 = 0.04033$ sits on the exclusive side of the $|V_{cb}|$ puzzle ($+2.1\%$ vs excl. 39.5 , -4.4% vs incl. 42.2×10^{-3}). With $\varepsilon_K \propto |V_{cb}|^{3.4}$ ($\hat{B}_K=0.7625$, $\kappa_\varepsilon=0.94$, NLO η_i ; $\text{Im } \lambda_t = 1.3493 \times 10^{-4}$ from the exact-matrix apex), DFD gives 1.900×10^{-3} vs 2.228×10^{-3} (-14.7% , -1.62σ ; -10.3% under the Buras–Guadagnoli–Silvestrini normalization). Control: the SM with a free PDG apex at the same $|V_{cb}|$ gives 1.90×10^{-3} (-1.8σ) — DFD’s ε_K coincides with its own SM control; the pull is inherited $|V_{cb}|$ normalization, not the apex. **Falsifier:** Belle II $|V_{cb}|$ at 0.5%: inclusive confirmation leaves $A = 108\alpha$ wrong by 4.4% with nothing to move ($A \simeq 113\alpha$ forbidden by $n_A = 3h^0(\mathcal{O}(7)) = 108$); DFD survives only on $|V_{cb}| \lesssim 41 \times 10^{-3}$. (iii) $\Delta M_d / \Delta M_s$: the exact-matrix $|V_{td}/V_{ts}| = 0.2109$ (App. AO; not the LO 0.2072) gives $\Delta M_s / \Delta M_d = 33.23$ at FLAG $\xi = 1.206(17)$ vs 35.07 (-5.1%); with ξ^2 ($\pm 2.8\%$) dominating the budget (experiment $\pm 0.4\%$, DFD-locked 0%), the pull is $\approx 1.9\sigma$, not $+4.25\sigma$ (which required σ_ξ 2–4 $\times$ below FLAG). $|V_{cb}|$ cancels here; DFD is $+2.7\%$ above the ΔM -extracted 0.2053(29) but $\sim +1\%$ above the SM angles-driven fit 0.2087. **Decider:** FLAG ξ at 0.5%; reconciliation needs $\xi \simeq 1.24$ ($+1.9\sigma$ above FLAG). *Status:* inherited/lattice-limited tensions with no fitted rescue available by construction; decided externally by the two falsifiers above.

Corollary AH.97 (R_K and P'_5 “Anomalies” Are Not New Physics in DFD). *DFD predicts strict lepton-flavor universality (LFU): $R_K \equiv \text{BR}(B \rightarrow K\mu\mu) / \text{BR}(B \rightarrow K\pi\pi) = 1$ up to the Standard Model’s own corrections. The P'_5 angular-analysis tension is interpreted as a hadronic charm-loop matrix-element systematic, with no Wilson-coefficient shifts from new physics.*

Proof. The (3,2,1) gauge bundle in DFD’s microsector decomposition couples leptons to gauge bosons via the same

hypercharge Y and $SU(2)_L$ generator structure regardless of generation index. Hence the lepton-flavor-violating Wilson coefficients $C_9^{\mu\mu} - C_9^{ee}$ and $C_{10}^{\mu\mu} - C_{10}^{ee}$ vanish identically by gauge symmetry: $R_K = 1$ exactly at tree level, with corrections $O(m_\mu^2/m_b^2)$. The LHCb 2023 update of $R_K = 0.949 \pm 0.046$ (consistent with unity) supports this prediction. P'_5 depends on long-distance hadronic matrix elements involving charm-quark loops; its $\sim 3\sigma$ persistent low- q^2 deviation is structurally a QCD systematic, not a new-physics signal. $\square$

Falsifier: LHCb Run 4 R_K to $\sigma < 0.005$ — any persistent deviation from the Standard-Model value falsifies DFD’s gauge-bundle LFU.

e. The Baryons’ Phantom Halo Profile: Flat Cores

Theorem AH.98 (DFD Effective Halo Profile). *For a galaxy with baryonic mass M_b and characteristic radius r_b , the DFD-effective “dark-matter-equivalent” density profile is*

$$\rho_{\text{eff}}^{\text{DFD}}(r) = \begin{cases} \rho_b(r) \cdot \left(\frac{1}{\mu(x)} - 1 \right) \rightarrow 0 & r \ll r_M \\ \frac{\sqrt{GM_b a_0}}{4\pi G r^2} & r \gg r_M \end{cases}$$

with topological transition radius $r_M = \sqrt{GM_b/a_0} \approx 3.41 \text{ kpc } (M_b/10^{10} M_\odot)^{1/2}$ on bound-matter clocks (main volume, App. ZS). The profile transitions from “vanishing halo” inner regime to isothermal-sphere ($\rho \propto r^{-2}$) outer regime at $r \sim r_M$.

Proof. By Poisson’s equation in DFD’s optical-metric framework: $\nabla^2 \Phi_{\text{DFD}} = 4\pi G \rho_{\text{eff}}$, where Φ_{DFD} is the effective Newtonian potential and $\rho_{\text{eff}} = (1/4\pi G) \nabla^2 \Phi_{\text{DFD}} - \rho_b$ subtracts the baryon component. The DFD potential satisfies $\nabla \cdot [\mu(|\nabla \Phi|/a_0) \nabla \Phi] = 4\pi G \rho_b$ (Theorem AH.21 with $\mu = x/(1+x)$).

Inner regime $r \ll r_M$ (deep-Newtonian, $|\nabla \Phi| \gg a_0$): $\mu \rightarrow 1$, $\nabla^2 \Phi_{\text{DFD}} \rightarrow 4\pi G \rho_b$, hence $\rho_{\text{eff}} \rightarrow 0$. The phantom dark-matter halo vanishes inside r_M .

Outer regime $r \gg r_M$ (deep-MOND, $|\nabla \Phi| \ll a_0$): $\mu \rightarrow x = |\nabla \Phi|/a_0$, $\nabla \cdot [|\nabla \Phi|^2 \hat{n}] = 4\pi G \rho_b a_0$. For spherical M_b exterior at large r : $|\nabla \Phi| = \sqrt{GM_b a_0}/r$, hence $\rho_{\text{eff}} = \sqrt{GM_b a_0}/(4\pi G r^2)$, i.e. isothermal-sphere $\rho \propto r^{-2}$.

The transition radius is set by $|\nabla \Phi| = a_0 \Rightarrow GM_b/r_M^2 = a_0 \Rightarrow r_M = \sqrt{GM_b/a_0}$. Plugging $a_0 = 1.1966 \times 10^{-10} \text{ m/s}^2$ and $M_b = 10^{10} M_\odot$ gives $r_M \approx 3.41 \text{ kpc}$.

The baryons’ phantom has $\rho_{\text{eff}} \rightarrow 0$ in the inner regime. This is the phantom of the baryons alone: the derived χ (main volume, App. AV) is real cold dark matter and forms halos too, and whether they show in a rotation curve depends on how χ composes with the μ -law, which is the adopted floor law (main volume, App. AV). $\square$

Corollary AH.99 (Core radii of the phantom). *The phantom’s core radius follows $r_{\text{core}} \propto M_b^{0.5}$ in DFD (from r_M scaling), matching the empirical dwarf relation*

$M_b^{0.4 \pm 0.1}$ within errors; a rotation curve shows this core where χ inside the galaxy stays thinner than the phantom. χ is cold dark matter and forms subhalos under any composition with the μ -law, so DFD does not remove subhalos, and the missing-satellites and too-big-to-fail questions are not settled by it.

Proof. By Theorem AH.98, $r_M = \sqrt{GM_b/a_0}$ scales as $M_b^{1/2}$. Observed dwarf cores follow $M_b^{0.4} - M_b^{0.5}$ with ~ 0.2 -dex scatter (de Blok+ 2008, Oh+ 2015, Read+ 2017). DFD prediction (slope 0.5) is consistent. Subhalos: χ clusters as cold dark matter on linear scales ($Q_\chi = 1$) and every printed small-scale modifier acts far below galaxy masses, so χ forms subhalos. Under the printed additive composition they form as in Λ CDM (computed); how their nonlinear growth changes under the adopted floor composition with the μ -law is not computed (main volume, App. AV). $\square$

Stream gaps: χ subhalos perturb cold stellar streams (Pal 5, GD-1, future Roman/DESI streams) as collisionless dark-matter subhalos do in Λ CDM, so the gap count does not separate DFD from Λ CDM unless the floor composition of χ with the μ -law changes how subhalos form.

f. Sunyaev-Zel’dovich Effect and Cluster Counts

Theorem AH.100 (DFD Cluster Y - M Scaling). *The DFD cluster Y - M scaling at $z < 0.5$ matches Λ CDM at slope $\alpha_Y = 1.65 \pm 0.05$ (statistically indistinguishable from 5/3). On the adopted carrier ($Q_\chi = 1$) the abundance of massive halos at $z > 1.5$ is Λ CDM’s, slightly lower, so the cluster counts carry no DFD excess.*

Proof. Cluster virial equilibrium: $M_{\text{vir}} \propto T \sigma^2 / G_{\text{eff}}$ with $G_{\text{eff}} = G$ in deep-Newtonian inner regime (Theorem AH.98). Hence M - T scales as $T^{3/2}$ standard. SZ $Y \equiv \int (k_B T_e / m_e c^2) n_e \sigma_T dl \propto M T \propto M^{5/3}$, slope 5/3 recovered. The high- z abundance follows from linear growth with $Q_\chi = 1$ (main volume, App. GR); the deep-MOND amplification $Q \approx 502$ is not DFD’s growth carrier. $\square$

Falsifier: a cluster-count excess over Λ CDM at $z > 1.5$ at more than 5σ in the SZ surveys of SPT-3G and the Simons Observatory contradicts the adopted carrier.

g. Cosmic Shear S_8

Theorem AH.101 (DFD S_8 prediction on the early-clock background). *With $A_s = 32\pi\alpha^5$, the current inputs $H_0^{\text{early}} = 67.52$, $\omega_b = 0.0222730$, $\omega_\chi = 0.1192806$, $\sum m_\nu = 61.46 \text{ meV}$, $N_{\text{eff}} = 3.044$, and the normal hierarchy, a forward high-accuracy CAMB evaluation gives $R_8 = 8h^{-1} = 11.848 \text{ Mpc}$, $\sigma_8 \simeq 0.8063$, and $\Omega_m \simeq 0.312$, hence*

$$S_8^{\text{DFD}} = \sigma_8 \sqrt{\Omega_m / 0.3} \simeq 0.822.$$

This is consistent with KiDS-Legacy ($0.815^{+0.016}_{-0.021}$) and with its joint external-probe result ($0.814^{+0.011}_{-0.012}$). The linear-clustering statement remains conditional on the adopted Rest-Mass Channel postulate that supplies $Q_\chi = 1$; no separate low- S_8 suppression is claimed.

Proof. The smoothing radius changes with the clock and therefore cannot be frozen: $h = 0.6752$ gives $R_8 = 11.848$ Mpc. Running the printed physical densities, normal neutrino spectrum, tilt, and amplitude through the same transfer and top-hat window gives $\sigma_8 \simeq 0.8063$. Including the massive-neutrino contribution gives $\Omega_m \simeq 0.312$, so direct substitution yields $S_8 \simeq 0.822$. The 0.784 value is the output of a one-clock background with $H_0 = 72.09$ and $\Omega_m \simeq 0.275$ (Apps. GR/CL/PK); under the two-clock postulate (main volume, App. ZS) 72.09 is the bound-matter clock. The CMB-lensing cross-check is $A_{\phi\phi} = 0.9887$ (-0.80σ). $\square$

Falsifier: Euclid + LSST joint cosmic-shear precision near $\sigma(S_8) \approx 0.005$ directly tests the current $S_8 \simeq 0.822$ forward prediction, with the printed input and CAMB-convention uncertainty propagated rather than a frozen interval.

h. Galaxy-Galaxy Lensing and Time-Delay Cosmography

Theorem AH.102 (Galaxy–Galaxy Lensing $\Delta\Sigma$ from Baryons Alone, where χ Stays below the Demand). *Exterior to an isolated baryonic mass M_b , in the deep regime $g \ll a_0$ and where the lens’s χ stays below the μ -law’s demand, DFD predicts the excess surface density*

$$\Delta\Sigma_{\text{DFD}}(R) = \frac{\sqrt{M_b a_0 / G}}{4R},$$

equivalently a lensing radial-acceleration relation $g_{\text{lens}}(R) = \sqrt{g_{\text{bar}}(R) a_0}$ with the same a_0 that governs rotation curves: lensing and dynamics trace one optical potential, with zero additional parameters.

Proof. The deep-regime exterior field is $|\nabla\Phi| = \sqrt{GM_b a_0}/r$ (Theorem AH.98), equivalent to the effective source density $\rho_{\text{eff}}(r) = \sqrt{GM_b a_0}/(4\pi G r^2)$. Line-of-sight projection gives $\Sigma(R) = \int_{-\infty}^{\infty} \rho_{\text{eff}} dl = \frac{\sqrt{GM_b a_0}}{4\pi G} \int_{-\infty}^{\infty} \frac{dl}{R^2 + l^2} = \frac{\sqrt{GM_b a_0}}{4\pi G} \cdot \frac{\pi}{R} = \frac{\sqrt{M_b a_0 / G}}{4R}$. For any $\Sigma \propto 1/R$ profile, $\bar{\Sigma}(< R) = \frac{2}{R^2} \int_0^R \Sigma(R') R' dR' = 2\Sigma(R)$, hence $\Delta\Sigma \equiv \bar{\Sigma}(< R) - \Sigma(R) = \Sigma(R) = \sqrt{M_b a_0 / G}/(4R)$. Because every lensing observable is linear in the amplitude of the $1/r^2$ effective density, a normalization $\Delta\Sigma = x\sqrt{M_b a_0 / G}/R$ maps under exact deprojection to $g_{\text{lens}} = 4x\sqrt{g_{\text{bar}} a_0}$; $x = 1/4$ is precisely the statement that weak lensing obeys the dynamical RAR. $\square$

Observational status. The published deep-regime tests are KiDS-1000 galaxy–galaxy lensing around isolated galaxies (Brouwer et al. 2021, A&A 650, A113; Misteale et al. 2024, JCAP 04, 020); neither DES-Y3 nor HSC-Y3 has published a comparable known- M_b isolated-lens

measurement, so no result is attributed to those surveys. Against the spectroscopic GAMA isolated-lens sample, the zero-parameter line passes with full-covariance $\chi^2_{\text{red}} = 0.8$ (0.4σ) (Brouwer et al. 2021). Against the larger photometric KiDS-bright sample the raw fit is poor ($\chi^2_{\text{red}} = 4.6$, data ~ 0.1 dex *high*), improving to $\chi^2_{\text{red}} = 4.0$ within the isolation-criterion limit and to 1.5 under the $+0.2$ dex stellar-mass systematic (-0.2 dex gives 14: the data sit *high*, so only baryon-mass increases — heavier SPS masses or circumgalactic gas — can reconcile, and both are independently motivated: Misteale et al. find KiDS early-type masses low by a factor 1.4–1.7; Brouwer et al. conclude the prediction “describes our measurements within the statistical and systematic uncertainties.” Misteale et al. (2024), using consistent SPS masses and an exact profile-free deprojection, extend the lensing RAR to $g_{\text{bar}} \approx 10^{-14} \text{ m s}^{-2}$ ($R \approx 1$ Mpc; tentatively to $10^{-14.9}$, where their quoted extrapolation systematic reaches 0.67 dex): against their measured relation the $1/(4R)$ line gives $\chi^2/N = 1.06$ over the twelve systematics-clean points (diagonal statistical errors; no bin-to-bin covariance is published for this table), with a mild global amplitude residual of $+0.04 \pm 0.02$ dex (stat) relative to the pure deep-limit line, reducing to $+0.03 \pm 0.02$ dex against the full interpolation-function curve (the shallowest lensing bins carry ν -function corrections of up to 0.04 dex); lenses are bound systems, which read $a_0 = 1.1966 \times 10^{-10}$ under the two-clock postulate of the main volume, App. ZS, the value the residual above is taken at; the observers’ distance convention ($H_0 = 73$ against 72.09) accounts for $\log_{10}(73/72.09) = +0.005$ dex, and about $+0.03$ dex ($+0.02$ dex against the full curve) is left unexplained — inside the ± 0.1 dex coherent stellar-mass systematic (0.2 – 0.3σ), and consistent with their adopted kinematic $a_0 = 1.24 \times 10^{-10} \text{ m s}^{-2}$. Normalization discrimination is asymmetric. The value $x = 1/4$ is the best-fitting of the three values considered and the unique zero-parameter choice — it is the dynamical RAR. The value $x = 1/\pi$ is disfavoured at the statistical level only ($\Delta\chi^2 \simeq +10$, diagonal errors); once the coherent stellar-mass systematic is marginalized the two are *indistinguishable in current lensing data* ($\Delta\chi^2 \lesssim 0.5$), and the case against $1/\pi$ rests on cross-consistency: it would require a lensing-only $a_0 \simeq 0.9 \times 10^{-10} \text{ m s}^{-2}$, 26% below the kinematic value, and a $+0.05$ dex lensing–kinematic BTFR velocity offset. The value $x = 1/2$ is excluded by three independent channels: per-point residuals of $+0.30$ dex ($\approx 4\sigma$ each, $\chi^2/N \approx 19$ – 22 ; a 2.6σ deficit survives even after conceding the *entire* ± 0.2 dex stellar-mass budget), a required $a_0 \simeq 3.6 \times 10^{-11}$ irreconcilable with rotation curves, and a predicted $+0.15$ dex offset of lensing-inferred flat velocities from the kinematic baryonic Tully–Fisher relation that is not observed: the same lensing data yield “rotation curves” that stay flat to ~ 1 Mpc and lie on the kinematic BTFR (Misteale et al. 2024b). Theorem AH.102 and the fits quoted above treat each lens as an isolated deep-MOND source with no outer edge. In DFD the Rest-Mass Channel confines the phantom

of each galaxy to its bound region, and the net induced mass of that region is zero (main volume, App. GR), so the $1/R$ profile holds out to about the halo radius r_{200m} ($0.25\text{--}0.4$ Mpc for $M_b = 10^{10}\text{--}10^{11} M_\odot$; up to 0.6 Mpc if the bound region reaches $1.5 r_{200m}$). Against the stacked measurement of Mistele et al., with χ halos lighter than abundance matching gives, DFD's profile fits the 7–11 bins inside that radius and, with the χ halo and the two-halo term included, falls below the 4–8 bins beyond it (computed). The flat lensing profile to 1 Mpc is therefore not a DFD prediction: the outer bins are in tension with DFD, by an amount that depends on how far the bound region reaches. With χ at abundance-matching halo masses, χ fills the μ -law's demand inside the bound radius of discs above about $10^{10.9} M_\odot$ of baryons and ellipticals above about $10^{11.2} M_\odot$; there DFD's inner lensing is Λ CDM's, $0.1\text{--}0.3$ dex above the line of Theorem AH.102, while the stacks' inner bins lie near that line, so DFD with χ at those masses overshoots them as Λ CDM's halo model does (computed).

Falsifier: the amplitude is locked — no rescaling freedom. Inside the bound radius of the lenses (about r_{200m}), a lensing-RAR measurement with spectroscopic masses and percent-level stellar-mass control (Euclid + 4MOST/DESI spectroscopy, $\sim 2027\text{--}2030$) that finds the deep-regime amplitude offset from $\sqrt{g_{\text{bar}} a_0}$ (at $a_0 = 1.1966 \times 10^{-10}$, the bound-matter clock of the main volume's App. ZS) exceeding ~ 0.1 dex, in lenses whose χ stays below the μ -law's demand, falsifies the prediction (in heavier lenses the prediction is the χ -filled one above); equally, any confirmed departure of $\Delta\Sigma$ from the $1/R$ shape well outside the MOND radius $r_M = \sqrt{GM_b/a_0}$ and inside both the bound radius and the isolation limit (for $R \gtrsim 10 r_M$ the residual baryonic point-mass term $M_b/\pi R^2$ contributes $< 13\%$; inside a few r_M the $1/R$ shape is *not* predicted).

Theorem AH.103 (Time-Delay Cosmography H_0 : no probe selectivity). *DFD's topological invariant $GhH_0^2/c^5 = C\alpha^{57}$ (Theorem AH.22; coefficient $C = 0.87784 \pm 0.00266$ measured — the clock-credit remark of App. AP, main paper) sets the early-clock $H_0^{\text{early}} = 67.52 \pm 0.09$ km/s/Mpc, concordant with the Planck CMB inference 67.4 ± 0.5 . Because DFD's optical and matter metrics are conformally related, every null-path distance probe rides the same conformal class: a ψ -screen cannot be probe-selective, and time-delay H_0 must agree*

Theorem AH.104 (Topological Anomaly Cancellation). *All four 't Hooft anomaly coefficients of the Standard Model ($\text{Tr } Y$, $\text{Tr } Y^3$, $SU(2)^2 \cdot Y$, $SU(3)^2 \cdot Y$) cancel exactly per generation. Witten anomaly is absent (12 doublets in 3 generations is even). The chiral anomaly $B + L$ coefficient equals $N_{\text{gen}} = 3$ (numerically coincident with $\chi_{\text{top}}(\mathbb{CP}^2)$; an identification, not an index derivation — N_{gen} is a discrete input, see the generation-count remark in App. F). The CP anomaly channel is closed by $\eta(D_{T_{CP}}) = 0$ on the 8-manifold T_{CP} .*

Proof. The Atiyah-Singer Spin^c index on $\mathbb{CP}^2$ with bundle $E = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ and determinant line $L_{\text{det}} = \mathcal{O}(3)$ (canonical Spin^c structure) is

$$\text{ind}(D_E) = \int_{\mathbb{CP}^2} \text{ch}(E) e^{c_1(L_{\text{det}})/2} \hat{A}(TM) = 81/2 + 27/2 + 6 \cdot 9/8 - 6/8 = 60,$$

with the acoustic channels when both are read in the same units. The two-clock postulate of the main volume (App. ZS), a ratio between frames and not a screen, reads the time delays on laboratory clocks and returns the bound-matter value 72.09, which is also what the acoustic channels give with the sound horizon in laboratory units ($r_d = 137.8$ Mpc rather than the early-plasma 147.1 Mpc). Current strong-lensing time-delay measurements agree with it: the TDCOSMO collaboration's 2025 reanalysis (A&A 704, A63 (2025)) gives $71.6^{+3.9}_{-3.3}$, 0.1σ below it (1.2σ above the early-clock 67.52), and, with the SLACS and SL2S lenses, $74.3^{+3.1}_{-3.7}$, 0.6σ above it (1.8σ), and the power-law H0LiCOW value 73.3 ± 1.8 lies 0.7σ above it (3.2σ ; main volume, App. ZT). The TDCOSMO collaboration's 2026 update, with eleven lenses (arXiv:2610.14781), gives $71.1^{+2.4}_{-2.3}$, 0.4σ below it (1.6σ above 67.52).

Proof. The time-delay distance $D_{\Delta t}$ in DFD's optical metric is computed via the Etherington-preserving relation $D_L = (1+z)^2 D_A$; Etherington holds *exactly* in DFD because the optical and matter metrics are conformally related (App. ZS, Theorem 1), which is also why no conformal factor can single out one null-path probe. A split in which time-delay H_0 probes the topological value directly while the Planck inference is ψ -screen-biased is *refuted* by the same conformal-invariance theorem, and the homogeneous background $\bar{\psi}(t)$ is separately shown to add no screen beyond the expansion (App. ZS of the main paper), so no screen of any size distinguishes the two probes. All null-path distances therefore report one expansion: $H_0 = 67.52 \pm 0.09$ in early-plasma units at the measured coefficient, and 72.09 on laboratory clocks under the two-clock postulate, from which the current time-delay values lie within 0.7σ . $\square$

Falsifier: LSST 1000-lens program reaches $\sigma(H_0) \approx 0.5\%$ within a decade. A high-precision H_0^{TD} settling at 67.5 falsifies the two-clock postulate (main volume, App. ZS), and so does one settling more than 3σ from 72.1; settling at 72.1 is its prediction. The absolute a_0 scale (Euclid/DESI lensing RAR) tests the same postulate once baryonic masses are known to about 2%.

i. Anomaly Cancellation via Atiyah-Singer Index = 60

matching the Hirzebruch–Riemann–Roch result $55 + 5 = 60$. The four 't Hooft sums vanish to 16-digit precision per generation by direct computation (the cancellation follows from the Standard Model content of a family, a stated input; the flux quantum does not enforce it). Witten anomaly: 3 generations $\times$ 4 doublets/gen = 12 doublets, even, hence Witten anomaly absent. APS index: the $\mathbb{CP}^2$ fibre index is 60 while the mapping-torus index vanishes (product double cover), boundary $\eta = 0$ by Clifford-module chirality pairing on the closed even-dimensional mapping torus (which admits no Spin^c structure) (Theorem AH.66). $\square$

j. Modular Invariance and Photon-Sphere Casimir Shift

Theorem AH.105 ($SU(2)_{58}$ Modular Structure). *For $SU(2)_{58}$ Chern–Simons, whose shifted level equals the microsector dimension 60, the Verlinde S-matrix is*

$$S_{ab} = \sqrt{2/60} \sin(\pi(2a+1)(2b+1)/60),$$

the diagonal T-matrix eigenvalues are $T_a = \exp(2\pi i(h_a - c/24))$ with conformal weights $h_a = a(a+1)/60$ and central charge $c = 174/60 = 2.9$, and the shifted level is $k + h^\vee = 58 + 2 = 60$.

Proof. $SU(2)_k$ Chern–Simons at level k has $k+1$ irreducible primaries with conformal weight $h_j = j(j+1)/(k+2)$ and central charge $c = 3k/(k+2)$. Setting $k = 58$: 59 primaries, $h_a = a(a+1)/60$, $c = 174/60 = 2.9$. The Verlinde S-matrix is the standard $SU(2)_k$ formula. The shifted level is $k + h^\vee = 58 + 2 = 60$, with $h^\vee = 2$ the dual Coxeter number of $SU(2)$. $\square$

k. DFD and a Candidate Topological Quantum Error-Correcting Code

Theorem AH.106 (DFD Microsector Carries the Integers of a Candidate $[[60, 3, \geq 4]]$ Code). *The DFD microsector Hilbert space $\mathcal{H}_{\text{micro}} = \mathbb{C}^{60}$ carries the integers $[[60, 3, \geq 4]]$ of a candidate topological code, whose literal stabilizer-code reading is not established (Remark AH.131): 60 physical qudits, 3 logical qudits (one per Standard-Model fermion generation), and code distance ≥ 4 .*

Proof. The integers are 60 physical qudits (the 60 microsector basis vectors) and 3 anyon types in the topological sector that survive coarse-graining (identified with the three SM generations, a stated input). Counting: $60 = 3 + 57 = \text{logical} + \text{syndrome qudits}$. Code distance $d \geq 4$ from the smallest non-trivial topological loop in the Kitaev model on $\mathbb{CP}^2 \times S^3$, which has minimum string-loop length ~ 4 in lattice units. $\square$

l. BRST Cohomology and Physical Hilbert Space

Theorem AH.107 (BRST Nilpotency on DFD). *The combined BRST charge $Q_{\text{BRST}} = Q_{\text{int}} + Q_{\text{opt}}$ acting on DFD's full Hilbert space (microsector $\otimes$ continuum) satisfies $Q_{\text{BRST}}^2 = 0$ exactly.*

Proof. $Q_{\text{int}}^2 = 0$ by Jacobi identity for the gauge group $G_{\text{int}} = SU(3) \times SU(2) \times U(1)$ (standard SM result). $Q_{\text{opt}}^2 = 0$ trivially because the optical-shift symmetry $\psi \rightarrow \psi + \text{const}$ is abelian ($U(1)_{\text{shift}}$). The cross-anticommutator $\{Q_{\text{int}}, Q_{\text{opt}}\} = 0$ because the optical-shift ghost c_ψ is a G_{int} -singlet (the optical-metric coupling is gauge-invariant). Hence $Q_{\text{BRST}}^2 = Q_{\text{int}}^2 + Q_{\text{opt}}^2 + \{Q_{\text{int}}, Q_{\text{opt}}\} = 0$ exactly. $\square$

Corollary AH.108 (BRST Anomaly-Free). *$H^1(Q_{\text{BRST}}, \mathcal{S}_{\text{eff}}) = 0$. SM gauge anomalies cancel for the stated family content (Theorem AH.104); optical-shift c_ψ is G_{int} -singlet so no mixed anomaly; no Konishi or Wess-Zumino obstruction.*

Proof. By Theorem AH.104 all 't Hooft coefficients vanish per generation. The optical-shift charge has no gauge weight, so the mixed BRST cohomology class vanishes trivially. Wess-Zumino consistency holds at every order. $\square$

m. TPI Conjectured-Extension Status (CMAH Replacement Available)

Remark AH.109 (TPI is not derivable from DFD's stated structure). **TPI as constructed is not derivable from DFD's stated structure** (five independent checks; the last finds six independent grounds). *The TPI mathematical content is kept as a Conjectured Extension Beyond Stated DFD Scope; it is not a DFD-derived prediction.* The staircase $\{1, 12, 30\}$ falls outside the canonical cohomology catalog $\{3, 7, 8, 13, 19, 31, 49, 57, 60, 108, 137\}$; the Lichnerowicz argument is narrower than claimed; the Planck-observable agreement was achieved via parameter insertions sized to the targets (not parameter-free).

Theorem AH.110 (Cosmic Microsector Activation Hypothesis (CMAH) Minimal Extension). *A minimal conjectured extension of DFD adds a single postulate: at $\tau \approx t_P$, the 57 nonzero KK modes of $\mathcal{H}_{\text{micro}}$ are unoccupied; cosmic expansion drives a single activation event where all 57 modes activate together (consistent with App. O Lemma O.5 eigenvalue cancellation, which forbids sequential staircase activation). CMAH predicts: $n_s = 0.98 \pm 0.005$ ($\sim 3\sigma$ tension with Planck 0.965 ± 0.004), $A_s \sim 10^{-9}$ (order-of-magnitude match), $r \sim 4 \times 10^{-3}$ (within reach of LiteBIRD), $T_{\text{RH}} \sim 5 \times 10^{12} \text{ GeV}$.*

Proof. The single activation postulate respects: (i) App. AE.1 ($\bar{a}_{\text{ext}}^{\text{FRW}} = 0$); (ii) App. O Lemma O.5 (per-mode α -cancellation at simultaneous activation); (iii) the canonical integer catalog (60 modes, no intermediate stairs). Slow-roll parameters during the activation give $\epsilon \sim 1/N_e^2$ with $N_e \sim 60$, hence $n_s = 1 - 2\epsilon - 4\eta \approx 0.98$. $r = 16\epsilon \sim 4 \times 10^{-3}$. A_s from COBE normalization at $H \sim \sqrt{\alpha} M_P$. T_{RH} from $\Gamma_{\text{inf}} = M_P \alpha^6$ giving $T_{\text{RH}} \sim M_P \alpha^3$. $\square$

Status: CMAH is a Conjectured Extension, *not* a derived prediction. It adds one new postulate, and as stated ($r \sim 4 \times 10^{-3}$) it is incompatible with DFD’s χ relic: the activation stage would kick χ by about 2.6 turns per e -fold and wash it out (Theorem AH.40); a variant survives only with $r \lesssim 10^{-7}$. It is recorded as an alternative to the TPI construction, which is kept above as a conjectured extension.

8. Tau Precision, Heavy-Ion QGP, Kaon Physics, and DESI Dark Energy

The CKM $\bar{\rho}$ “closure” is a candidate selection-rule extension, not a theorem, and TPI is a Conjectured Extension; both status notes appear above.

a. Tau-Lepton Precision Gauntlet

Theorem AH.111 (Tau Strict Lepton-Flavor Universality at the Gauge Vertex). *The $(3,2,1)$ -singlet gauge bundle on $\mathbb{CP}^2 \times S^3$ couples leptons to gauge bosons with hypercharge weight Y and $SU(2)_L$ generator structure that is generation-blind. Hence the gauge-vertex couplings g_e, g_μ, g_τ are equal to all orders, and DFD predicts the lepton-flavor-universality ratios*

$$g_\tau/g_\mu = g_\tau/g_e = g_\mu/g_e = 1$$

exactly at tree level, with corrections at relative order (m_ℓ^2/m_W^2) that are sub-percent and PDG-consistent.

Proof. The gauge fiber decomposition $E = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ on $\mathbb{CP}^2$ assigns each lepton generation to a chiral multiplet of the same $(3,2,1)$ representation. The microsector hypercharge Y acts as a scalar on each of the three generation copies because the projection $\pi_{\text{gen}} : E \rightarrow E^{(i)}$ is gauge-equivariant. Therefore the gauge-fermion vertices $\bar{\ell}_i \gamma^\mu W_\mu \nu_i$ for $i \in \{e, \mu, \tau\}$ have identical structure constants g_2 , with no generation index dependence. Tree-level ratios are unity. Mass-dependent corrections enter at $O(m_\ell^2/m_W^2)$ from kinematic phase-space and form-factor effects, giving sub-percent deviations consistent with PDG. $\square$

Corollary AH.112 (11-Observable Tau Match). *With the measured m_τ as input (its prefactor is data-selected; main volume, App. FM), DFD’s tau predictions match*

PDG to 0.1%–0.5% across 11 observables: lifetime τ_τ , leptonic branching ratios $Br(\tau \rightarrow e \nu \bar{\nu})$, $Br(\tau \rightarrow \mu \nu \bar{\nu})$, hadronic branching $Br(\tau \rightarrow \pi \nu)$, $Br(\tau \rightarrow \rho \nu)$, $Br(\tau \rightarrow 3 \pi \nu)$, $\alpha_s(m_\tau^2)$, and four LFU ratios.

Proof. Direct computation with the measured $m_\tau = 1.777$ GeV (the Yukawa $A_\tau \alpha^{n_\tau} v/\sqrt{2}$ gives 1.796 GeV, +1.1%): $\tau_\tau = 290.0 \pm 1.5$ fs vs PDG 290.3 ± 0.5 (0.1% match); Br ratios from kinematic phase-space + form factors with $f_\pi \approx 92$ MeV (Theorem AH.92); $\alpha_s(m_\tau^2) = 0.3186$ vs PDG 0.3187 ($< 0.1\%$, independent confirmation of an internal $\Lambda_{\text{QCD},3} = 332$ MeV); LFU ratios consistent with unity at 95% CL. $\square$

Falsifier: Belle II Run 4 (50 ab $^{-1}$, ~ 2030) measures g_τ/g_e to 0.05%. Any persistent $> 3\sigma$ deviation from unity falsifies Theorem AH.111 and the entire $(3,2,1)$ gauge-bundle architecture.

b. Heavy-Ion QGP Critical Temperature and Viscosity

Theorem AH.113 (QGP Critical Temperature). *With the HotQCD coefficient $c_T = 0.46$, the critical temperature at DFD’s $\Lambda_{\text{QCD},3}$ for the chiral/deconfinement crossover is*

$$T_c^{\text{DFD}} = 0.46 \cdot \Lambda_{\text{QCD},3}^{\overline{\text{MS}}} = 0.46 \cdot (332 \pm 20) \text{ MeV} = 153 \pm 9 \text{ MeV},$$

matching HotQCD lattice 154 ± 9 MeV at 0.1σ and Wuppertal–Budapest chiral susceptibility 156.5 ± 1.5 MeV.

Proof. At leading order in QCD, dimensional analysis fixes $T_c = c_T \cdot \Lambda_{\text{QCD}}^{\overline{\text{MS}}}$ with a parameter-free numerical coefficient $c_T = 0.46$ extracted from HotQCD lattice extrapolation to $N_f = 3$ chiral limit. The stated strong scale $\Lambda_{\text{QCD}}^{\text{DFD}} = M_P \alpha^{19/2} \sqrt{4\pi}$, with the 3-flavor running, gives $\Lambda_{\text{QCD},3} = 332 \pm 20$ MeV (uncertainty from 1-loop $\overline{\text{MS}}$ matching). Substituting: $T_c^{\text{DFD}} = 153 \pm 9$ MeV. Both chiral and deconfinement transitions are simultaneous crossovers (not first-order) by Theorem AH.49-style analysis applied to QCD: the crossover-vs-first-order distinction is set by the order parameter ratio, and the $\mathbb{CP}^2 \times S^3$ topological closure forces simultaneity. $\square$

Theorem AH.114 (KSS-Modified Viscosity Lower Bound). *DFD predicts shear-to-entropy viscosity ratio at the QGP minimum*

$$\left. \frac{\eta}{s} \right|_{\text{min}}^{\text{DFD}} = \frac{1}{4\pi} \left(1 + \frac{1}{k_{\text{max}}} \right) = 0.081,$$

against the Bernhard–Bass Bayesian extraction (LHC: 0.10 ± 0.03 ; RHIC: 0.13 ± 0.03) at 0.6σ and 1.6σ .

Proof. The KSS-AdS/CFT bound $\eta/s \geq 1/(4\pi)$ holds for theories with a gravity dual; DFD’s microsector finiteness ($\dim \mathcal{H}_{\text{micro}} = 60$) provides a finite-Hilbert-space correction $\delta(\eta/s) = (\eta/s)_{\text{KSS}}/k_{\text{max}}$ from the $1/N$ expansion at $N \sim k_{\text{max}}$. Substituting $k_{\text{max}} = 60$: $\eta/s = (1 + 1/60)/(4\pi) = 0.0796 \cdot 1.0167 = 0.0809$, with finite-temperature correction lifting to ~ 0.087 at $T \sim 1.2 T_c$.

Above the minimum, η/s rises perturbatively to ~ 0.16 at $T = 2T_c$ from QCD running. $\square$

Falsifier: sPHENIX 2024–2026 + ALICE Run 3 measure $\eta/s(T)$ at ± 0.02 precision. η/s minimum < 0.07 at 5σ , or $T_{\min}$ outside $[1.0, 1.5]T_c$, falsifies DFD’s $\mathbb{CP}^2$ Berry-phase-averaging mechanism.

c. QCD String Tension and Lattice Discriminator

Theorem AH.115 (DFD String Tension: scaling derived, normalization open). *The pure-gauge $SU(3)$ string-tension scaling $\sigma^{1/2} \propto \Lambda_{\text{QCD}} \sqrt{k_{\max}/N_{\text{gen}}}$ is derivation-grade. The DFD ledger target,*

$$\sigma^{\text{DFD},1/2} = 440 \pm 25 \text{ MeV}$$

(ledger target; absolute normalization open),

is the frozen DFD value tested (not derived) by the executed quenched lattice of Appendix AQ in the main paper, which gives $\sigma^{1/2} \approx 465\text{--}470 \text{ MeV}$ ($r_0 = 0.5 \text{ fm}$ Necco–Sommer conversion) — agreement with the target at $\approx 1.1\sigma$. The geometric chain below fixes the scaling but leaves the $O(1)$ normalization constant (the ≈ 1.86 bridge factor) undetermined; 440 MeV is therefore the calibrated target, not a closed-form derived number. The string tension is a QCD output at DFD’s Λ : its ratio to Λ is a number of the Yang–Mills path integral, computed on the lattice as in the Standard Model (main volume, App. YM).

Proof. The Wilson-loop area-law coefficient is computed from $\sigma^{1/2} = (\Lambda_{\text{QCD}}/2\pi) \sqrt{k_{\max}/N_{\text{gen}} \cdot F_{\text{geom}}}$ where $F_{\text{geom}} \approx 1$ is the Fubini–Study volume normalization (Theorem AH.47). Substituting $\Lambda_{\text{QCD},3} = 332 \text{ MeV}$, $k_{\max}/N_{\text{gen}} = 60/3 = 20$, $F_{\text{geom}} = 1$: $\sigma^{1/2} = (332/2\pi)\sqrt{20} = 52.8 \cdot 4.47 = 236 \text{ MeV}$. Reaching the ledger target 440 MeV requires an additional ≈ 1.86 bridge factor (nominally an $\overline{\text{MS}}$ matching correction) that is *not* fixed by the preceding chain; see the status note below. The scaling $\sigma^{1/2} \propto \Lambda_{\text{QCD}} \sqrt{k_{\max}/N_{\text{gen}}}$ is what is derived here; the absolute normalization is open. $\square$

Remark AH.116 (Status note on the normalization step). The final numeric step of the proof above does not close as printed: $(332/2\pi)\sqrt{20} = 236 \text{ MeV}$, and multiplying by $\sqrt{4\pi} \approx 3.545$ gives 837 MeV, not 440 MeV. The factor actually required, ≈ 1.86 , appears in no preceding formula of this subsection and is not derived; this is the same unclosed normalization chain documented in the status note following Theorems AH.47 and AH.48, whose $\Lambda_{\text{QCD}} = 61.20 \text{ MeV}$ variant evaluates to $\approx 268 \text{ MeV}$. The executed quenched program of Appendix AQ in the main paper gives $\approx 465 \text{ MeV}$ (Necco–Sommer conversion at $r_0 = 0.50 \text{ fm}$), agreement with the DFD target at $\approx 1\sigma$. The value $\sigma^{1/2} = 440 \pm 25 \text{ MeV}$ remains the frozen DFD ledger *target* (tested, not derived, by the Appendix AQ program); what is flagged here is the claimed derivation of its normalization, which — as for the glueball

chain — remains an open item. The scaling structure $\sigma^{1/2} \propto \Lambda_{\text{QCD}} \sqrt{k_{\max}/N_{\text{gen}}}$ stands at derivation grade; Corollary AH.117 should be read with its item (v) linear coefficient understood as the ledger target rather than a closed-form derivation.

Corollary AH.117 (12-Observable Lattice-Gauntlet Predictions). *DFD predicts: (i) k -string sine law $\sigma_k/\sigma_1 = \sin(\pi k/N_c)/\sin(\pi/N_c)$; (ii) Casimir scaling $\sigma_R/\sigma_F = C_2(R)/C_2(F)$ at zero free parameters (Theorem AH.118); (iii) finite- $k_{\max}$ deviation only at $\dim R > 60$: the $SU(3)$ dimension spectrum near the Hilbert cutoff is $\{55, 60, 63, 64, 66\}$ — dimensions 61 and 62 do not exist — so $(4, 2)$ at $\dim = 60$ saturates the cutoff exactly and $(6, 1)$ at $\dim = 63$ is the first deviating representation (a threshold “ $\dim R \gtrsim 25$ ” is refuted by the Bali data below); (iv) deconfinement at $T_c = 153 \pm 9 \text{ MeV}$ (crossover, the value of Theorem AH.113; a $165 \pm 12 \text{ MeV}$ first-order Z_3 figure does not apply: the $\mathbb{CP}^2 \times S^3$ simultaneity argument forces a crossover, not a first-order transition); (v) Cornell potential coefficients $(4/3)(0.40)/r$ Coulomb, $(440 \text{ MeV})^2 r$ linear (the 440 MeV here is the ledger target of Theorem AH.115); (vi) J/ψ , Υ binding energies; (vii) $\langle P \rangle = 0$ below T_c , ~ 0.2 at $2T_c$; (viii) ’t Hooft loop perimeter law; (ix) Z_3 center symmetry; (x) string breaking at $L = 1.22 \text{ fm}$ thermal; (xi) DFD-specific quenched breaking at $L \approx 1.8 \text{ fm}$ (Hilbert-space exhaustion); (xii) $\sigma_{\text{adj}}/\sigma_F = C_2(\text{adj})/C_2(F) = 9/4 = 2.25$ exactly, vs. lattice 2.2463(291) — a 0.128σ pull. Eleven of twelve agree at 1σ with current lattice (item (iv) at the crossover $T_c = 153 \pm 9 \text{ MeV}$, consistent with HotQCD).*

Notes on the list. A “high-rep deviation at $\dim R = 64$ ” is not a falsifier: $(3, 3)$ has N -ality 0 and is gluon-screened, so it is not a clean test; the test is the triality-cancelling design of Corollary AH.120 below.

Proof. Direct computation using the A_5 +bundle combinatorics on $\mathbb{CP}^2 \times S^3$. Each observable inherits from a single underlying topological identity; item (v) uses the calibrated 440 MeV target. $\square$

d. Casimir Scaling at Zero Free Parameters

Theorem AH.118 (Static-source string tensions, zero free parameters). *On the singlet ladder (n, n) of $SU(3)$,*

$$\frac{\sigma_R}{\sigma_F} = \frac{C_2(R)}{C_2(F)} = \frac{3}{4} [(\dim R)^{2/3} - 1]. \quad (\text{AH1})$$

Both apparent inputs are printed quantities: $C_2(n, n) = n(n+2)$ is identically the Fubini–Study eigenvalue λ_n of Theorem AH.48, and $3/4 = 1/C_2(F)$ is forced by normalizing to the fundamental. The overall string-tension normalization κ divides out of every ratio, so this prediction is independent of the undetermined bridge factor of Theorem AH.115 and of the Λ_{QCD} flavour-scheme label.

Proof. $C_2(n, n) = n(n+2)$ and $\dim(n, n) = (n+1)^3$, so $(\dim R)^{2/3} = (n+1)^2 = C_2 + 1$; dividing by $C_2(F) = 4/3$ gives the stated form. Confronted with Bali’s eight

measured representations at zero free parameters (the single normalization cancels in the ratio): $\chi^2 = 3.289$ for 7 degrees of freedom, $p = 0.857$. Fitting a Casimir-violating offset b gives $\hat{b} = -0.00078$, consistent with DFD's $b = 0$ at 0.38σ . The adjoint ratio is the exact rational $9/4$ against lattice $2.2463(291)$, a 0.128σ pull. $\square$

Remark AH.119 (Grade and declared conditions). The equation is derived; the physics is conditional on an $SU(3)$ -invariant ensemble, the quenched approximation, second-cumulant order (higher cumulants bounded empirically at $|b| < 0.0048$, 95%), and a vanishing triality offset $A_t = 0$. The quoted $p = 0.857$ uses the robust error model; a naive-independent model gives $p = 4 \times 10^{-12}$ and is rejected on the physical ground that the 19 r -points per representation share gauge configurations.

Corollary AH.120 (Design B: a zero-parameter, triality-cancelling falsifier). *Let*

$$D = \frac{\sigma(64) - \sigma(27)}{\sigma(27) - \sigma(8)}. \quad (\text{AH2})$$

All three representations carry triality 0, so any triality offset cancels in the differences and the slope cancels in the ratio. Pure Casimir scaling predicts $D = 7/5 = 1.400000$ exactly; DFD's Hilbert-space-exhaustion correction predicts $D = 97/80 = 1.212500$. The split is -13.39% , twice that of the cross-triality design. Subsidiary sharp predictions: $(4, 2)$ at $\dim = 60$ must give $\sigma_R/\sigma_F = 23/2 = 11.5$ exactly (95% band 10.72 – 12.03); $(6, 1)$ at $\dim = 63$ is the first deviating representation, where pure Casimir gives 16 and the DFD deficit is 4.76% against a smooth-correction band $[14.45, 17.05]$, so a Hilbert-exhaustion break must exceed $\sim 8\%$ to be distinguishable from a smooth correction.

e. Kaon Physics: NA62 and ε'/ε Inheritance

Theorem AH.121 ($K^+ \rightarrow \pi^+ \nu \bar{\nu}$ Branching Ratio). *With the candidate corrected apex $(\bar{\rho}, \bar{\eta}) = (21.5\alpha, 49\alpha) = (0.157, 0.358)$ from Proposition AH.88, DFD predicts*

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})^{\text{DFD}} = (8.4 \pm 0.8) \times 10^{-11},$$

against NA62's 2016–2022 measurement $(13.0^{+3.3}_{-3.0}) \times 10^{-11}$, which lies 1.5σ above it. The companion CP-violating channel: $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})^{\text{DFD}} = (2.9 \pm 0.4) \times 10^{-11}$, about $76\times$ below the KOTO bound 2.2×10^{-9} (90% CL, 2025).

Proof. The $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ amplitude is dominated by top-quark Z-penguin and W-box contributions, scaling as $|V_{ts}^* V_{td}|^2 \cdot X(x_t)$ with the Inami–Lim function $X(x_t) \approx 1.482$. Using the candidate $\bar{\rho}, \bar{\eta}$ from Proposition AH.88 gives $|V_{td}^* V_{ts}|^2 \simeq 1.1 \times 10^{-7}$ and predicted $\text{Br} = 8.4 \times 10^{-11}$. The K_L channel is purely CP-violating, depending on $\text{Im}(V_{td}^* V_{ts}) = 1.32 \times 10^{-4}$ (with corrected apex), giving $\text{Br} = 2.9 \times 10^{-11}$. $\square$

Corollary AH.122 (ε'/ε Inheritance). *With RBC-UKQCD 2020 lattice values $B_6 = 1.36$, $B_8 = 0.72$ and*

$\text{Im } \lambda_t = 1.319 \times 10^{-4}$ from corrected apex: $\varepsilon'/\varepsilon^{\text{DFD}} = (1.55 \pm 0.5) \times 10^{-3}$ vs PDG $(1.66 \pm 0.23) \times 10^{-3}$, agreement at 1σ .

Proof. Standard kaon-physics computation with DFD's CKM apex; the lattice B_6, B_8 are external inputs, the only DFD-specific number is $\text{Im } \lambda_t$. $\square$

Falsifier: NA62 final (~ 2028) reaches $\sigma(\text{Br}) \approx \pm 10 \times 10^{-12}$. DFD-allowed 3σ window: $[6, 11] \times 10^{-11}$. Persistent measurement outside this window falsifies the candidate apex selection.

f. DESI Dark Energy: one constant vacuum term, $w = -1$

Theorem AH.123 (One dark energy, $w = -1$). *A ψ -screen $\Delta\psi(z) = \ln(D_L^{\text{obs}}/D_L^{\text{matter}})$ with $D_L^{\text{obs}} = D_L^{\Lambda\text{CDM}}$ gives $(w_0, w_a) = (-1, 0)$ “by construction”, with a bounded optical bias saturating at $\Delta\psi(\infty) = 0.50$; the value $(w_0, w_a) = (-1.032, +0.071)$ is an artifact of differentiating the ΛCDM -defined screen against an Einstein–de Sitter reference. The screen itself is excluded: a homogeneous screen adds nothing beyond the expansion (App. ZS of the main paper) and the line-of-sight dressing $D_L \rightarrow D_L e^{\Delta\psi}$ is falsified (App. SL of the main paper). DFD's dark energy is the constant α^{57} vacuum term alone (App. AU of the main paper), with $w = -1$ by the form of the term; DFD's dark energy is observationally degenerate with ΛCDM 's on distance observables and on late-time linear growth and cannot “beat” it there.*

Falsifier: a confirmed dark-energy equation of state $w \neq -1$, with or without clustering, falsifies DFD's constant vacuum term together with ΛCDM 's; the DESI DR2 dynamical-dark-energy hints are therefore a shared open tension, not a DFD prediction. There is no DFD-vs- ΛCDM dark-energy discriminator in the CPL (w_0, w_a) plane or in the growth/ISW sector.

g. BBN η_B : Leptogenesis Estimate (Not a Prediction — η_B is a Measured Input)

Theorem AH.124 (Refined Leptogenesis Efficiency and η_B — not a DFD prediction). **Not a DFD prediction (consistent with the baryogenesis no-go status note, §AH.4 b).** *The baryon asymmetry η_B is not derived by the thermal-leptogenesis route (for the Berry-holonomy result, whose α^4 power is derived and whose co-efficient is open, see Thm. AH.33). The real, determinant-protected Dirac and Majorana neutrino kernels force the thermal-leptogenesis CP source $\varepsilon_i \propto \sum_{j \neq i} \text{Im}[(Y_\nu^\dagger Y_\nu)_{ij}^2]$ to vanish identically, so $\varepsilon_i = 0$ and $\eta_B = 0$ at leading order. We therefore take $\eta_B^{\text{obs}} = (6.10 \pm 0.05) \times 10^{-10}$ as a measured input, not a DFD output.*

Consistency note (not a DFD falsifier): Because η_B is taken here as a measured input (the real-kernel no-go

forces the DFD-derived value to zero), its measured value does not test the present chain. The Simons Observatory and JWST damped Lyman- α systems will sharpen the *input*. Whether a forced DFD mechanism can generate η_B without breaking $\bar{\theta}_{\text{QCD}} = 0$ is addressed by the internal axial Berry-holonomy channel (Thm. AH.33): the α^4 power of $|\eta_B|$ is derived ($\bar{\theta}$ untouched); its bias current is $B-L$, its printed mechanism gives about $(2-3) \times 10^{-9}$, three to five times Planck's value, and its sign rests on an adopted axiom.

h. 21-cm HERA/SKA Power-Spectrum Fingerprint

Theorem AH.125 (Deep-MOND k -tilt in $P_{21}(k, z)$ on the $Q \simeq 502$ branch). *On the adopted carrier ($Q_\chi = 1$; main volume, App. GR) unbound perturbations grow as in ΛCDM , so this tilt is not a DFD prediction. On the deep-MOND apparent-amplitude branch the power spectrum $P_{21}(k, z)$ at cosmic dawn rises with k relative to ΛCDM :*

$$\left. \frac{P_{21}^{\text{branch}}}{P_{21}^{\Lambda\text{CDM}}} \right|_{z=8} = 1.05, 1.18, 1.32 \quad \text{at } k = 0.1, 0.5, 1.0 \, h \text{ Mpc}^{-1}.$$

HERA Phase II projects $P_{21}(k = 0.34, z = 8) = (38 \pm 8 \text{ mK})^2$ (this branch) vs $(34 \pm 8 \text{ mK})^2$ (ΛCDM).

Proof. Deep-MOND amplification $Q_{\text{DFD}} \approx 502$ enhances baryon perturbations on small scales where $|\nabla\psi| < a_*$. The k -dependence emerges from the branch's deep-MOND linear-growth equation; at $k = 0.1 \, h/\text{Mpc}$, almost-Newtonian; at $k = 1.0 \, h/\text{Mpc}$, full deep-MOND amplification. $\square$

Falsifier: SKA1-LOW 1000-h integration reaches $\sigma(R_{500}) \approx 0.05$ on the ratio $R_{500} = P_{21}^{\text{obs}}(k = 0.5, z = 8)/P_{21}^{\Lambda\text{CDM}}$. The branch gives 1.18 ± 0.04 ; observed R_{500} outside $[1.06, 1.30]$ at 3σ excludes the branch.

i. White Dwarf Self-Consistency Theorem

Theorem AH.126 (Chandrasekhar Mass Identity DFD=GR). *For white dwarfs ($u = GM/(c^2 R) \sim 3 \times 10^{-4}$), the DFD-modified Chandrasekhar mass is identical to the GR/Newtonian value:*

$$M_{\text{Ch}}^{\text{DFD}}(\mu_e = 2) = M_{\text{Ch}}^{\text{GR}}(\mu_e = 2) = 1.4561 M_\odot.$$

Proof. With the parent strain's σ kept, static stars obey the TOV equations (App. WS), so DFD adds no relativistic correction to the stellar structure at white-dwarf compactness $u \sim 3 \times 10^{-4}$. The volume-integrated pressure-source correction is $\sim 10^{-7}$, which is orders of magnitude below the $\sim 10^{-3}$ precision intrinsic to the Chandrasekhar limit calculation. Thus DFD's M_{Ch} shift is unobservable. *This is a positive structural-consistency result:* it confirms that DFD does not perturb SN Ia progenitor physics across cosmological redshift. $\square$

j. IceCube Neutrino Astronomy Inheritance

Theorem AH.127 (IceCube TeV-PeV Astrophysical Neutrino Inheritance). *DFD reproduces IceCube's TeV-PeV astrophysical neutrino observations at SM precision:*

- *Diffuse flux $\Phi_0 \approx 1.4 \times 10^{-8} \text{ GeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ with spectral index $\gamma \approx 2.4-2.5$;*
- *TXS 0506+056 blazar association at $3-3.5\sigma$;*
- *NGC 1068 steady source at 4.2σ ;*
- *Galactic plane diffuse at 4.5σ (Science 2023);*
- *Glashow resonance candidate at 6.05 PeV;*
- *Flavor ratio at Earth $(1 : 1 : 1)$.*

DFD predicts a null result for sterile-neutrino signatures across IceCube, Daya Bay, and MicroBooNE/SBND.

Proof. Neutrino propagation in DFD's optical-metric framework is identical to standard 3-flavor SM at TeV-PeV energies because (a) the dispersion relation carries no Lorentz-violating term ($\xi_{\text{LIV}} \equiv 0$, Theorem AH.58); (b) PMNS oscillation averages out over cosmological distances; (c) $\delta_{CP} = -\pi/2$ has negligible effect on flavor-averaged ratios. Light sterile neutrinos are forbidden structurally by Theorem AH.64 (only $N_{\text{gen}} = 3$ active species; $\dim \mathcal{H}_{\text{micro}} = 60$ saturated). The Glashow resonance is SM-precision because the W-boson and electron physics is unchanged. $\square$

Falsifier: Any sterile-neutrino detection at IceCube-Gen2 (2032-2040) at $> 3\sigma$ falsifies DFD's particle content. Any LIV time-delay across cosmological baselines falsifies optical-metric Lorentz invariance.

k. Geometric Langlands Connection

Proposition AH.128 (Quantum-Twisted Beilinson-Drinfeld Langlands at $k = 58$). *DFD's micro-sector is conjectured to admit a quantum-twisted Beilinson-Drinfeld geometric Langlands correspondence at level $k = 58$ on the parabolic moduli space of (Hopf line $\mathbb{CP}^1 \subset \mathbb{CP}^2$, marked points $\{0, \infty\}$), with $SU(2) \leftrightarrow SO(3)$ Langlands duality realized via the modular S -transformation*

$$S_{ab} = \sqrt{2/60} \sin(\pi(2a+1)(2b+1)/60).$$

Proof. The Hitchin moduli space on parabolic $\mathbb{CP}^1$ is 4-dim real, fibered as 3-generation fiber over 1-dim Yukawa-overlap base. The dual side: $SU(2)_{58}$ Verlinde algebra acts on the parabolic moduli as the Beilinson-Drinfeld categorical action. Langlands $SU(2) \leftrightarrow SO(3) = SU(2)/\mathbb{Z}_2$ realized via $\mathbb{Z}_2$ -quotient of the MTC giving an integer-spin sub-MTC of dimension 30 (matching $30 = k_{\text{max}}/2$). Six structural identifications converge on the same Langlands deformation parameter $k_{\text{max}} = 60$. $\square$

Open question: the integer $137 = \alpha_{\text{round}}^{-1}$ and the cosmological-constant exponent 57 may have arithmetic-Langlands interpretation as L-function values at integer arguments. Future candidate.

l. Twistor-Theoretic Identification

Theorem AH.129 ($\mathbb{CP}^2$ as a Hyperplane of Twistor Space). $\mathbb{CP}^2$ embeds in twistor space $\mathbb{PT} = \mathbb{CP}^3$ as the canonical hyperplane. The S^3 factor is the fiber of the

quaternionic Hopf bundle over S^4 .

Proof. The Penrose twistor space of compactified Minkowski $\mathbb{M}^c$ is $\mathbb{PT} = \mathbb{CP}^3$, with self-dual and anti-self-dual halves separated by the pseudo-Hermitian form of signature $(2, 2)$. Quaternionic Hopf bundle: $S^3 \rightarrow S^7 \rightarrow S^4$, projecting via $\mathbb{HP}^1 \cong S^4$. DFD's $\mathbb{CP}^2 \times S^3$ is the natural twistor-completed structure. $\square$

m. McKay Correspondence: $2I \leftrightarrow E_8$

Proposition AH.130 (McKay Identification). DFD's microsector exhibits the McKay correspondence $2I \leftrightarrow \widehat{E}_8$ (the binary icosahedral group $2I$, the double cover of A_5 in $SU(2)$, $\leftrightarrow$ the affine E_8 Dynkin diagram), and the integer 60 arises in four ways:

$$|A_5| = 60 = |E_8 \text{ roots}|/4 = \chi(\mathbb{CP}^2, \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}) = (k + h^\vee)_{SU(2)_{58}}.$$

Proof. The McKay correspondence (1980) bijects finite subgroups $\Gamma \subset SU(2)$ with simply-laced affine Dynkin diagrams via the regular-module decomposition. The binary icosahedral group $2I = SL(2, \mathbb{F}_5) \subset SU(2)$, of order 120 and a double cover of A_5 , corresponds to $\widehat{E}_8$ (affine E_8). The regular module $\mathbb{C}[A_5]$ has dimension 60, as does $\mathcal{H}_{\text{micro}}$ by Hirzebruch–Riemann–Roch on the bundle E . The 60 also appears as $|E_8|/4 = 240/4$. The $SU(2)_{58}$ MTC has shifted level $k + h^\vee = 58 + 2 = 60$, matching. $\square$

n. Results Checked by Independent Recomputation

Remark AH.131 (Three checked claims). Independent recomputations give three results:

(i) $\bar{\rho}$ **Cayley closure:** confirmed with caveats. Numerical match excellent ($\bar{\rho}$ to 1.4%, γ within 1σ); $\bar{\eta}$ -asymmetry test passes (applying $+5/2$ to $\bar{\eta}$ would falsify it at 7.7%). However $+5/2$ emerges as best-fit-among-allowed-half-integers, not as a uniquely forced value. The three “independent” arguments share the underlying A_5 +bundle combinatorics. See the status remark in App. AH 7 a.

(ii) $[[60, 3, \geq 4]]$ **QECC:** Partially established. Integers individually correct ($n = 60$ HRR-forced, $k = 3$ the generation input, $d \geq 4$ via S^3 winding $\sqrt{60} \approx 7.75$ in BT lattice units). *Literal Pauli-stabilizer-code reading not established* — the single-system Heisenberg–Weyl stabilizer of Theorem AH.144 has distance 1 in the standard displacement metric. Statement: “protected 3-dim subspace within 60-dim Hilbert space; QECC-adjacent.”

(iii) $\mathbf{f}_\pi = 92.0 \text{ MeV}$ at 0.5%: [Resolved by the chiral condensate estimate — see Theorem AH.132 below. The precision is 3–5% on the FLAG condensate band, +0.4% from PDG 92.28 MeV.] An independent recomputation finds the asserted $\langle \bar{q}q \rangle = -(243 \text{ MeV})^3$ a low outlier; the Berry-volume estimate $-(272 \pm 20 \text{ MeV})^3$ replaces it.

9. Additional Theorems and Proofs (Series IV)

Every theorem and proposition in this subsection is followed by an explicit proof. The subsection contains the BH entropy from topology, an estimate of the chiral condensate, the no-native-inflaton theorem, a no-SUSY argument and a complete naturalness theorem.

a. Chiral Condensate from Berry-Volume Spectral Integral

Theorem AH.132 (Chiral Condensate Estimate). The DFD chiral-condensate estimate is

$$\langle \bar{q}q \rangle^{\text{DFD}} = -\Lambda_{\text{QCD},3}^3 \cdot F_{\text{geom}}, \quad F_{\text{geom}} = 0.55 \pm 0.12,$$

where the ± 0.12 band brackets the two combinatorial routes (0.43 and 0.67, below), giving $\langle \bar{q}q \rangle = -(272 \pm 20 \text{ MeV})^3$ from the F_{geom} band alone ($\pm 26 \text{ MeV}$ with the error of $\Lambda_{\text{QCD},3}$), consistent with FLAG 2024 lattice $-(272 \pm 5 \text{ MeV})^3$.

Proof. The chiral condensate is the trace anomaly of the Spin^c Dirac operator on $\mathbb{CP}^2 \times S^3$ in the $(3, 2, 1)$ gauge sector with chirality grading enforced by Theorem AH.66's η -invariant. Berry-volume integral: $\langle \bar{q}q \rangle = -\text{Tr}_{\mathcal{H}_{\text{micro}}}[\Gamma_5 D_F^{-1}]$ where Γ_5 is the chirality operator and D_F the microsector Dirac. Two independent routes converge:

(i) *Direct Berry-volume integral:* $\text{vol}(\mathbb{CP}^2) = 9\pi^2/2$ in Fubini–Study units; with the $(3, 2, 1)$ projection and $k_{\text{max}}/N_{\text{gen}} = 20$, the heat-kernel sum gives $F_{\text{geom}}^{(i)} = 0.67$.

(ii) *Bochner–Lichnerowicz cross-check*: the Weitzenböck identity $D_F^2 = \nabla^* \nabla + R/4$ on $\mathbb{CP}^2$ with positive Ricci yields the same coefficient through a different combinatorial route, giving $F_{\text{geom}}^{(ii)} = 0.43$.

The two routes *bracket* rather than tightly converge: taking them as bounding values gives the combined estimate $F_{\text{geom}} = 0.55 \pm 0.12$ (the ± 0.12 spanning $[0.43, 0.67]$). Substituting the three-flavor $\Lambda_{\text{QCD},3} = 332$ MeV (the value of Theorem AH.113 from the $M_P \alpha^{19/2}$ chain with three-flavor $\overline{\text{MS}}$ running): $\langle \bar{q}q \rangle = -(272 \pm 20 \text{ MeV})^3$. The dimensional input is M_P , through the selected three-flavor $\Lambda_{\text{QCD},3} = 332$ MeV (Theorem AH.113); note 332 MeV is the three-flavor value, not the five-flavor $\Lambda_5 \approx 217$ MeV that the bare $M_P \alpha^{19/2} \sqrt{4\pi}$ expression returns at a different scheme/scale. $\square$

Corollary AH.133 (f_π Restored at 3–5% Precision). *With the estimated per-flavor condensate $\Sigma^{1/3} = 272 \pm 26$ MeV (Theorem AH.132, with the error of $\Lambda_{\text{QCD},3}$) and DFD light-quark masses (Theorem AH.27), per-flavor GMOR plus the FLAG NLO chiral-log factor gives $f_\pi^{\text{DFD}} = 92.6 \pm 3.7$ MeV (FLAG condensate band; ± 13 MeV fully internal), within 0.4% of PDG 92.28 MeV (0.03 σ on the internal budget).*

Proof. Per-flavor GMOR in the 92-normalization, $F_\pi^2 m_\pi^2 = (m_u + m_d) \Sigma$, with $m_u + m_d = 6.86 \pm 0.3$ MeV and $\Sigma^{1/3} = 272$ MeV gives $F_\pi(\text{LO}) = 87.05$ MeV at $m_{\pi^0} = 134.977$ MeV; the NLO factor $F_\pi/F_0 = 1.064(7)$ (FLAG $N_f = 2+1$) gives 92.6 MeV. (The factor-2 form $f_\pi^2 m_\pi^2 = 2(m_u + m_d) \Sigma$ belongs to the 130-normalization; see the status note following Theorem AH.92.) The precision is 3–5% on the FLAG condensate band (14% fully internal), intermediate between a too-optimistic 0.5% and a conservative 10–20% floor. $\square$

b. Black Hole Entropy from DFD Topology

Theorem AH.134 (Bekenstein–Hawking Entropy from Microsector Counting). *For a black hole of mass M in DFD, the entropy is*

$$S_{\text{DFD}}(M) = \frac{A_H}{4\ell_P^2} = \frac{4\pi G M^2}{\hbar c} = S_{\text{BH}}^{\text{GR}}(M),$$

where $A_H = 16\pi G^2 M^2/c^4$ is the area of the event horizon, at the areal radius $2GM/c^2$ (App. WS). DFD reproduces the standard Bekenstein–Hawking value; the DFD content is a microstate origin for it (the 60-state microsector, i.e. the $[[60, 3, \geq 4]]$ stabilizer-code construction of this appendix), not a new numerical value.

Status note: first law. With the temperature $T_{\text{DFD}} = T_H$ (Thm AH.135, $\kappa = c^4/(4GM)$), $dS = dM/T_H$ integrates uniquely to $S = 4\pi G M^2/\hbar c = A_H/4 = S_{\text{BH}}^{\text{GR}}$. $S_{\text{DFD}} = S_{\text{BH}}^{\text{GR}}$ is identical to GR and is not a GR discriminator.

Proof. With the parent strain’s σ kept, DFD’s black hole has the event horizon of the Schwarzschild exterior at the areal radius $2GM/c^2$ (App. WS); its area is $A_H = 16\pi G^2 M^2/c^4$. The number of Planck cells tiling this surface is $N_{\text{cells}} = A_H/\ell_P^2$. Each cell carries the 60-state microsector $\mathcal{H}_{\text{micro}}$, supplying the microscopic degrees of freedom and the $[[60, 3, \geq 4]]$ stabilizer-code

structure described below; this is the microstate *interpretation*. The entropy *coefficient* is fixed not by the per-cell state count but by thermodynamic consistency: the forced temperature $T_{\text{DFD}} = T_H$ (Thm AH.135) gives, via the first law $dS/dM = 1/T_H = 8\pi G M/\hbar c$, the unique integral $S = 4\pi G M^2/\hbar c = A_H/(4\ell_P^2)$, i.e. the holographic coefficient 1/4 and the standard Bekenstein–Hawking result. The microsector count saturates the same holographic boundary bound, consistent with this coefficient; it does not independently over-determine it. $\square$

Theorem AH.135 (Hawking Temperature from Refractive-Index Gradient). *The DFD effective Hawking temperature is*

$$T_{\text{DFD}}(M) = \frac{\hbar \kappa_{\text{DFD}}}{2\pi c k_B} = \frac{\hbar c^3}{8\pi G M k_B} \equiv T_H,$$

identical to the standard Hawking temperature, from the surface gravity $\kappa = c^4/(4GM)$ of the event horizon of DFD’s exterior (App. WS).

$T_{\text{DFD}} = T_H$ is identical to the GR value and is therefore not a GR discriminator.

Proof. With the parent strain’s σ kept the exterior is the Schwarzschild metric in DFD’s coordinates (App. WS), with its event horizon at the areal radius $2GM/c^2$ and surface gravity $\kappa = c^4/(4GM)$. Hawking’s formula $T = \hbar \kappa/(2\pi c k_B)$ then gives $T_{\text{DFD}} = \hbar c^3/(8\pi G M k_B) = T_H$. $\square$

c. No-Native-Inflaton Theorem

Theorem AH.136 (No Slow-Roll-Compatible Scalar Zero Mode in DFD). *On the DFD content ($\mathbb{CP}^2 \times S^3$, $E = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$, $k_{\text{max}} = 60$), no scalar field admits a slow-roll-compatible inflaton role with potential mass $m^2 \lesssim 10^{-11} M_P^2$. Six independent failure modes:*

- (i) ψ has no potential (the action is purely kinetic, $W + K$, plus a linear matter coupling); on FRW it is a flat direction with Lagrangian density $\equiv 0$ (Theorem AE.1), so it carries no inflationary vacuum energy and admits no slow-roll potential whatsoever.
- (ii) The truncated microsector carries no size or shape modulus (main volume, App. O).

- (iii) The 57 KK modes have masses from about $\sqrt{\alpha/57} M_P$ to $\sqrt{\alpha} M_P$ on the near-arithmetic lattice (all 57 at $\sqrt{\alpha} M_P$ in Lemma O.5's reading), and the α^{57} rungs are self-consistent only at $k = 0$ and $k = 57$, so no staircase of intermediate stages exists.
- (iv) $h^{2,0}(\mathbb{CP}^2) = 0$ kills complex-structure deformations.
- (v) $b_1(K) = 0$ on $K = \mathbb{CP}^2 \times S^3$ kills harmonic 1-forms.
- (vi) $k_{\max}$ and Chern-Simons levels are integer-quantized (no continuous-modulus inflaton).

Nine further routes derived from the printed structure supply no accelerating stage with an exit and no source of the primordial perturbations: the spectral action (its vacuum is lowest at zero internal mass, with no exit, and a continuous internal space runs away or sits at a maximum), the conformally coupled Higgs, the χ and Chern-Simons sector, thermal and holographic fluctuations, the trace anomaly, anisotropic scaling, relaxation of the internal spectrum, the E-transport clock and the Majorana sector (main volume, App. FK).

Proof. (i) The DFD scalar action (Theorem AE.1) contains no potential $V(\psi)$: it is purely kinetic, $\frac{a_0^2}{8\pi G} [W(|\nabla\psi|^2/a_*^2) + K(\Delta)]$, plus the linear matter coupling $-\frac{c^2}{2}\psi(\rho - \bar{\rho})$. On FRW, App. AE.1 gives $|\nabla\bar{\psi}| = 0$ and $\Delta = 0$, so with the normalizations $W(0) = K(0) = 0$ the FRW Lagrangian density vanishes identically and $\bar{\psi}(t)$ is a flat direction (the Euler-Lagrange equation holds for any $\bar{\psi}(t)$). A slow-roll inflaton requires a potential $V \simeq \text{const} > 0$ to supply the inflationary vacuum energy, with slow-roll parameters $\epsilon = \frac{\bar{M}_P^2}{2} (V'/V)^2$, $\eta = \bar{M}_P^2 V''/V$; with $V \equiv 0$ these are undefined and the FRW energy density is zero. Hence ψ is excluded a fortiori—not because it is too heavy, but because it has no potential to roll down. (Note: this is not justified by an off-FRW vacuum mass $m_\psi^2 \sim M_P^2$, which would be a mislabel: $c^4/(32\pi G) \sim M_P^2$ —the high-gradient coefficient of $|\nabla\psi|^2$ —is the Newtonian stiffness ($\mu \rightarrow 1$), whereas the perturbation stiffness $M_{ij} = \mu(\bar{x})\delta_{ij} + \mu'(\bar{x})\hat{g}_i\hat{g}_j$ of App. U is μ -suppressed to zero at the matter-free FRW vacuum $\bar{x} \rightarrow 0$, where $\mu(x) \rightarrow x$ in deep MOND (Cor. AE.2); the stiffness is positive-semidefinite, not Planck-massive. The “no potential” argument above is strictly stronger and is what excludes ψ .)

(ii) On the truncated microsector the internal space is finite and carries no size or shape modulus; on a continuous internal space the shape sits at the Einstein point $\tau_* = 1/\sqrt{3}$, a minimum of the curvature potential for the connection Laplacian, where $\Phi''/\Phi \approx 2.94$ is a curvature ratio rather than a mass (main volume, App. O). The volume is not held there: with the connection Laplacian that fixes α it runs away along $e^{-\lambda\phi}$, $\lambda = \sqrt{14}/3 < \sqrt{2}$, an accelerating stage with no exit ($n_s = -6$, $r = 12.4$), and with the Dirac operator volume and shape sit at a

maximum with $\eta = -2$, where the roll is fast; neither is a slow-roll stage (main volume, App. FK).

(iii) Lemma O.5 (per-mode α -cancellation) takes the 57 nonzero KK modes at $\Lambda_{\text{top}} = \sqrt{\alpha} M_P$; on the near-arithmetic lattice their masses spread from 1.4×10^{17} to 1.04×10^{18} GeV, so they do not activate at one scale. A staircase would need intermediate rungs. A mode heavier than the Hubble rate is integrated out and contributes one factor of α (main volume, App. O), so a rung with k such modes is self-consistent only if its own Hubble rate leaves exactly k modes heavier than itself, and on the lattice that holds only at $k = 0$ and $k = 57$ (computed; main volume, App. FK). No sequential staircase can produce slow-roll inflation.

(iv) For complex-structure deformations of $\mathbb{CP}^2$ to provide an inflaton, $h^{2,0}$ would need to be positive. Direct Hodge-number computation: $h^{2,0}(\mathbb{CP}^2) = 0$. No inflaton from complex moduli.

(v) Harmonic 1-forms on K are counted by $b_1(K) = b_1(\mathbb{CP}^2) + b_1(S^3) = 0 + 0 = 0$. No inflaton from gauge-equivalent flat connections.

(vi) $k_{\max} = 60$ is integer-valued; Chern-Simons levels are integer-quantized. Continuous parameter modulation of $k_{\max}$ during cosmic evolution would violate the Spin^c index theorem instantaneously. No continuous-modulus inflaton.

Six independent failures rule out slow-roll-compatible inflaton in DFD. The TPI staircase $\{1, 12, 30\}$ specifically violates (iii) and (vi). $\square$

Remark AH.137 (Status of TPI and CMAH). This theorem closes the TPI question. The TPI mathematical content is kept, with the status recorded in Remark AH.109; CMAH (Theorem AH.110) is retained as a conjectured extension beyond DFD's stated scope.

d. SUSY Forbidden in the Microsector

Theorem AH.138 (SUSY Structurally Forbidden at Every Mass Scale). *Supersymmetric extensions of DFD are forbidden at any mass scale below M_P by six closure arguments.*

Proof. (1) *Hilbert-space saturation*: $\dim \mathcal{H}_{\text{micro}} = 60$ is exactly saturated by 3 generations $\times$ (5 hypercharged species + KK tower segment) per Theorem AH.13. Adding superpartners would require $\dim \mathcal{H}_{\text{micro}} \geq 65$, violating the Atiyah-Singer index $\chi(\mathbb{CP}^2, E) = 60$.

(2) *Spin^c integrality*: $q_1 = 3$, which follows from the isotropy charge $6Y$ under the electroweak identification (main volume, App. F, the remark on the flux quantum), fixes E through the conditional determinant-line count. SUSY would require $q_1 \neq 3$ with extra superpartner charges incompatible with this minimal structure.

(3) *Decoupling failure*: Naive “decoupled SUSY” fails because $\mathcal{H}_{\text{micro}}$ is closed under the spectral action; superpartners cannot be heavy without enlarging the algebra.

(4) *E₈ embedding mismatch*: DFD’s connection to E_8 is categorical via McKay (not a gauge embedding). There is no broken-GUT phase from which SUSY remnants could persist.

(5) *Spin-0 partner enumeration*: The 5 hypercharged species + 3 right-handed neutrinos saturate the Y-charge slots; squarks and sleptons have no place.

(6) *Spin-1/2 partner enumeration*: Gauginos and higgsinos would require additional fermion zero modes; $\ker D_F$ is 3-dimensional, exhausted by SM generations. $\square$

Corollary AH.139 (Strongest Falsifiable BSM-Negative Prediction). *Detection of any squark, gluino, neutralino, chargino, slepton, sneutrino, gravitino, higgsino, or any other supersymmetric partner at HL-LHC, FCC-hh, or any future collider falsifies DFD outright.*

Proof. Direct: by Theorem AH.138, no SUSY partner can exist at any mass scale below M_P . Discovery at any energy below $M_P \sim 10^{19}$ GeV is a contradiction. $\square$

e. Complete Naturalness Theorem

Theorem AH.140 (Structural Determination of the Dimensionless Microsector). *Within the dimensionless microsector, the printed hierarchy exponents and ratios are determined by derived or discretely selected structure on $\mathbb{CP}^2 \times S^3$ together with $\dim \mathcal{H}_{\text{micro}} = 60$. No continuous parameter is fitted to a predicted shape except the baryon density. The full physical corpus carries the four cosmological calibrations recorded in App. FK of the main volume: the measured vacuum coefficient R , the amplitude A_s , the coefficient of $\omega_i h^2$, and the shared astrophysical τ .*

Proof. The proof combines the following results:

- Λ : $G\hbar H_0^2/c^5 = C\alpha^{57}$ (Theorem AH.23); exponent 57 from the primed-determinant ratio, coefficient C measured on the early clock and $C = 1$ on bound-matter clocks under the two-clock postulate (main volume, App. ZS).
- Higgs hierarchy: $v = M_P \alpha^8 \sqrt{2\pi}$ (Theorem AH.17); 17 orders forced by 4-step cohomology filtration.
- Strong CP: $\bar{\theta} = 0$ at tree level, loop-bounded (Theorem AH.66); the det-orthogonal tree-level protection together with the η -invariant argument, with $\theta_{\text{bare}} = 0$ fixed by the spectral action given that nothing is added to it for the gauge fields.
- Fermion masses: $m_f = A_f \alpha^{n_f} v / \sqrt{2}$ (Theorem AH.27); 1.42% leading-order mean error with seven data-selected prefactors (main volume, App. FM).

- $\alpha = 1/137.036$: Chern-Simons + Toeplitz at $k_{\text{max}} = 60$ (Theorem AH.26); the identification of its hypercharge-trace coefficient with $1/e^2$ is not derived (main volume, the convention-locked closed form for α).
- $N_{\text{gen}} = 3$: a stated input, numerically $\chi_{\text{top}}(\mathbb{CP}^2) = 3$ (main volume, App. F).

The theorem is the conjunction of these dimensionless-sector statements; it does not override the normalization ledger of App. FK. The remaining open frontier is the origin of the primordial perturbations and the horizon problem, which enter through one stated starting condition (main volume, App. Q) and for which the theory states no mechanism; per Theorem AH.136, DFD has no native inflaton. The pre-BBN optical engine would inflate the optical metric only with a negative-pressure source and a reheating stage that the printed theory does not contain, and then only below $H \simeq 4 \times 10^{10}$ GeV (main volume, App. Q), and the χ relic rules out any appended stage that would leave a detectable tensor signal, CMAH as stated included ($r \lesssim 10^{-7}$; Theorem AH.40). $\square$

Corollary AH.141 (Anthropic Principle Displaced). *The standard alternative to “why these constants?” is the anthropic principle plus multiverse selection. DFD’s structural determination provides a deterministic mechanism: the hierarchy exponents are fixed by derived or discretely selected structure, not selected from an ensemble.*

Proof. By Theorem AH.140, the dimensionless hierarchy exponents and ratios are fixed by derived or discretely selected structure; the continuous inputs of App. FK remain. Predictivity scoreboard: windows allow ranges (e.g., α in $[1/200, 1/100]$ for chemistry); the closed form gives $1/137.036$ specifically. Five orders sharper. $\square$

f. Holographic Principle: Photon-Sphere Bulk-Boundary Duality

Theorem AH.142 (Bulk-Boundary Duality at Any Compact Mass, conjectural). *For any compact mass M in DFD, the bulk theory (flat $\mathbb{R}^3 \times \mathbb{R}_t$ with $\hat{\psi}$, $\hat{h}_{ij}^{\text{TT}}$, microsector) is dual to a 2D $SU(2)_{58}$ WZW boundary CFT on the photon-sphere $\partial B_{r_{\text{ph}}}$ at the areal radius $3GM/c^2$ of DFD’s exterior, with central charge $c_{\text{BD}} = 174/60 = 2.9$. Operator-operator bijection:*

$$O_{\partial}(\sigma) = \lim_{r \rightarrow r_{\text{ph}}^+} (r - r_{\text{ph}})^{-\Delta} O_{\text{bulk}}(r, \sigma).$$

Proof. The $SU(2)_{58}$ MTC, whose shifted level equals the microsector dimension (Theorem AH.75), provides the boundary CFT structure: 59 primaries with conformal weights $h_a = a(a+1)/60$, central charge $c = 3 \cdot 58/(58+2) = 174/60 = 2.9$. The photon sphere $\partial B_{r_{\text{ph}}}$ admits a natural 2D CFT structure from the optical-metric induced metric. Bulk-boundary correspondence: bulk operators

at radius r project to boundary operators with conformal weight Δ_a via the optical-radial scaling. Reconstruction kernel in HKLL-style:

$$\phi_{\text{bulk}}(r, \sigma) = \int dA_{\text{opt}}(\sigma') K^{\text{DFD}}(r, \sigma; \sigma') \phi_{\partial}(\sigma'),$$

with $K^{\text{DFD}}(r, \sigma; \sigma') = (n(r)/n(r_{\text{ph}}))^{-\Delta_a} P_a(\cos \gamma)$ where γ is the angular separation. Valid outside the event horizon. $\square$

g. CKM $\bar{\rho}$: Both Paths Recorded

Remark AH.143 (Apex paths A and B: status open). Neither Path A (+5/2 Cayley correction $\Rightarrow \bar{\rho} = 21.5\alpha = 0.157$) nor Path B (integer 22 reassignment $\Rightarrow \bar{\rho} = 22\alpha = 0.161$) is data-independent. Both routes use ~ 1 bit of PDG-driven selection from a structurally-admissible set. Path A has deeper structural grounding (4 convergent perspectives: $\mathbb{Z}_3$ cohomology, MTC $\text{SU}(2)_{58}$ Verlinde, Connes Spin^c canonical shift, Cayley graph (A_5, S) gen-1 distance) and preserves the uniform selection rule across channels. Path B has cleaner arithmetic (0.91% vs 1.39%) but breaks the bundle-filtration rule. Path A is kept as a “Conjectural Selection-Rule Extension”, with Path B cross-referenced as the alternative. Discrimination is experimental: hyper-Belle / FCC-ee at $\sigma(\bar{\rho}) \leq 0.0012$ will decide at $> 3\sigma$. The integers 31, 108 and 49 of λ , A and $\bar{\eta}$ are kept as suggestive of cohomological origin; the strengthened branch of Appendix AO uses $(31, 108, 43/2, 49)\alpha$, whose apex is the Path A value.

h. QECC: Explicit Pauli-Stabilizer Construction

Theorem AH.144 (Heisenberg-Weyl Stabilizer Subgroup of $M_{60}(\mathbb{C})$). *The microsector algebra $M_{60}(\mathbb{C})$ contains a Heisenberg–Weyl subgroup $\text{HW}(60) = \langle X_{60}, Z_{60} \rangle$ generated by the $\mathbb{Z}_{60}$ clock and shift pair, with abelian stabilizer subgroup $S_{\text{DFD}} = \langle Z_{60}^3 \rangle$ of order 20. The joint +1 eigenspace equals $\ker D_F = \text{span}\{|0\rangle, |20\rangle, |40\rangle\}$, the 3-generation logical subspace. Logical Pauli pair $(\bar{X}, \bar{Z}) = (X_{60}^{20}, Z_{60})$ acts as qutrit Pauli on the 3-dim code.*

Proof. The cyclic group $\mathbb{Z}_{60}$ provides clock and shift generators X_{60}, Z_{60} acting on $\mathbb{C}^{60}$ with $X_{60}Z_{60} = \omega Z_{60}X_{60}$ where $\omega = e^{2\pi i/60}$. Order: both have order 60. Abelian subgroup: Z_{60}^3 has order 20 and commutes with itself trivially. Joint +1 eigenspace: vectors $|n\rangle$ with $\omega^{3n} = 1$, i.e. $n \in \{0, 20, 40\}$ — 3-dimensional. This matches $\ker D_F = \mathbb{C}^3$. Logical operators: $\bar{X} = X_{60}^{20}$ permutes $\{|0\rangle, |20\rangle, |40\rangle\}$ cyclically (qutrit shift); $\bar{Z} = Z_{60}$ phases them by $\omega^{20n} = e^{2\pi i n/3}$ (qutrit clock). $\bar{X}\bar{Z} = \omega^{20}\bar{Z}\bar{X} = e^{2\pi i/3}\bar{Z}\bar{X}$ — qutrit Pauli algebra. $\square$

Remark AH.145 (Caveats). The construction is a measurement-theoretic Gottesman stabilizer code; it is not a Hamiltonian Kitaev code because $[D_F, S_{\text{DFD}}] \neq 0$

as operators (only subspace preservation holds). Code distance $d = 1$ in the standard HW-displacement metric, $d \geq 4$ in the Toeplitz-mode-jump metric. The literal $[[60, 3, \geq 4]]$ reading holds in the Toeplitz metric only.

Lemma AH.146 (Evaluation-reading dichotomy: every embedding yields $a = 9$ or fails). *Let s_1, s_2, s_3 span $\ker D_F$ and let $\iota : \ker D_F \hookrightarrow H^0(\mathbb{CP}^2, \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5})$ be any embedding into the written microsector ambient. Write $\text{ev}_x(s_i) = (p_i(x), c_i) \in \mathcal{O}(9)_x \oplus \mathbb{C}^5$. Then exactly one of two horns holds: (i) $\text{rank}\{c_1, c_2, c_3\} = 3$, in which case ev is fibrewise injective, the evaluation subbundle is trivial $\mathcal{O}^{\oplus 3}$, its determinant twist vanishes, and the charged determinant line is $\det(\ker D_F \otimes L_Y) = \mathcal{O}(9)$ identically — the same answer as the fixed-kernel reading; or (ii) $\text{rank}\{c_i\} \leq 2$, in which case some nonzero combination is a pure $\mathcal{O}(9)$ section, vanishing on a nonempty degree-9 curve, so constant rank fails and no rank-3 evaluation subbundle exists (the all-in- $\mathcal{O}(9)$ assignment favored by the determinant-line bookkeeping is the extreme rank ≤ 1 sub-case). Hence $a = 9$ in every well-defined reading. The lone variant that escapes both horns — saturating the rank-2 case to the subsheaf $\mathcal{O}^{\oplus 2} \oplus \mathcal{O}(9)$, giving $a = 18$, $k_{\text{max}} = 195$ — abandons the constant-rank subbundle definition and reproduces exactly the $q_1 = 6$ alternative already excluded by the measured α (App. F, Rem. on independence).*

Remark AH.147 (Basis convention). For definiteness we adopt the lexicographic dictionary: indices 0–54 of the microsector basis label the 55 degree-9 monomial sections of $\mathcal{O}(9)$, indices 55–59 the five trivial matter-type lines — consistent with the determinant-line bookkeeping (App. F: the generations are carried inside $\mathcal{O}(9)$; the $\mathcal{O}^{\oplus 5}$ are the matter-type lines). Under this convention $\ker D_F = \text{span}\{|0\rangle, |20\rangle, |40\rangle\}$ lies in the $\mathcal{O}(9)$ column and horn (ii) applies in its rank- ≤ 1 form.

i. Microsector Mass Spectrum: Direct-Detection Locked Out

Theorem AH.148 (Microsector Mass Lattice). *The 60 microsector modes split as 3 chirality zero modes (the SM generations) plus 57 nonzero Toeplitz–Dirac eigenvalues forming a near-arithmetic lattice*

$$m_n^2 \approx (n - 3) \cdot \Lambda_{\text{top}}^2 / 57, \quad n = 4, \dots, 60.$$

The lightest nonzero mode is $m_4 \approx \Lambda_{\text{top}} / \sqrt{57} \approx 1.4 \times 10^{17} \text{ GeV}$, thirteen orders above the LHC and twelve above a 100 TeV collider.

Proof. The Spin^c Dirac operator D_F on $\mathbb{CP}^2 \times S^3$ at $k_{\text{max}} = 60$ has 3 zero modes (kernel) plus 57 positive eigenvalues distributed approximately uniformly between $\sim \Lambda_{\text{top}} / \sqrt{57}$ and Λ_{top} . With $\Lambda_{\text{top}} = \sqrt{\alpha} M_P = 1.04 \times 10^{18} \text{ GeV}$: $m_4 = 1.4 \times 10^{17} \text{ GeV}$. No mode is sub-TeV. The bound $m_4 \geq \sqrt{\alpha/k_{\text{max}}} \cdot M_P$ holds for the printed spectrum, which the rigidity premise (layer 7 of Theorem AH.179) holds fixed. $\square$

Corollary AH.149 (No Direct Lab Production). *No microsector mode is producible at any current or planned collider. The best analog test is a 60-state condensed-matter system on trapped-ion or superconducting hardware.*

Proof. Direct: $m_4 \gg E_{\text{cm}}^{\text{FCC}} \approx 100 \text{ TeV}$. $\square$

j. Amplitude Bootstrap Saturation

Theorem AH.150 (DFD Saturates Minimum-Spectral-Gap Bootstrap Inequality, conjectural). *Among SM-compatible UV completions, DFD saturates the minimum-spectral-gap inequality: the lightest non-zero state has mass $m_{\text{gap}} = \sqrt{\alpha/57} M_P$, the smallest gap consistent with Spin^c integrality and $(3, 2, 1)$ gauge partition. The leading amplitude EFT coefficient c_3 on the $R_{\mu\nu\rho\sigma}^2$ operator is $c_3 = 1/6$ forced by two independent derivations:*

(i) *Witten degree-1 twistor-string symmetry factor $1/3!$.*

(ii) *One-loop Toeplitz quadrature on $\mathcal{H}_{\text{micro}}$ (Theorem AH.37).*

Proof. Spectral gap: $m_{\text{gap}} = m_4 = \sqrt{\alpha} M_P / \sqrt{57}$ from Theorem AH.148; this is the minimum gap consistent with $\dim \mathcal{H}_{\text{micro}} = 60$ and the Spin^c index. Each of (i) and (ii) gives $c_3 = 1/6$ via different combinatorics. The convergence is non-trivial because the two derivations use distinct categorical structures. $\square$

Corollary AH.151 (Primordial gravitational-wave helicity asymmetry). *The $c_3 = 1/6$ coefficient gives a primordial gravitational-wave background the helicity asymmetry $A_h = c_3 (H/M_P)^2$ at the rate H of the stage that froze it. The χ relic caps that rate at $H \lesssim 4 \times 10^{10} \text{ GeV}$ (main volume, App. Q), where $A_h \lesssim 2 \times 10^{-18}$, and the background itself is $r \lesssim 10^{-7}$ (main volume, App. SM); no helicity signal is predicted for LISA.*

Proof. At $H = 4.3 \times 10^{10} \text{ GeV}$, $(H/M_P)^2 = 1.2 \times 10^{-17}$ and $A_h = 2.1 \times 10^{-18}$. A rate $H \sim 10^{-5} M_P$ would give $A_h = \frac{1}{6}(10^{-5})^2 = 1.7 \times 10^{-11}$, and the relic excludes it (computed). $\square$

k. Dark-Matter Null Portfolio

Theorem AH.152 (DFD Predicts Null Results Across 45 DM Searches). *DFD predicts zero signal across all current-reach dark-matter direct-detection, indirect-detection, axion-haloscope, sterile-neutrino, hidden-photon, and modulation experiments. DFD's cold dark matter is the derived χ -matter field (App. AV), a non-thermal, Standard-Model-neutral pseudoscalar with no photon coupling (its photon coefficient is $E = 0$, derived in App. AV); it sources none of their recoil or haloscope channels. The set of null predictions therefore covers every current search (~ 45 experiments). With $m_\chi \simeq 32 \text{ eV}$ and $E = 0$, χ has no decay line and no conversion line: any*

dark-matter photon line at 16.0 eV (77.5 nm , two-photon decay) or axion-like signal at 32.005 eV (38.74 nm , conversion), at any strength, falsifies it. This breadth makes DFD multiply falsifiable: a confirmed halo-recoil signal in any current-reach channel falsifies it.

Proof. DFD has no postulated BSM particle dark matter (Theorem AH.64, items i–vi); its cold dark matter is the derived χ -matter field (App. AV), the harmonic b_3 three-form on $S^3 = SU(2)$, which is Standard-Model-neutral and non-thermal and so produces no recoil or haloscope signal in the channels below. Galactic phenomenology is separately covered by the μ -law with $a_0 = 2\sqrt{\alpha} c H_0^{\text{loc}}$ (Theorem AH.21; main volume, App. ZS); inside galaxies the two compose by the adopted floor law with a stated three-dimensional filling rule (App. AV). The chiral index $\dim \mathcal{H}_{\text{micro}} = 60$ is saturated by SM matter; χ is a bosonic harmonic mode, not one of the 60. DFD's σ_8 of record, 0.806, is that of the cold χ carrier with $Q_\chi = 1$ (main volume, App. GR); the value 0.811 from Ω_b with the optical-metric amplification $Q_{\text{DFD}} \approx 502$ (Theorem AH.7) belongs to the baryon-only construction of the main volume's App. AE, which is not DFD's growth carrier. Null prediction is structural across: xenon TPCs (XENON-nT, LZ, PandaX, DARWIN), argon (DEAP-3600, DarkSide), spin-dependent (PICO-500), low-mass (SuperCDMS, DAMIC-M, SENSEI), modulation (DAMA, ANAIS, COSINE, SABRE-South), axion (ADMX, MAD-MAX, IAXO), sterile- ν (KATRIN, JUNO, DUNE, MicroBooNE), hidden-photon, indirect γ -ray (Fermi-LAT, HESS, MAGIC, CTA), cosmic-ray (AMS-02, GAPS), neutrino (IceCube-Gen2, KM3NeT). $\square$

Corollary AH.153 (Single-Statement Falsifier). *A single confirmed positive detection of any halo-DM signal (cross-experiment confirmed, $> 5\sigma$, halo-consistent recoil/coupling) at any of the 45+ current-reach experiments falsifies DFD's current-reach-null closure at theorem level (the derived χ -matter field, with no photon coupling, predicts no such signal).*

Proof. DFD's claim is single-statement: zero halo-recoil/haloscope signal from any thermal WIMP or current-reach axion-like particle. The derived χ -matter field has $\rho_\chi \neq 0$ ($\Omega_\chi h^2 \simeq 0.12$; App. AV) but is non-thermal and Standard-Model-neutral, so it sources none of these channels; its cold clustering is tested cosmologically (CMB third-peak height), not in direct detection. Any cross-confirmed such recoil detection contradicts DFD. ΛCDM by contrast can absorb any null result by shifting search parameters; DFD cannot. $\square$

l. Axiomatic Foundation

The inputs of the graded predictions, the postulates, adopted axioms, identifications, discrete selections and continuous calibrations, are listed in the input tables of App. FK of the main volume.

10. Additional Theorems and Proofs (Series V)

This subsection covers the master-action restatement, the single-invariant theorem and the two-axiom minimum lemma.

a. Master DFD Lagrangian: Single-Equation Statement

Theorem AH.154 (Master DFD Action). *The micro-sector, gauge and Einstein–Hilbert content of DFD is organized by the Connes-style spectral action*

$$S_{\text{DFD}} = \text{Tr } f(D/\Lambda_\star) + \langle \Psi, D\Psi \rangle$$

where $D = D_{M_4} \otimes 1 + \gamma_5 \otimes D_F$ is the full Dirac operator on $M_4 \times \mathbb{CP}^2 \times S^3$, $\Lambda_\star = \sqrt{\alpha} M_P$, f is a heat-kernel cut-off, and $\dim \mathcal{H}_F = 60$. The expression fixes the claimed dimensionless microsector structure with no continuous shape fit; conversion to the physical cosmological prediction uses the four cosmological calibrations of App. FK of the main volume (the vacuum coefficient R , the amplitude A_s , the ω_b coefficient, and τ).

Proof. Heat-kernel expansion $\text{Tr } f(D/\Lambda) = \sum_n a_n(D^2) f_n \Lambda^{4-2n}$ gives:

- a_0 coefficient: a Planckian vacuum term (App. WS); reading it as the α^{57} term of Theorem AH.23 is a conjecture that needs the whole D_F -dependent vacuum suppressed with it.
- a_2 coefficient: Einstein-Hilbert with $G^{-1} \propto \alpha M_P^2$ as the Newton constant.
- a_4 coefficient: Yang-Mills terms for the gauge fields of the inner fluctuations, whose group

Proof. The integer $\Omega = 60$ arises in four ways: (a) Hirzebruch-Riemann-Roch index on $\mathbb{CP}^2$ with bundle E ; (b) icosahedral group order $|A_5| = 60$; (c) E_8 root system $|R_{E_8}|/4 = 240/4 = 60$; (d) shifted level $k + h^\vee = 58 + 2$ of the $SU(2)_{58}$ MTC. With the inputs of App. FK of the main volume, and with $\theta_{\text{bare}} = 0$ fixed by the spectral action given that nothing is added to it for the gauge fields, the printed formulas read:

$$\begin{aligned} \alpha^{-1} &= (\pi^{3/2}/24) \cdot 10 \cdot \Omega \cdot ((\Omega + 3)/(\Omega + 4)) \cdot [1 + 7/(80 \cdot 4095)] = 137.036, \quad 7/(80 \cdot 4095) = 1/(13 \Omega^2) \\ G\hbar H_0^2/c^5 &= C\alpha^{\Omega - N_{\text{gen}}} = C\alpha^{57} \\ v/M_P &= \alpha^8 \sqrt{2\pi} \quad m_R/M_P = \alpha^3 \\ m_f &= A_f \alpha^{n_f} v/\sqrt{2} \quad (\text{Cayley graph on } A_5) \\ \bar{\theta} &= 0 \quad (\text{tree level; loop-induced } \lesssim 10^{-16}) \quad N_{\text{gen}} = 3 \quad (\text{input; numerically } \chi_{\text{top}}(\mathbb{CP}^2)). \end{aligned}$$

□

c. Wheeler-DeWitt Equation in DFD

Theorem AH.156 (Wheeler-DeWitt Closed-Form Ground State). *The DFD Wheeler-DeWitt equation $\hat{H}_{\text{DFD}}\Psi = 0$ (equivalently $\hat{H}'\Psi = E_0\Psi$ with $E_0 =$*

$SU(3) \times SU(2) \times U(1)$ is selected, not forced (Theorem AH.214), + Higgs potential (Theorem AH.17). The conjectured $\lambda_H = 1/8$ of the main volume (App. Z) is stated at an electroweak matching scale, not at $\Lambda_\star$, where the measured Higgs mass needs a negative quartic, -0.011 ± 0.003 (computed).

- $\langle \Psi, D\Psi \rangle$: Dirac kinetic + Yukawa from off-diagonal D_F via Berry-bundle overlap integrals (Theorem AH.27).

The 3 generations are a stated input identified with $\chi_{\text{top}}(\mathbb{CP}^2) = 3$ (main volume, App. F). $\bar{\theta}_{\text{QCD}} = 0$ (tree level; loop-induced $\lesssim 10^{-16}$) from the det-orthogonal tree-level protection together with η -invariant vanishing on the CP mapping torus, with $\theta_{\text{bare}} = 0$ fixed by the spectral action given that nothing is added to it for the gauge fields (Theorem AH.66). The proof aggregates all prior theorems. □

b. Single Master Topological Invariant

Theorem AH.155 (One Integer Organizing the Hierarchy Exponents). *The printed hierarchy exponents of the Standard Model and the cosmological constant are organized by a single topological integer*

$\Omega \equiv \dim_{\mathbb{C}} \mathcal{H}_{\text{micro}} = \chi(\mathbb{CP}^2, \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}) = 60$, together with one external dimensional anchor (e.g., M_P or H_0^{loc}), with the inputs listed in App. FK of the main volume.

$\alpha^{57} M_P^4$ under the additive-constant convention of Theorem AH.177 — under the lock of the lapse to the scale factor a constant added to the Hamiltonian acts as a curvature term ($w = -1/3$), not as vacuum energy, so the two notations agree only when E_0 is carried as a vacuum

term of the action (computed); that theorem's eigenfunction and eigenvalue steps are asserted, not derived, per its status note, §AH.12b) has the proposed ground state

$$\Psi_0[\psi, \Xi] = \mathcal{N} \exp(-S_E[\psi, \Xi]/\hbar),$$

the Boltzmann weight of the Euclidean DFD action. The exponent of the cosmological constant appears in the Wheeler-DeWitt eigenvalue $\alpha^{57} M_P^4$, about 14 times the observed density (scope note of Theorem AH.177); the ground state fixes no amplitude for the primordial perturbations (main volume, App. FK).

Proof. DFD's flat spatial arena $g_{ij} = \delta_{ij}$ is non-dynamical, and in the homogeneous minisuperspace the traceless strain of the parent tensor vanishes by symmetry; only $\hat{\psi}$ and the 60-dim microsector are quantized. This eliminates the supermetric pathologies of standard Wheeler-DeWitt. The constraint reduces to an elliptic functional eigenvalue problem on $L^2(\mu_\beta)$ with respect to the Parisi-Wu invariant measure. Strong monotonicity of $\mu(x) = x/(1+x)$ (App. U) provides coercivity; the status note of Theorem AH.177 records the gaps. Direct check: $\hat{H}_{\text{DFD}} \mathcal{N} e^{-S_E/\hbar} = 0$ via the standard Schrodinger-action Euclidean correspondence. Cosmological constant emerges as the lowest eigenvalue of the FRW-restricted Hamiltonian, computed via App. O determinant ratio. Vacuum ψ fluctuations on the scale of today's horizon are of order $H_0/M_P \approx 1.2 \times 10^{-61}$, far below the observed 10^{-5} , which has no mechanism in DFD (main volume, App. FK). $\square$

Theorem AH.158 (All-Orders Bound on the Vacuum Deposit).

$$\rho_{\text{dep}} = \frac{3}{8\pi} \alpha^{57} M_P^4 [1 + R(\alpha)], \quad |R(\alpha)| \leq \frac{\alpha^2}{1 - \alpha^2} + \exp(-8\pi^2/\alpha) \approx 5 \times 10^{-5}.$$

This is the $C = 1$ deposit; the observed dark-energy density is $(0.6042 \pm 0.0027) \rho_{\text{dep}}$, its coefficient measured and underived (main volume, App. AU). The 10^{122} fine-tuning problem is closed at theorem grade in its exponent, for the microsector vacuum piece. (Scope: the bound governs the 60-mode microsector trace only; the Standard-Model matter-sector vacuum loop — external to the microsector, of order $\alpha^{32} M_P^4$ — is not bounded by this theorem, nor is the Planckian a_0 vacuum term of the spectral action (Theorem AH.154), and no full-spectrum vacuum sum rule is derived. The matter loop and the a_0 term are each assumed to cancel, and the matter-piece cancellation remains the universal open cosmological-constant problem; see the scope remark in App. AU.)

Proof. Tree level: $\rho_{\text{dep}}^{(0)} = (3/8\pi) \alpha^{57} M_P^4$ from Theorem AH.23. One-loop: $|\delta\Lambda^{(1)}/\Lambda| \leq \alpha^2$ from Wilsonian regulation (Theorem AH.1). n -loop bound: $|\delta\Lambda^{(n)}/\Lambda| \leq \alpha^{2n}$ by induction with per-loop suppression from finite microsector trace. Geometric series sums to $\alpha^2/(1 - \alpha^2) \approx 5 \times 10^{-5}$. Non-perturbative contributions: instantons of action $S_{\text{inst}} = 8\pi^2/\alpha \approx 10820$ give $e^{-10820} \approx 10^{-4699}$, negligible. Domain walls forbidden by $\pi_2(\mathcal{V}) = 0$. Sphalerons thermally suppressed at $T = 0$. Total all-orders correction bounded by 5×10^{-5} . $\square$

d. Single-Vacuum Theorem (No Landscape)

Theorem AH.157 (DFD Has Exactly One Vacuum). The DFD vacuum manifold has dimension zero modulo gauge transformations and discrete symmetries:

$$\dim(\mathcal{V}_{\text{DFD}}/\sim) = 0.$$

There is no DFD landscape.

Proof. Five-step composition of prior closure theorems: (1) manifold $\mathbb{C}P^2 \times S^3$ is unique (Theorem AH.9); (2) bundle $E = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ follows from the determinant-line arithmetic at the derived $q_1 = 3$ (main volume, App. F, the remark on the flux quantum); (3) the Toeplitz cap $k_{\text{max}} = 60$ follows from $N_{\text{gen}} = 3$, $q_1 = 3$ and the five hypercharged multiplet types (main volume, App. F); (4) the ψ vacuum is unique up to the constant shift of its flat direction (the scalar action carries no potential term in ψ ; Theorem AH.136, sub-(i)); (5) microsector ground state is unique by App. F gauge emergence, with the internal Dirac operator held at its printed spectrum by the rigidity premise (layer 7 of Theorem AH.179); without that premise the inner fluctuations of $M_{60}(\mathbb{C})$ would make its 60 eigenvalues dynamical (main volume, App. FK). Ten classes of counter-vacua (alternative manifolds, alternative bundles, alternative complex structures, alternative Toeplitz truncations, alternative gauge partitions, alternative Spin^c structures, alternative metric deformations, alternative boundary conditions, alternative microsector fillings, alternative ψ vacua) are excluded given the stated selections and the rigidity premise.

Comparison with string theory: $\dim(\mathcal{V}_{\text{string}}) \sim 500\text{--}1000$ continuous moduli plus $\sim 10^{500}$ discrete choices; DFD: 0 continuous + 1 discrete. The string-landscape problem does not arise in DFD. $\square$

e. Vacuum Deposit: All-Orders Theorem

f. *EW-Planck Hierarchy: No-Tuning Theorem*

Theorem AH.159 (Hierarchy Problem Eliminated, Not Solved). *DFD does not solve the hierarchy problem by tuning a parameter; it eliminates the bare-parameter form of the question by deriving v from topology: the bare parameter m_H^2 does not exist in DFD's parameter list. Radiative corrections to v from the microsector satisfy $|\delta v/v| \leq O(\alpha^2) \approx 5 \times 10^{-5}$ at all loop orders. (Scope: the external 4D Coleman–Weinberg loop over noncompact momenta is not regulated by the microsector trace — see the status note of §AH.2a — so radiative stability of the derived v against the external matter loop remains the universal open hierarchy problem, shared with all QFT coupled to gravity; what is eliminated is the tuned-bare-parameter formulation.)*

Proof. $v = M_P \alpha^8 \sqrt{2\pi}$: the exponent 8 is forced ($\dim X + 1$, Theorem AH.17); the $\sqrt{2\pi}$ prefactor is fitted, and no printed rule fixes it (main volume, App. K). The Higgs mass is $m_H^2 = 2\lambda_H v^2$, with $\lambda_H = 1/8$ a dimension-counting conjecture (main volume, App. Z). Wilsonian regulation at $\Lambda_{\text{top}} = \sqrt{\alpha} M_P$ (Theorem AH.1) plus per-mode α -cancellation (Lemma AH.2) gives $|\delta v/v| \leq O(\alpha^2)$ at one loop. All-orders sum: convergent geometric series $\alpha^2/(1 - \alpha^2)$. Compare with SUSY: SUSY cancels δm_H^2 via symmetry, requires TeV superpartners (LHC null), suffers a "little hierarchy" 10^{-2} – 10^{-4} . DFD eliminates the parameter m_H^2 itself; no new physics below $\Lambda_{\text{top}} \approx 10^{18}$ GeV. $\square$

g. *All-Orders QG Amplitude Finiteness*

Theorem AH.160 (Universal Amplitude Finiteness, conjectural). *Every DFD scattering amplitude at every loop order L with every external leg count n is finite.*

Proof. Six-step proof: (1) $\mathcal{H}_{\text{micro}}$ is 60-dimensional via Atiyah–Singer; (2) continuum 4-momentum integration is bounded by Λ_{top} (no Landau pole, Theorem AH.16); (3) combined integration is a finite sum of finite terms (Toeplitz quadrature); (4) per-loop bound is sub-exponential; (5) UV divergences absent (no Goroff–Sagnotti R^3 counterterm because integrand is finite trace, not divergent loop integral); (6) IR divergences cancel by standard Bloch–Nordsieck/KLN. RG flow freezes above Λ_{top} . $\square$

Corollary AH.161 (DFD vs String Theory UV-Finiteness). *DFD and string theory approach UV-finiteness by different mechanisms: string theory uses an infinite Regge tower with modular invariance; DFD uses a finite tower of 60 microsector states. DFD's gravity sector carries one measured coefficient, the early-clock coefficient $C = 1/\lambda^2$ (main volume, Apps. FK and ZS); string has compactification moduli at ~ 500 – 1000 .*

Proof. Parameter counting: see Theorem AH.157. $\square$

h. *Quantum Measurement Problem, Structural Reduction*

Theorem AH.162 (Structural Reduction of the Measurement Problem). *In DFD, the optical metric $n = e^\psi$ is sourced by the c-number expectation $\langle \Psi | \hat{\rho} | \Psi \rangle$ via an elliptic PDE with monotone μ , giving a unique optical metric. No metric branching occurs; the Many-Worlds interpretation is structurally excluded. The Born-rule form follows from Gleason's theorem once the complex structure $J_\star$ is supplied, and its value is conditional on $J_\star$ (Theorem AH.178); the measurement postulate is imported (main volume, App. FK).*

Proof. By Theorem AH.24, the elliptic equation has a unique weak solution ψ for any L_{loc}^2 source (Browder–Minty with strict monotonicity of $\mu(x)x$; main volume, App. U). Hence the optical metric is unique. There is no branching. Decoherence in the position basis follows from the standard ρ -coupling in the optical-metric prescription; pointer states are position eigenstates. Born rule: Gleason's theorem fixes the form of the measure on the projection lattice of $\mathcal{H}_{\text{micro}}$ ($\dim = 60 \geq 3$) once $J_\star$ supplies the inner product; the value is conditional on $J_\star$ (Theorem AH.178). Tree-level intrinsic gravitational collapse is $\tau = \infty$ (opposite of Diosi–Penrose $\tau \sim \hbar/E_{\text{grav}}$). $\square$

Falsifier: MAQRO experiment (10^{-13} kg test mass, space-vacuum, 100 s coherence time): DFD predicts sustained interference fringes; Diosi–Penrose predicts collapse at 10^{-3} s. This sustained-fringe prediction is the direct consequence of the no-branching theorem above (one c-number-sourced optical metric, no per-branch self-force), consistent with the open-problems section; the per-branch deep-MOND-enhanced collapse alternative is *excluded* by the two-body structure of the derived vertex (main volume, App. ZU).

i. *Direct Microsector Lab Analog*

Theorem AH.163 (Quantum-Hardware-Buildable DFD Microsector Simulator). *A $[[60, 3, \geq 4]]$ stabilizer code (distance ≥ 4 in the Toeplitz metric, 1 in the standard displacement metric; Theorem AH.181) implementing DFD's microsector structure can be constructed on existing quantum hardware (Quantinuum H2-1 trapped ions, IBM Heron, or Google Willow).*

Proof. Map each microsector mode to a qudit basis state in a 60-dim Hilbert space. Implement Spin^c Dirac D_F as a block-diagonal Hamiltonian with eigenvalues $\{0, 0, 0, \lambda_4, \dots, \lambda_{60}\}$ via a 19-laser AC-Stark protocol. Stabilizer measurement of $\langle Z_{60}^3 \rangle$ via Knill ancilla phase-kickback. $\square$

j. *AdS/CFT Without AdS: A Photon-Sphere Holographic Duality*

Theorem AH.164 (Photon-Sphere Holographic Duality on Flat Spacetime, conjectural). *For any compact mass M in DFD with photon sphere at the areal radius $3GM/c^2$, the bulk partition function with Dirichlet data $\partial\phi$ on $\partial B_{r_{\text{ph}}}$ equals the $SU(2)_{58}$ WZW boundary partition function with source $J = \partial\phi$:*

$$Z_{\text{bulk}}[g_{ij}^{\partial}, \partial\phi] = Z_{\partial}^{SU(2)_{58}}[J(\sigma) = \partial\phi(\sigma)],$$

to all orders in the asymptotic expansion as $r \rightarrow r_{\text{ph}}^+$. The boundary central charge is $c_{\text{BD}} = 174/60 = 29/10 = 2.9$.

Proof. The induced optical metric on $\partial B_{r_{\text{ph}}}$ carries an emergent $SO(3,1)$ Möbius conformal structure intrinsic to the round S^2 with Weyl factor $e^{2\psi(r_{\text{ph}})} = 9$ on the static exterior, where $e^{\psi} = (1 - 2GM/c^2 R)^{-1} = 3$. This conformal symmetry is *not* inherited from any bulk isometry; it lives on the boundary. The boundary CFT is $SU(2)_{58}$ WZW (level $k = 58$, central charge $c = 3k/(k+2) = 2.9$). Bulk fields project to boundary primary operators of conformal weight $\Delta_a = a(a+1)/60$ via radial scaling; HKLL-style reconstruction kernel $K^{\text{DFD}}(r, \sigma; \sigma') = (n(r)/n(r_{\text{ph}}))^{-\Delta_a} P_a(\cos \gamma)$ valid outside the event horizon. Partition-function identity follows from Witten-style bulk-boundary correspondence applied to the optical-metric path integral. $\square$

Corollary AH.165 (Local vs Global Holography). *DFD has local holography (one CFT per object); AdS/CFT has global holography (one boundary CFT for the whole bulk). DFD's boundary CFT is strictly more rigid: $k = 58$ fixed by $\chi(\mathbb{CP}^2, E) = 60$, no free parameters versus AdS's continuous N, R, g_s .*

Proof. Direct: each photon sphere supports its own boundary CFT independently; AdS supports a single global one. Rigidity comparison: count parameters. $\square$

k. *Two-Axiom Minimal Statement of DFD*

Theorem AH.166 (Single-Axiom Reduction Impossible; Two-Axiom Minimum). *The DFD theory cannot be reduced to a single axiom (conformal self-similarity of spectral triples blocks it: $D \rightarrow \lambda D$ with $\Lambda \rightarrow \lambda\Lambda$ gives an isomorphic spectral action). The minimum axiomatic rank for the internal spectral object is two: one structural axiom selecting the spectral object, one anchoring axiom selecting the dimensional scale. (The complete theory additionally posits the flat $\mathbb{R}^3$ arena and the optical map $n = e^{\psi}$, and imports the quantum seed — the numeric $\hbar$ and an indifference/typicality bridge for Born; the global irreducible-input count is therefore larger. The “two” is the irreducible rank of the internal-index sublayer, not of the whole framework.)*

Proof. Single-axiom impossibility: any structural axiom characterizing the DFD spectral data $(\mathcal{A}, \mathcal{H}, D)$ is

invariant under $D \rightarrow \lambda D$ with $\Lambda \rightarrow \lambda\Lambda$ (rescaling). Hence the dimensional scale cannot be fixed by any structural axiom alone; one anchoring axiom is needed.

Two-axiom minimal statement:

(SA) Structural: *DFD = spectral action of the real KO-dim-4 spectral data (Theorem AH.77) over $C^\infty(\mathbb{R}^3 \times \mathbb{R}_t) \otimes \text{End}(\text{Berezin-Toeplitz Hilbert space})$ for a compact Spin^c Kähler-times-Lie-group manifold with $\chi_{\text{top}}(M_c) = 3$, positive Ricci curvature, $\pi_1(M_g) = 1$, $\pi_3(M_g) = \mathbb{Z}$ and minimal dimension.*

(AA) Anchoring: *One external scale $\Lambda_\star \in \{M_P, v, H_0^{\text{loc}}, \dots\}$.*

Comparison with Einstein-Hilbert: GR's effective axiomatic rank is also 2 (4-diff-invariant scalar action + Newton's G). $\square$

11. Additional Theorems and Proofs (Series VI)

This subsection treats *precision-frontier overdetermination and comparisons with rival quantum-gravity programs*. It contains: (i) a closed-form α^{-1} at $\Omega = 60$, exact under two stated choices, whose integers alone lie 21 ppm from CODATA 2022 and which lies 0.005 ppm from it with the open bracket coefficient; (ii) sub-meV neutrino spectrum closure with NuFIT 6.0 $\chi^2(2) = 0.021$ at zero continuous parameters (one data-selected branch choice among four equally natural fractions; see the branch-selection remark of Appendix X in the main paper); (iii) a +0.23% (α/π) sector-resolved Casimir coefficient, a conjecture outside DFD's claims (Theorem AH.171); (iv) a comparative documentation block listing concrete empirical falsifiers for DFD that the four rival programs do not currently predict at theorem grade, with the scorecard presented as documentation, not as a uniqueness or superiority claim; (v) a cross-check of 27 live dual-route pairs, each agreeing below 1 σ .

a. *Closed Form for the Fine-Structure Constant*

Theorem AH.167 (Closed-form α). *The closed-form polynomial in the master invariant Ω ,*

$$\alpha^{-1}(\Omega) = \frac{\pi^{3/2}}{24} \text{Tr}(Y^2) \Omega \frac{\Omega + 3}{\Omega + 4} \left[1 + \frac{7}{80(\Omega + 4)^2 - 80} \right],$$

evaluated at $\Omega = 60$ with $\text{Tr}(Y^2) = 10$ on the $(3, 2, 1)$ gauge-bundle structure of $\mathbb{CP}^2 \times S^3$, gives $\alpha_{\text{DFD}}^{-1} = 137.03599985412\dots$. The CODATA 2022 measured value is $\alpha^{-1} = 137.035999177(21)$. The residual is $\alpha_{\text{CODATA}}^{-1} - \alpha_{\text{DFD}}^{-1} = -6.8 \times 10^{-7}$ (-0.0049 ppm, matching the convention-locked -0.005 ppm statement of the main paper); this is $\sim 32\times$ CODATA's quoted uncertainty of 2.1×10^{-8} , i.e. not an agreement within current experimental error.

Proof. The $\pi^{3/2}/24$ prefactor is the heat-kernel normalization of the main volume's exact construction: the $\zeta(0)$ of the $\mathbb{CP}^2$ charge trace, $-1/24$ per unit charge squared, times the terminating heat trace $(\sqrt{\pi}/4)\Lambda^3$ of the ρ -shifted Casimir on S^3 , with the heat-kernel regulator. It is not a Spin^c Hirzebruch–Riemann–Roch number, since those are rational. The $\text{Tr}(Y^2) = 10$ factor counts the standard-model hypercharge invariant on the three-generation $(3, 2, 1)$ bundle. With the standard $Y_Q = 1/6$ normalization (Slansky 1981 p. 96), $\text{Tr}(Y^2)|_{\text{one gen}} = 6 \cdot (1/6)^2 + 3 \cdot (2/3)^2 + 3 \cdot (-1/3)^2 + 2 \cdot (-1/2)^2 + 1 \cdot (-1)^2 = 1/6 + 4/3 + 1/3 + 1/2 + 1 = 10/3$; summed over three generations gives 10; its identification with $1/e^2$ rather than with the hypercharge coupling is not derived (main volume, the convention-locked closed form for α). In the factor $\Omega(\Omega + 3)/(\Omega + 4)$, the ratio $(\Omega + 3)/\Omega$ is the exact quantum-to-classical ratio of the $\mathbb{CP}^2$ hypercharge sec-

ond moment and $\Omega + 4$ is the Toeplitz dimension (main volume, convention-locked closed form). The coefficient of the $7/[80(\Omega + 4)^2 - 80]$ correction is a stated rational, $1/13$ at $\Omega = 60$, whose provenance is open: the $\mathbb{CP}^2$ Toeplitz sector is exact at every level and supplies no such term. Substituting $\Omega = 60$, $\text{Tr}(Y^2) = 10$, $\Omega + 4 = 64$, $80 \cdot 64^2 - 80 = 327600$ yields the displayed value to 14 digits via standard arbitrary-precision arithmetic. $\square$

The closed form has no continuous parameter, but its last 21 ppm rests on the bracket coefficient, whose provenance is open, and the 5 ppb figure is a property of the closed form rather than a test of the bracket.

b. Sub-meV Neutrino Spectrum Closure

Theorem AH.168 (Sub-meV neutrino-spectrum closure). *Branch-B exponents $k = \alpha^{-3/11}$, $r = \alpha^{-7/20}$ together with the 14/13 finite-dimensional priming factor on the right-handed Majorana scale $M_R = M_P \alpha^3$ predict the absolute neutrino mass spectrum at sub-meV precision:*

$$m_1 = 2.342 \text{ meV}, \quad m_2 = 8.963 \text{ meV}, \quad m_3 = 50.157 \text{ meV},$$

$$\Sigma m_\nu = 61.462 \text{ meV}, \quad m_{\beta\beta} = 9.1514 \text{ meV}, \quad m_{\beta\beta} \in \{0.13, 2.33, 3.25, 5.45\} \text{ meV},$$

with normal ordering, and $\delta_{CP} = -\pi/2$ from the μ - τ reflection, a structural postulate (main volume, App. FK); the four $m_{\beta\beta}$ values are the discrete Majorana-phase branches of the canonical TM1 chain. Confrontation with NuFIT 6.0 gives $\text{pull}_{21} = -0.031\sigma$, $\text{pull}_{31} = -0.141\sigma$, $\chi^2(2) = 0.021$, $p = 0.99$. DFD lands precisely on the global-fit best point with zero continuous parameters. (The rounded-prediction evaluation of Appendix X gives $\chi^2 = 0.025$; 0.021 is the full-precision-spectrum value — both at zero continuous parameters. Branch-selection status: the A–B binary is data-selected per the branch-selection remark of Appendix X of the main paper.)

Proof. The seesaw composition $m_\nu \sim m_D^2/M_R$ with Dirac-Yukawa $m_D = M_P \alpha^k$ and Majorana scale $M_R = M_P \alpha^r$ yields $m_\nu = M_P \alpha^{2k-r}$ at tree level. Branch B fixes $k = \alpha^{-3/11}$ exponent and $r = 3$ via Theorem AH.28. The 14/13 priming factor is $d_\nu/(d_\nu - 1)$ with $d_\nu = a + 5 = 14$ (main volume, App. X). Numerically: $m_3 = (14/13) \pi M_P \alpha^{14} = (14/13) \cdot (1.221 \times 10^{19} \text{ GeV}) \cdot (3.815 \times 10^{-30}) = 50.157 \text{ meV}$, with the mass ratios $m_2/m_1 = \alpha^{-3/11}$ and $m_3/m_2 = \alpha^{-7/20}$. The lower masses follow from the diagonal entries of the PMNS matrix with $\theta_{12} = 34.35^\circ$, $\theta_{23} = 45.0^\circ$ (maximal; the μ - τ reflection forces $\theta_{23} = \pi/4$ bundled with $\delta_{CP} = -\pi/2$, a sharp falsifiable prediction in soft $\sim 2\sigma$ tension with NuFIT 6.0, octant unresolved), $\theta_{13} = 8.509^\circ$. The normal-ordering forcing follows because $m_3 > m_2 > m_1$ in the geometric sector decomposition (App. X.8) is incompatible with inverted ordering. The NuFIT 6.0 χ^2 residual evaluates to 0.021 on two degrees of freedom (the two mass-splittings Δm_{21}^2 and $|\Delta m_{31}^2|$), giving $p = 0.99$. $\square$

Falsifier: JUNO's mass-ordering determination at 3σ (~ 2032). Inverted ordering falsifies the entire microsector architecture with no patch. A Simons Observatory + DESI + Euclid measurement of $\Sigma m_\nu > 80 \text{ meV}$ at 3σ (~ 2032) likewise falsifies.

c. Comparative Structural Profile of DFD vs. Four Major UV-Completion Programs

This subsection documents DFD's comparative structural profile against four major UV-completion programs (string theory, loop quantum gravity, causal set theory, asymptotic safety) on a fixed list of structural and empir-

ical criteria. It is presented as a comparative documentation, not as a uniqueness theorem; a uniqueness claim would require an external precise definition of “candidate unified theory.”

Proposition AH.169 (Comparative structural profile: DFD vs. four UV-completion programs). *The four leading UV-completion programs (string/M-theory, loop quantum gravity, causal set theory, asymptotic safety) are scored against DFD on seven structural-and-empirical criteria below. DFD scores positively on all seven; no listed rival scores positively on all seven. This documents the comparative status; it is not a uniqueness theorem.*

1. No continuous parameter fitted to a predicted shape

except the baryon density (main volume, App. FK).

2. No landscape problem (Single-Vacuum Theorem AH.157).
3. A single spectral action for the microsector, gauge and Einstein–Hilbert content (Theorem AH.154).
4. A single topological invariant ($\Omega = 60$, Theorem AH.155).
5. A two-axiom minimal statement (Theorem AH.166).
6. The SM + Λ + H_0 exponent ladder derived, with the inputs of App. FK.
7. 25+ sharp empirical falsifiers within 25 years.

Proof. The seven criteria are individually established by the cited theorems for DFD. The comparison is by direct enumeration over the four rival programs.

vs. String Theory. String theory has $\sim 10^{500}$ vacua (criterion 2: no); does not predict the SM gauge group (3, 2, 1) structurally (subsumed under criterion 6: no); does not predict fermion masses; does not predict α , Λ , or H_0 ; relies on selection. Has neither a single master action that determines all SM constants (criterion 3: partial only), nor a single topological invariant (criterion 4: no). Its Yukawas depend on continuous moduli (criterion 1: no). Number of theorem-grade falsifiers within 25 years: zero (criterion 7: no). DFD scores positively on all 7 criteria where string theory scores positively on at most 1 (the master action partial credit).

vs. Loop Quantum Gravity. LQG does not structurally predict the SM gauge group, fermion masses, Higgs mechanism, Λ , or α (criterion 6: no). No single master action that derives the SM (criterion 3: no). LQG’s Wheeler–DeWitt formal results and BH entropy from spectral count are real, but the comparison on criterion 6 is decisive: LQG has not closed the SM continuous-parameter prediction problem.

vs. Causal Set Theory. CST matches DFD on discrete cardinality at the fundamental level ($k_{\max} = 60$ Toeplitz quantization vs. causal-set count) and on a small-number Λ outcome (Sorkin’s $\Lambda \sim 1/\sqrt{N}$ vs. DFD’s α^{57}). CST is the closest competitor on Λ alone but does not extend to the full SM spectrum (no fermion masses, gauge group, Higgs, or α at theorem grade); criterion 6: partial only.

vs. Asymptotic Safety. AS shares DFD’s UV-finiteness ambition but assumes the existence of a non-Gaussian fixed point (NGFP). AS has not delivered SM closure (no fermion masses, Higgs, gauge group at theorem grade).

A summary scorecard per criterion is straightforward; positive scores are recorded only where the listed program achieves the structural outcome at theorem grade. The comparative profile (rather than a numerical scorecard) is presented in qualitative form: DFD predicts each of the seven listed criteria; the four rival programs each predict

a proper subset of them. The criteria are author-selected and not intended as a uniqueness claim. *The comparative profile, not the uniqueness claim, is the substantive content of this subsection.* $\square$

d. Cross-Cutting Consistency Theorem

Theorem AH.170 (Cross-cutting consistency of dual-route predictions). *The historical register contains 28 cases in which two predictions reached the same observable by different routes. Twenty-seven remain live and agree at sub- 1σ . The 28th was the optical screen route to H_0 , which fails (see the proof), so it is not a dual-route confirmation. Scope. This is a statement about the enumerated pairs only. It does not assert that the corpus contains no internal inconsistency of any other kind.*

Proof. By the master-action theorem (Theorem AH.154) and the single-invariant theorem (Theorem AH.155), the live pairs descend from the common structural source $\mathcal{L}_{\text{DFD}}$ and $\Omega = 60$. Explicit cross-check gives residuals below 1σ for the 27 surviving pairs. The 28th comparison had a 2.4σ residual, but its optical-screen H_0 arm fails: exact conformal invariance makes the global screen null, and its $C = 1$ clock is the bound-matter clock of the two-clock postulate (main volume, App. ZS), not the early clock. It remains in the historical enumeration only, not in the live consistency score. $\square$

No single-number probability claim is attached to the dual-route consistency check; predictions descending from a common heat-kernel expansion are not statistically independent, so a joint- χ^2 argument over them is not appropriate. The 27 live dual-route pairs are presented as internal-consistency cross-checks rather than as an aggregate-probability statement.

e. Sector-Resolved Casimir Modification (Leading Order)

Theorem AH.171 (Leading-order sector-resolved Casimir slope). *DFD’s optical-metric coupling ($n = e^\psi$) implies a leading-order modification to the standard QED Casimir force in a sector-resolved (metal/dielectric) parallel-plate geometry of magnitude*

$$\Delta F/F = \alpha/\pi \approx 2.32 \times 10^{-3} = 0.232\%,$$

arising directly from the sector-resolved optical-metric coupling: gauge-field zero-point fluctuations couple via the optical metric $\tilde{g}_{\mu\nu} = e^{2\psi} g_{\mu\nu}$ while material-bound fluctuations couple via the matter metric $\eta_{\mu\nu}$.

Proof. The standard QED Casimir energy density between parallel plates at separation d is $u_{\text{QED}} = -\pi^2 \hbar c / (720 d^4)$. In DFD, the gauge-field vacuum responds to $\tilde{g}_{\mu\nu}$ while the matter sector (responsible for the boundary conditions) responds to $\eta_{\mu\nu}$. For a sector-resolved geometry at $d \sim 100$ nm with local boundary-induced

refractive contrast $\Delta\psi_{\text{bdy}} \sim \alpha/(4\pi)$ (this is the standard one-loop QED self-energy scale at the boundary), the relative sector-resolved correction is

$$\Delta F/F = 4\Delta\psi_{\text{bdy}} = 4\alpha/(4\pi) = \alpha/\pi \approx 0.232\%.$$

This is the parallel-plate leading-order estimate. $\square$

The present prediction is the leading-order $\alpha/\pi = 0.232\%$ value; the falsifier window is widened correspondingly to reflect the geometric uncertainty in the boundary-condition sector. A future explicit Lifshitz-formula derivation may sharpen the central value.

Status: The geometric prefactor of the boundary sector is unresolved, so the coefficient is a conjecture outside DFD’s claims. A sub-0.1% precision Casimir-force measurement in sector-resolved (metal/dielectric) geometry that finds a modification outside $[+0.15\%, +0.50\%]$ refutes the conjecture, not the theory.

f. Quantum-Classical Mass Scale

Theorem AH.172 (Quantum-classical macroscopic-superposition threshold: a conjecture outside DFD’s claims). *DFD predicts that quantum coherence is sustained at all mesoscopic mass scales currently accessible by matter-wave or nano-mechanical interferometry.*

Proof. By the structural-reduction-of-measurement theorem (Theorem AH.162), the optical metric $n = e^\psi$ is uniquely sourced; no metric branching occurs. Decoherence in the position basis follows from the standard ρ -coupling at rate $\Gamma_{\text{DFD}} \sim Gm^2/(\hbar d) \cdot F(d/\lambda_{\text{dB}})$. For an object of mass m coupled via N internal microsector modes (with $N = \Omega = 60$), the macroscopic-superposition threshold corresponds to $\Gamma_{\text{DFD}} \cdot \tau_{\text{exp}} \approx 1$ at experimentally relevant times $\tau_{\text{exp}} \sim 1$ s. $\square$

Implication: DFD predicts no spontaneous macroscopic-superposition collapse at any currently testable scale. This contrasts sharply with Diosi–Penrose collapse (predicting collapse at $\tau \sim \hbar/E_{\text{grav}}$ for $m \gtrsim 10^{-13}$ kg). MAQRO is the decisive test (cf. Theorem AH.162).

g. Mathematical-Structure Connections (Riemann, Moonshine, Triality)

Proposition AH.173 (Riemann zeta connection at $\Omega = 60$, conjectural). *The DFD primed-determinant ratio at $k_{\text{max}} = 60$, formally*

$$\det(D^2)|_{\text{DFD}} = \exp[-\zeta'_{D^2}(0)],$$

is connected to $\zeta_{\text{Riemann}}(s)$ via the Voros–Cartier framework: the trivial zeros $s = -2, -4, -6, \dots$ correspond to the heat-kernel coefficients $a_2, a_4, a_6, \dots$ of the master action. Whether the non-trivial Riemann zeros at $s = 1/2 + iE_n$ have a DFD-spectral interpretation (Berry–Keating Hamiltonian conjecture) is open.

Proposition AH.174 (Moonshine connection, conjectural). *The McKay–Conway–Norton ADE classification chain $|A_5| = 60 \rightarrow E_8 \rightarrow \text{Leech lattice} \rightarrow \text{Monster}$ situates $\Omega = 60$ at the foot of the moonshine correspondence. Whether this is a deep mathematical fact (DFD structurally encoding the moonshine–monster categorical equivalence) or a numerological coincidence is open.*

Proposition AH.175 (Atiyah–LeBrun–Witten triality, conjectural). *DFD sits at the intersection of three deep mathematical-physics structures: (i) the spectral side (Connes spectral triple, Theorem AH.154); (ii) the twistor side (the flag-manifold twistor space $\text{SU}(3)/T^2$ of $\mathbb{CP}^2$); (iii) the categorical side (Reshetikhin–Turaev $\text{SU}(2)_{58}$ TQFT on S^3). A rigorous triality theorem situating DFD within Atiyah’s 2018 program, LeBrun’s twistor work, and Witten’s spectral-action contributions remains open.*

Future-work obligations: prove or refute each conjectural connection at theorem grade.

h. Audit of Existing Data

Five observational streams compared with DFD’s values. This is not a discovery claim; specific measurements within the 2026–2030 window can decide each case:

1. NA62 (2016–2022) $K^+ \rightarrow \pi^+ \nu \bar{\nu} = (13.0^{+3.3}_{-3.0}) \times 10^{-11}$, above DFD’s allowed window $[6, 11] \times 10^{-11}$ (Theorem AH.121) at its central value.
2. HERA Phase I 21-cm power-spectrum limits consistent with standard heating, as DFD predicts (its black holes radiate at Hawking’s rate) — a consistency check, *not* a near-term discriminator.
3. FLAG 2024 lattice $|\langle \bar{q}q \rangle|^{1/3} = 272 \pm 5$ MeV (0σ DFD per Theorem AH.132).
4. ATLAS $-1.2 < \kappa_\lambda < 7.2$ (95% CL), consistent with DFD’s $\kappa_\lambda = 1$, which is the Standard Model’s value.
5. NuFIT 6.0 mass splittings $\chi^2(2) = 0.021$ against DFD predictions at zero continuous parameters (one data-selected binary branch choice; App. X, branch-selection remark).

12. Mathematical-Foundation Closures and High-Precision Predictions

This subsection presents nine results on remaining structural gaps in the program, each with its grade: the Atiyah–LeBrun–Witten triality on the DFD slice, the Wheeler–DeWitt closed-form ground state, structural reduction of the measurement problem, the eight-layer single-vacuum rigidity, the photon-sphere bulk-boundary

partition-function identity, the A_5 -equivariant sharpening of the QECC reading (which keeps it conditional and partially established: the $[[60, 3, \geq 4]]$ Pauli-stabilizer structure holds, but the literal-Hamiltonian realization remains unverified), the gen-1 fermion mass refinement, the sphaleron-rate prefactor, and the PTA amplitudes against the supermassive-binary background. Three named structural negative results (the Riemann no-go, the Moonshine no-equivalence, and the sphaleron κ_{SM} measurement bottleneck) are recorded alongside.

a. Atiyah–LeBrun–Witten Triality

Theorem AH.176 (ALW triality on the DFD slice, conditional). *There exist explicit functors $F_1 : \text{Spec}_{\text{DFD}} \rightarrow \text{TW}_{\text{DFD}}$ (spectral $\rightarrow$ twistor), $F_2 : \text{TW}_{\text{DFD}} \rightarrow \text{Cat}_{\text{DFD}}$ (twistor $\rightarrow$ Reshetikhin–Turaev modular tensor category), $F_3 : \text{Cat}_{\text{DFD}} \rightarrow \text{Spec}_{\text{DFD}}$ (categorical $\rightarrow$ Frobenius algebra $\rightarrow$ spectral data), such that the triality identity*

$$F_3 \circ F_2 \circ F_1 \cong \text{Id}_{\text{Spec}_{\text{DFD}}}$$

holds at the strict-fixed-point level on the DFD slice (the spectral data over $\mathbb{CP}^2 \times S^3$ with $\chi_{\text{top}}(\mathbb{CP}^2, E) = 60$, Theorem AH.77).

Proof. F_1 is the composition: Connes reconstruction $\rightarrow$ LeBrun twistor functor (using the anti-self-dual structure of $\mathbb{CP}^2$) $\rightarrow$ Spin^c-determinant lift to $\text{SU}(3)/T^2 \rightarrow \mathcal{O}(d)$ -bundle restriction $\rightarrow$ real-form σ . F_2 is the composition: holomorphic Chern–Simons at level 58 on twistor space $\rightarrow$ partition function on $S^3 \sim \sqrt{2/60} \sin(\pi/60) \rightarrow$ the Reshetikhin–Turaev modular tensor category $\text{Rep}_q(\text{SU}(2)_{58})$ at $q = e^{2\pi i/60}$. F_3 is the composition: Fuchs–Runkel–Schweigert reconstruction of a Frobenius algebra from the MTC $\rightarrow$ modular generator D_C with eigenvalues $a(2a+3)/120 \rightarrow$ FS-indicator real structure $\rightarrow$ spin-parity grading $\rightarrow$ spectral data.

The strict-fixed-point property uses the Borel–Weil–Bott canonical isomorphism: $H^0(\mathbb{CP}^2, \mathcal{O}(9)) \cong V_{(9,0)}$ of $\text{SU}(3)$, which descends to $\text{SU}(2)$ -representations matching the level-58 truncation pointwise. The composition $F_3 \circ F_2 \circ F_1$ on Spec_{DFD} returns the same spectral data by direct computation via Atiyah–Singer at F_1 , the Witten Chern–Simons identity at F_2 , and Reshetikhin–Turaev surgery at F_3 . $\square$

Status. The triality is conditional: its F_1 leg is the conditional twistor theorem (Theorem AH.230), and a rigorous triality theorem remains open (Proposition AH.175). Uniqueness across broader slices (e.g., for arbitrary Spin^c Kähler manifolds with $\chi(M_c) = 3$) is open. The technical core at the F_2 stage (BV-quantized factorization-algebra rigor for holomorphic Chern–Simons) follows Costello’s framework.

b. Wheeler–DeWitt Closed-Form Ground State, Derivation Status

Theorem AH.177 (Wheeler–DeWitt ground state in DFD). *The functional $\Psi_0[\psi, \Xi] = \mathcal{N} \exp(-S_E[\psi, \Xi]/\hbar)$ on the DFD configuration space $\mathfrak{C}_{\text{DFD}} = H^s(\mathbb{R}^3; \mathbb{R}) \oplus H^s(\mathbb{R}^3; \mathbb{C}^{60})$ (with internal manifold $X = \mathbb{CP}^2 \times S^3$ and twisting bundle $\mathbf{E} = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$, $\chi(X, \mathbf{E}) = 60$) satisfies the Wheeler–DeWitt constraint $H_{\text{WD}}\Psi_0 = E_0\Psi_0$ with eigenvalue $E_0 = \alpha^{57} M_P^4 = 1.59 \times 10^{-122} M_P^4$. (Scope of the match: the agreement with the observed cosmological constant is at the exponent level, $10^{-122.9} - E_0$ itself is $\sim 14\times$ the observed ρ_Λ/ρ_P . The physical clock dictionary is $(H_0 t_P)^2 = C\alpha^{57}$, where the exponent is derived but $C = 0.87784 \pm 0.00266$ is measured; it gives the early clock $H_0^{\text{early}} = 67.54 \pm 0.10 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and the cross-channel value is 67.52 ± 0.09 ; bound-matter clocks read $C = 1$, $H_0^{\text{loc}} = 72.09$, under the two-clock postulate; see Apps. AP/ZS. Under the lock of the lapse to the scale factor an eigenvalue of this Hamiltonian acts as a curvature term, not as vacuum energy, so $\alpha^{57} M_P^4$ can be the dark energy only as a vacuum term of the action.)*

Proof. The DFD Wheeler–DeWitt constraint operator is $H_{\text{WD}} = -\hbar^2 \delta^2/\delta\psi^2 + V[\psi, \Xi]$ with $V[\psi, \Xi]$ the spectral-action potential on $\mathbb{CP}^2 \times S^3$. The Euclidean action $S_E[\psi, \Xi]$ is positive, coercive, and convex on $\mathfrak{C}_{\text{DFD}}$ (via monotonicity of the spectral-action potential and subcritical growth). Functional differentiation of Ψ_0 gives $\delta\Psi_0/\delta\psi = -\Psi_0 \delta S_E/\delta\psi/\hbar$ and $\delta^2\Psi_0/\delta\psi^2 = \Psi_0[(\delta S_E/\delta\psi)^2/\hbar^2 - \delta^2 S_E/\delta\psi^2/\hbar]$. Substituting into the WD equation and using the Hamilton–Jacobi identity $(\delta S_E/\delta\psi)^2 = V[\psi, \Xi] - E_0$ on-shell yields the eigenvalue equation. Uniqueness follows from Faris–Lavine (self-adjointness of $-\Delta + V$ for coercive V) and Perron–Frobenius (positivity of the ground state). The eigenvalue is extracted by primed-determinant ratio at gauge-normalized microsector coupling $g = \alpha$ (so that $\det'(g\Delta) = g^{57} \det' \Delta = \alpha^{57} \det' \Delta$): $E_0 = \det'(D^2)|_{\text{DFD}}/M_P^4 = \alpha^{57} M_P^4$. $\square$

Status note. The printed substitution drops a term: differentiating Ψ_0 twice gives $-\hbar^2 \delta^2\Psi_0/\delta\psi^2 = \Psi_0[-(\delta S_E/\delta\psi)^2 + \hbar \delta^2 S_E/\delta\psi^2]$, so $H_{\text{WD}}\Psi_0 = E_0\Psi_0$ requires $\hbar \delta^2 S_E/\delta\psi^2$ to be a ψ -independent constant; the proof never shows this. The eigenvalue $E_0 = \alpha^{57} M_P^4$ via the primed-determinant ratio is likewise **asserted, not derived**: $\det'(D^2)|_{\text{DFD}}$ is never computed above.

CMB consequences. The dictionary reading, a Bunch–Davies vacuum on Ψ_0 with slow-roll parameters $\epsilon_W = \alpha/(4\pi)$ and $\eta_W = 0$ and a microsector loop correction with $N_* \approx 27.7$, gives $n_s = 1 - 28.7\alpha/(2\pi) = 0.96667$, which is $1 - 2/60$, since $28.7\alpha/2\pi = 2/60$ to the printed digits, and $A_s \approx 2.10 \times 10^{-9}$. DFD has no mechanism for either. The tilt is a stated number; the mechanism is open. The amplitude A_s is calibrated, with $32\pi\alpha^5$ a numerical match (main volume, App. FK). The tilt lies within 0.5σ of the CMB alone, and the combination of Planck, ACT DR6, CMB lensing and DESI gives

0.974 ± 0.003 , about 2.4σ above it. On these premises the tensor ratio would be $16\epsilon_W = 4\alpha/\pi \approx 9.3 \times 10^{-3}$; the main volume does not adopt that dictionary (App. SM), and the χ relic bounds the primordial B-mode ratio at $r \lesssim 10^{-7}$, so $4\alpha/\pi$ is not a B-mode prediction.

Status of A_s (a calibrated amplitude). The value $A_s \approx 2.10 \times 10^{-9}$ above is the de Sitter dictionary result $A_s = H_*^2/(8\pi^2\epsilon_W \bar{M}_P^2) = 32\pi\alpha^5 = 2.080 \times 10^{-9}$ (-0.82σ vs Planck), with $\epsilon_W = \alpha/(4\pi)$ and the primordial (Wheeler–DeWitt) scale on the locked Majorana rung $M_R = M_P\alpha^3$. The exponent α^5 (the squared α^3 rung) holds only inside the dictionary at H_* , where no DFD stage freezes perturbations (main volume, App. FK), and the coefficient 32π is *not* first-principles derived — it rests on the H_* – M_R normalization, for which the corpus supplies no de Sitter horizon, internal-volume (S^3/CP^2), or spectral-action derivation. A Planck-mass subtlety is involved: $M_R = 4.74 \times 10^{12}$ GeV uses the *non-reduced* $M_P = 1.22 \times 10^{19}$ GeV, while the formula carries the *reduced* $\bar{M}_P = M_P/\sqrt{8\pi}$; under consistent conventions the required ratio is $H_*/M_R = \sqrt{8\pi}$ (mixing the two masses gives $H_* = \sqrt{8\pi} M_R \Rightarrow A_s = 8.3 \times 10^{-11}$, a spurious “ -67σ ”, and setting $H_* = 8\pi M_R$ double-counts the conversion). A_s is therefore one calibrated amplitude, and $32\pi\alpha^5$ a numerical match; see App. AT (“new predictions”). Relatedly, the printed $\eta_W = 0$ with $N_* \approx 27.7$ is a mislabel: $1 - 28.7\alpha/2\pi$ is the slow-roll form $1 - 2/N_e$ with $N_e = 60$, and DFD has no potential from which to compute a slow-roll parameter (main volume, App. FK), so the tilt is a stated number; the mechanism is open.

Falsifier. The tilt $n_s = 1 - 2/60 = 0.96667$ (a stated number; the mechanism is open), with no running, and no detectable primordial tensors ($r \lesssim 10^{-7}$) is the joint observational test for the Simons Observatory and LiteBIRD: a tilt outside 0.957 – 0.977 at the Simons Observatory’s precision, or a detected primordial tensor signal, falsifies it (main volume, App. FK and App. SM). The value $f_{NL}^{\text{equil}} = 0.007$ of the de Sitter dictionary has no printed source and is not a prediction.

c. Quantum Measurement Problem, Structural Reduction

Theorem AH.178 (Structural reduction of the measurement problem, rigorous). *The DFD elliptic optical-metric PDE*

$$\nabla \cdot [\mu(|\nabla\psi|/a_*)\nabla\psi] = -\kappa \langle \Psi|\hat{\rho}|\Psi \rangle$$

admits a unique weak solution when $\mu(x)x$ is strictly monotone; existence is by Browder–Minty (main volume, App. U). There is no metric branching; the position basis is einselected as the DFD pointer basis. The Born-rule functional form $\text{Tr}(\rho P)$ on $\mathcal{H}_{\text{micro}}$ ($\dim = 60 \geq 3$) follows from Gleason’s theorem once a complex inner product is supplied—which DFD obtains from the compatible complex structure J_* ($h = g + i\omega$), unique given the Hessian positivity that the main volume’s App. QM argues but

does not prove; Gleason itself cannot select the inner product. The Born value $|\langle \phi|\Psi \rangle|^2$ then follows from envariance/typicality, conditional on J_* .

Proof. Existence of weak solutions follows from the Browder–Minty theorem, uniqueness from the strict monotonicity of $\mu(x)x$, and $C_{\text{loc}}^{1,\alpha}$ regularity from quasilinear elliptic theory (main volume, App. U). No-branching: the optical metric is uniquely determined by the c-number expectation $\langle \Psi|\hat{\rho}|\Psi \rangle$, and any branching of Ψ into orthogonal components contributes additively to the c-number source, sourcing a single (averaged) optical metric. The pointer basis is position because the ρ -coupling is local in position space.

Decoherence rates: $\Gamma_{\text{DFD}} = \xi_{\text{DFD}} m^2 d^2 F(d/\lambda_{\text{dB}})$ with ξ_{DFD} from the massless elliptic Green’s function of the optical metric (Appendix A; the optical ψ is long-range MOND, *not* Yukawa-screened). The smallness reflects the c-number no-branching suppression of the paragraph above (no per-branch self-force), *not* a ψ mass. Numerical (dominant c-number reading): single atom $\sim 10^{-43} \text{ s}^{-1}$ (stable on cosmological timescales); C_{60} fullerene $\sim 10^{-38} \text{ s}^{-1}$; MAQRO test mass (10^{-13} kg) $\sim 10^{-20} \text{ s}^{-1}$ (sustained interference for 100 s); macro-cat (10^{23} atoms) $\sim 1 \text{ s}^{-1}$.

Born rule, status: Gleason’s theorem fixes only the *functional form* of the probability measure on the projection lattice $\mathcal{L}(\mathcal{H}_{\text{micro}})$ ($\dim = 60 \geq 3$) *given* a complex inner product; it does *not* derive Born from scratch, since it presupposes the very complex inner product / projection lattice in question (the same 60 real dimensions admit different positive-definite inner products giving different “Gleason” weights for one ray). DFD supplies the inner product from the compatible complex structure J_* ($h = g + i\omega$), unique given the argued Hessian positivity, and the Born *value* $|\langle \phi|\Psi \rangle|^2$ then follows from envariance/typicality, conditional on J_* (consistent with the Born ledger of the QM-status appendix, App. QM). $\square$

Falsifier. MAQRO experiment at 10^{-13} kg test mass with 100 s coherence time decisively distinguishes DFD ($\tau_{\text{dec}} \sim 10^{20} \text{ s}$, sustained interference) from Diosi–Penrose ($\tau_{\text{dec}} \sim 10^{-3} \text{ s}$, collapse). The sustained-interference prediction is fixed by the no-branching theorem (c-number sourcing, no per-branch self-force), consistent with the open-problems section; the deep-MOND-enhanced per-branch collapse alternative is excluded by the two-body structure of the derived vertex (main volume, App. ZU).

d. Single-Vacuum Theorem, Rigidity Layers

Theorem AH.179 (Eight-layer single-vacuum rigidity). *The DFD vacuum moduli space $V_{\text{DFD}}/\sim$ (where $\sim$ is the closure of gauge-equivalence, discrete-symmetry equivalence, and spectral-data equivalence) has $\dim = 0$ and cardinality 1. The uniqueness traces to eight rigidity statements; layer 1 is the manifold theorem, layer 6 conditional and layer 7 a stated premise (see the note below).*

Proof. The eight rigidity layers, with the deepest atomic fact $\chi_{\text{top}}(\mathbb{CP}^2) = 3$ at the foundation:

1. Manifold uniqueness $\mathbb{CP}^2 \times S^3$ (Appendix AB, Theorem AB.1).
2. $k_{\text{max}} = 60$ forced by $\chi(\mathbb{CP}^2, E) = 60$ via Hirzebruch–Riemann–Roch.
3. Flux quantum $q_1 = 3$, derived from the isotropy charge $6Y$ (main volume, App. F, the remark on the flux quantum); the allowed values are 3λ with λ odd.
4. Bundle decomposition $(a, n) = (9, 5)$ forced by minimal-padding.
5. Spin^c structure unique on $\mathbb{CP}^2$ (by $w_2(\mathbb{CP}^2) = c_1 \bmod 2$).
6. Real KO-dim 4 spectral triple unique up to isomorphism (Connes’ axioms), conditional on a carrier that meets order zero, which $\mathbb{C}^{60}$ does not (Theorem AH.77).
7. No-go for continuous deformation $D \rightarrow D + \delta D$ at Λ_* fixed (preserved spectral data force isolated points). This layer is a stated premise, not a consequence of the spectral action: it holds D_F at its printed spectrum and admits as physical only the inner fluctuations of $\text{End}(\mathbf{E})$, which carry the gauge fields (their group is selected, not forced: Theorem AH.214, Remark AH.215), not those of $M_{60}(\mathbb{C})$ (Theorem AH.77), which would make the 60 eigenvalues dynamical; the 57-level spectrum is not a stationary point of the spectral action, whose stationary spectra have one nonzero level inside the plateau and at most two beyond it for the cutoffs tested (main volume, App. FK).
8. Cardinality 1 in the connected component (unique limit point in the moduli of bundles satisfying 1–7).

Each layer except layer 6, conditional as stated, and layer 7, a stated premise, is a theorem-grade rigidity statement; the conjunction forces $\dim(V_{\text{DFD}}/\sim) = 0$ and $|V_{\text{DFD}}/\sim| = 1$. The deepest atomic fact driving the chain is $\chi_{\text{top}}(\mathbb{CP}^2) = 3$ (identified with the discrete input $N_{\text{gen}} = 3$; the derived $q_1 = 3$ then propagates through the bundle decomposition; status remarks: App. F, on the generation count and q_1).

Note: layer 1 is not independent. Layer 1 as printed is Theorem AB.1, the manifold-uniqueness theorem itself, so invoking it as one of eight “independent” rigidity layers supporting single-vacuum rigidity is circular. This theorem has **seven** independent layers (2–8), each conditional on the manifold $\mathbb{CP}^2 \times S^3$ having already been fixed by Theorem AB.1, which is itself conditional on the $p = 1$ lens-space selection recorded in the uniqueness appendix of the main volume. The conclusion $\dim(V_{\text{DFD}}/\sim) = 0$, $|V_{\text{DFD}}/\sim| = 1$ is unaffected — layers 2–8 alone force

it once the manifold is fixed — but the independence count and the logical base must be stated as above, not as “eight independent layers” resting on nothing prior.

Comparison: string theory (hundreds of continuous moduli and $\sim 10^{500}$ flux choices, $\dim > 0$); M-theory (G_2 moduli, $\dim > 0$); LQG (non-unique kinematic vacua); SM EFT (26 continuous parameters); DFD ($\dim = 0$, $|V/\sim| = 1$). $\square$

e. AdS/CFT-without-AdS Bulk-Boundary Identity

Theorem AH.180 (Photon-sphere bulk-boundary partition function identity, conjectural). *For any compact mass M in DFD with photon sphere at the areal radius $3GM/c^2$, the bulk path integral with Dirichlet data $(g_{ij}^\partial, \partial\phi)$ on $\partial B_{r_{\text{ph}}}$ equals the $SU(2)_{58}$ WZW boundary partition function:*

$$Z_{\text{bulk}}[g_{ij}^\partial, \partial\phi] = Z_{\partial}^{SU(2)_{58}}[J(\sigma) = \partial\phi(\sigma)]$$

to all orders in the asymptotic expansion as $r \rightarrow r_{\text{ph}}^+$. The boundary central charge is $c = 3 \cdot 58/(58 + 2) = 174/60 = 2.9$. The HKLL-style reconstruction kernel is valid outside the event horizon (not just near r_{ph}).

Proof. The bulk on-shell expansion in $\rho = r - r_{\text{ph}}$ admits proper holographic renormalization counterterms. The boundary $SU(2)_{58}$ WZW partition function uses the affine-Lie-algebra Sugawara construction with characters $\chi_a(\tau)$ and modular S -matrix at $k = 58$. In the order-by-order partition-function identity the bulk Knizhnik–Zamolodchikov equations emerge from DFD bulk equations expanded near r_{ph} . Conformal weights $\Delta_a = a(a + 1)/60$ derive from Sugawara at level $k = 58$; primary fields $a \in \{0, 1/2, 1, \dots, 29\}$ tabulate explicitly. The HKLL reconstruction kernel $K^{\text{DFD}}(r, \sigma; \sigma') = (n(r)/n(r_{\text{ph}}))^{-\Delta_a} P_a(\cos \gamma)$ uses hypergeometric radial profiles $g_a(r) = (r_{\text{ph}}/r)^{\Delta_a + (a) \cdot 2} F_1(\dots)$. Global validity for $r > 0$ uses piecewise definition with analytic continuation across the photon sphere. $\square$

f. QECC (partially established): Pauli-Stabilizer Structure, Literal-Hamiltonian Realization Conditional

Theorem AH.181 ($[[60, 3, \geq 4]]$ Pauli-stabilizer structure; literal-Hamiltonian QECC conditional). *The DFD microsector Hilbert space $\mathcal{H}_{\text{micro}} = \mathbb{C}^{60}$ carries an A_5 -equivariant Pauli-stabilizer structure with parameters $[[60, 3, \geq 4]]$: stabilizer group $S_{\text{DFD}} = \langle Z_{60}^3 \rangle \subset \text{HW}(60)$, $|S_{\text{DFD}}| = 20$, whose $+1$ -eigenspace is the 3-dimensional code subspace $\mathcal{C} = \text{span}\{|0\rangle, |20\rangle, |40\rangle\}$, identifiable with the 3 SM generations as logical qutrits; logical Pauli operators $(\bar{X}, \bar{Y}, \bar{Z})$ form the qutrit Pauli group on $\mathcal{C}$, with code distance $d \geq 4$ in the SM-gauge-quotient metric. The $[[60, 3, \geq 4]]$ stabilizer structure holds; the literal-Hamiltonian QECC realization is conditional (partially established, consistent with the assessment in App. AH 8n*

of the CKM/ τ companion): $[D_F, S_{\text{DFD}}] \neq 0$ as operators and the code distance is 1 in the standard displacement metric (caveat to Theorem AH.144), so the literal-Hamiltonian and Knill–Laflamme readings are not yet established.

Proof. The Heisenberg–Weyl group $\text{HW}(60)$ has 3600 elements with generators X_{60}, Z_{60} satisfying $X_{60}Z_{60} = e^{2\pi i/60} Z_{60}X_{60}$. The Spin^c Dirac operator D_F on $\mathbb{CP}^2 \times S^3$ is A_5 -equivariant (since the binary icosahedral $2I = \text{SL}(2, \mathbb{F}_5)$ acts on $\mathbb{CP}^2 \times S^3$ by isometries). Character theory of A_5 decomposes $\mathcal{H}_F = \mathbb{C}[A_5]$ into five character-isotypic components, of dimensions 1, 9, 9, 16 and 25 (the squares of the irreducible dimensions 1, 3, 3, 4, 5); the identification of the channel space with $\mathbb{C}[A_5]$ is one of the stated structural postulates (main volume, App. Z). The three-dimensional code space comes instead from the choice of stabiliser: the eigenspaces of Z_{60}^3 on $\mathbb{C}^{60}$ are twenty spaces of dimension 3, so the commutant of $S_{\text{DFD}} = \langle Z_{60}^3 \rangle$ is $\bigoplus_{j=0}^{19} M_3(\mathbb{C})$, with $|S_{\text{DFD}}| = 20$; the dimension 3 is $\gcd(3, 60)$, set by the choice of Z_{60}^3 .

Code subspace $\mathcal{C} = \ker(Z_{60}^3 - \mathbb{I}) = \text{span}\{|0\rangle, |20\rangle, |40\rangle\} \cong \mathbb{C}^3 = 3$ generations. Logical $\bar{X}$ permutes basis $|0\rangle \rightarrow |20\rangle \rightarrow |40\rangle \rightarrow |0\rangle$; logical $\bar{Z}$ acts as $|0\rangle \rightarrow |0\rangle$, $|20\rangle \rightarrow e^{2\pi i/3}|20\rangle$, $|40\rangle \rightarrow e^{4\pi i/3}|40\rangle$. Together they satisfy $\bar{Z}\bar{X} = \zeta \bar{X}\bar{Z}$ with $\zeta = e^{2\pi i/3}$, the qutrit Pauli relation.

Code distance $d \geq 4$ in the SM-gauge-quotient metric (via the S^3 winding $\sqrt{60} \approx 7.75$ in Berezin–Toeplitz lattice units). The $[[60, 3, \geq 4]]$ stabilizer structure holds; the literal-Hamiltonian QECC realization (and hence the Knill–Laflamme verification for error sets up to weight 3, with the quoted threshold $p_{\text{th}} \approx 0.022$) is conditional, because $[D_F, S_{\text{DFD}}] \neq 0$ as operators and the code distance is 1 in the standard displacement metric (caveat to Theorem AH.144). Pending that, the conservative reading is “protected 3-dimensional subspace within a 60-dimensional Hilbert space; QECC-adjacent” (partially established). $\square$

g. Light-Fermion Generation-1 Mass Refinement

Theorem AH.182 (Sector-specific Toeplitz priming for gen-1 fermion masses). *The 3.32% electron-mass residual is closed to 0.0091% via the sector-specific finite-d Toeplitz priming factor $F_e = 30/31$. The same mechanism, with sector-specific Toeplitz dimensions d , applies to up- and down-quark masses with $F_u = 45/44$ (inverted direction, conjugate-Higgs) and $F_d = 57/58$ (positive direction). The mean absolute error across nine charged fermions drops from 1.42% (leading order) to 0.61%.*

Proof. The Toeplitz heat-kernel expansion at finite dimension d contributes a sub-leading $1/d$ correction to the leading-order mass formula $m_f = M_P \alpha^{k_f} F_{\text{geom}}^{(f)}$. The direction (multiplicative or divisive) is fixed by whether the sector couples to the Higgs in the direct or conjugate orientation. For the lepton (direct-Higgs) sector with

$d_e = k_{\text{max}}/2 + 1 = 31$ (chirality-projected half-spectrum + null channel), $F_e = (d_e - 1)/d_e = 30/31$. For the up-quark (conjugate-Higgs) sector with $d_u = a \cdot n = 9 \cdot 5 = 45$, $F_u = d_u/(d_u - 1) = 45/44$. For the down-quark (direct-Higgs) sector with $d_d = (k_{\text{max}} - N_{\text{gen}}) + 1 = 58$, $F_d = (d_d - 1)/d_d = 57/58$.

Numerical evaluation:

- $m_e^{\text{DFD}} = 0.5109523 \text{ MeV}$ vs. CODATA $0.5109989 \text{ MeV} \rightarrow -0.0091\%$.
- $m_u^{\text{DFD}} = 2.1599 \text{ MeV}$ vs. FLAG 2024 $2.16 \pm 0.07 \rightarrow -0.005\%$ (within 1σ).
- $m_d^{\text{DFD}} = 4.6699 \text{ MeV}$ vs. FLAG 2024 $4.70 \pm 0.05 \rightarrow -0.640\%$ (within 1σ).

The same $\{30/31, 45/44, 57/58, 14/13\}$ priming family applies to the four primed sectors $\{e, u, d, \nu\}$, with $F_\nu = 14/13$ producing the sub-meV neutrino spectrum (Theorem AH.168). Heavier-fermion stability: no priming applies at gen-2/gen-3 because the $1/d$ mechanism enters only at maximum geodesic distance $n_f = 2.5$ from the gen-1 Higgs vertex. $\square$

Cross-cutting consistency. The priming family is consistent across the four primed sectors $\{e, u, d, \nu\}$.

h. Sphaleron-Rate Prefactor

Theorem AH.183 (Sphaleron-rate prefactor: structure, not value). *The DFD electroweak sphaleron-rate prefactor is*

$$\kappa_{\text{DFD}} = \kappa_{\text{SM}} \cdot [1 + \delta_{\text{B-T}}],$$

where $\delta_{\text{B-T}}$ receives a UV contribution $\delta_{\text{UV}} = (T/M_P)^2 \sim 10^{-32}$ (structurally negligible, modes above $k_{\text{max}} = 60$ absent), a boundary contribution $\propto \alpha_W a_3$ from the a_3 Berezin–Toeplitz heat-kernel coefficient on $\mathbb{CP}^2 \times S^3$, and a generation contribution $\propto \alpha_W \ln(M_R/M_P)/(4\pi)$, which is negative for $M_R = M_P \alpha^3$. **The numerical value of $\delta_{\text{B-T}}$ is an open item:** the two contributions summed to $\delta_{\text{B-T}} = +0.067 \pm 0.04$ require mutually inconsistent values of α_W ($1/14.5$ vs $1/117.5$) and the second carries a sign error, so that sum does **not** hold. With $\delta_{\text{B-T}} = 0$, $\kappa_{\text{DFD}} = \kappa_{\text{SM}} = 16.5 \pm 2.0$ (D’Onofrio–Rummukainen–Tranberg 2014 + Bödeker–Moore 2000). The lattice κ_{SM} uncertainty is the binding bottleneck per the structural negative result of §AH 12j. The result here is the structure of κ_{DFD} , not its value; η_B is a measured input here because the thermal-leptogenesis route is closed — the real determinant-protected lepton kernel forces its CP source $\varepsilon_i = 0$ (baryogenesis no-go, §AH 4b). The internal axial Berry-holonomy channel separately derives the α^4 power of $|\eta_B|$; its bias current is $B-L$, its printed mechanism gives about $(2-3) \times 10^{-9}$, three to five times Planck’s value, and its sign rests on an adopted axiom (Thm. AH.33).

Proof. The Klinkhamer–Manton sphaleron solution lives in the $SU(2)_L \times U(1)_Y$ Higgs sector at the EWPT. With DFD’s optical metric $n = e^\psi$, the static-saddle equations modify E_{sph} by $\zeta = \langle e^\psi \rangle_T^{1/2}$. At $T_c = 159.5$ GeV (matching lattice SM crossover), $|\zeta - 1| \lesssim 10^{-32}$, structurally negligible.

The prefactor decomposition $\delta_{\text{B-T}} = \delta_{\text{UV}} + \delta_{\text{boundary}} + \delta_{\text{gen}}$:

- $\delta_{\text{UV}} = (T/M_P)^2 \sim 10^{-32}$ (UV modes above $k_{\text{max}} = 60$ absent in DFD).
- $\delta_{\text{boundary}} = (3/2)\alpha_W a_3$ with $a_3 = 9/(8\pi)$ from HRR on bundle E over $\mathbb{CP}^2 \times S^3$: at the physical $\alpha_W(T_c) \approx 1/29.6$ this gives $+0.018$, not $+0.037$ (which requires $\alpha_W = 1/14.5$).
- $\delta_{\text{gen}} = 3\alpha_W \ln(M_R/M_P)/(4\pi)$ with $M_R = M_P\alpha^3$: since $\ln(M_R/M_P) = 3\ln\alpha < 0$, this term is *negative* (-0.119 at $\alpha_W = 1/29.6$), not $+0.030$ (which requires a negative α_W , i.e. a sign error).

The sum $\delta_{\text{B-T}} = 0.067 \pm 0.04$ does **not** hold: no single α_W reproduces the two contributions $+0.037$ and $+0.030$. The structure stands; the value is open. $\square$

i. PTA Amplitudes and the Supermassive-Binary Background

Theorem AH.184 (PTA amplitudes and the supermassive-binary A_GWB). *At $\gamma = 13/3$ the NANOGrav 15-yr amplitude $A = 2.4_{-0.6}^{+0.7} \times 10^{-15}$ agrees with the supermassive-binary value $A_{\text{GWB}} = (2.40 \pm 0.30) \times 10^{-15}$ from the SMBH binary mass function. The prediction is the standard supermassive-binary estimate; it does not discriminate DFD from Λ CDM astrophysics.*

Proof. The DFD A_{GWB} prediction follows from: (i) SMBH binary mass function (Schechter fit at $z = 2$ with $M_\star = 6.3 \times 10^9 M_\odot$); (ii) Phinney’s circular-inspiral formula $A_{\text{GWB}}^2 \propto \int dM n(M) M^{5/3}$. Result: $A_{\text{GWB}}^{\text{DFD}} = 2.40 \times 10^{-15}$ at 1-yr period.

The five PTA amplitudes are: NANOGrav 15-yr $A = 2.4_{-0.6}^{+0.7}$; EPTA DR2new $A = 2.5 \pm 0.7$; PPTA DR3 $A = 2.04_{-0.22}^{+0.25}$; IPTA DR2 $A = 2.8_{-0.8}^{+1.2}$; CPTA DR1 $\log_{10} A = -14.4_{-2.8}^{+1.0}$ for a free spectral index (the other four at $\gamma = 13/3$). Spectral index $\gamma = 13/3 = 4.33$ matches DFD; cosmic strings predict $\gamma = 5$; FOPT $\gamma = 4-5$. $\square$

Falsifier. SKA-PTA by 2032 measures γ to ± 0.02 . If γ persistently lies outside $[4.0, 4.6]$, DFD’s nanohertz theorem is structurally falsified.

j. Structural Negative Results

The following three results are recorded as *anti-theorems*: structural negative findings, recorded as the TPI and CKM results are (Remarks AH.109 and AH.91).

a. Anti-Theorem (Riemann Hypothesis no-go). Self-adjointness of the DFD Hamiltonian H_{DFD} on $\mathcal{H}_{\text{micro}}$ does not imply the Riemann Hypothesis. Three independent obstructions: (i) finite-rank self-adjointness $\neq$ infinite-dim self-adjointness; (ii) the $\Omega = 60$ axiom forbids the $k_{\text{max}} \rightarrow \infty$ limit needed for genuine Hilbert–Pólya; (iii) density agreement $\neq$ pointwise eigenvalue equality (residuals up to ± 0.09 visible). *Positive partial result preserved:* the Weyl counting density of $\{E_n\}$ matches Riemann’s $N_\zeta(E)$ exactly including the $7/8$ Maslov phase, and the trivial-zeros / heat-kernel coefficient correspondence is rigorous.

b. Anti-Theorem (Moonshine no-equivalence). The chain $\Omega = 60 \rightarrow A_5 \rightarrow \hat{E}_8 \rightarrow \text{Leech} \rightarrow \text{Monster}$ does not lift to a categorical equivalence. $\text{rank}(\mathcal{C}(SU(2)_{58})) = 59 \neq 1 = \text{rank}(\text{Mod}(V^\natural))$ since $V^\natural$ is holomorphic. No braided monoidal equivalence possible, no faithful tensor functor possible. The chain breaks at the first link ($\Omega = 60 \leftrightarrow A_5$), which is purely cardinality-matching. *Positive partial result preserved:* $A_5 \subset \text{Monster}$ via Wilson 1986 contributing $T_{5A}(\tau)$ Hauptmodul through prime 5 of A_5 — but this distinguishes 5, not 60.

c. Anti-Theorem (Sphaleron κ_{SM} bottleneck). The DFD sphaleron-rate prediction is bottlenecked by lattice κ_{SM} uncertainty (currently ± 2.0 on $\kappa_{\text{SM}} = 16.5$). Sub-1% lattice precision is required for a sub-1% test of the Berezin–Toeplitz boundary correction; current state-of-the-art is 12%. The bottleneck is the lattice, not DFD-internal phenomenology (Theorem AH.183).

13. Origin, Multi-Route Electromagnetism, Quantum Gravity, and Materials-Science Predictions

This subsection presents fifteen results covering the deepest origin questions, multi-route derivations of electromagnetism, the non-perturbative quantum-gravity path integral plus S-matrix and the observable metric, the three-layer back-reaction system, and materials-science predictions. One structural negative result is recorded: the “five proofs of $\Omega = 60$ ” are four routes to the number 60.

a. Origin of the Universe in DFD

Theorem AH.185 (Wheeler–DeWitt ground state as the cosmogenic state). *The Wheeler–DeWitt ground state $\Psi_0[\psi, \Xi] = \mathcal{N} \exp(-S_E[\psi, \Xi]/\hbar)$ on the DFD configuration space is the proposed cosmogenic solution to the master constraint $H_{\text{WD}}\Psi_0 = E_0\Psi_0$ with $E_0 = \alpha^{57} M_P^4$ (read as in the scope note of Theorem AH.177). The start state rests on two stated inputs, the no-boundary state with nothing inserted and the smooth beginning, which carries the arrow of time (main volume, App. Q). The cosmological arrow of time is welded to the dynamical complex structure $J_\star$ via the optical arrow $\dot{\psi}_0 > 0$ (sub-statement (ii)), not to the static $\mathbb{CP}^2$ Kähler structure. The global*

state does not dissolve the horizon problem: it fixes the ψ fluctuation sector and the internal S^3 , not the plasma (sub-statement (iii)).

Proof. The cosmogenic claim follows from three sub-statements:

(i) **No-boundary condition.** The DFD configuration space $\mathfrak{C}_{\text{DFD}}$ has no preferred temporal boundary; Ψ_0 is defined globally without specifying initial data on a Cauchy slice.

(ii) **T-asymmetry from the optical arrow on the dynamical J_* , not the static $\mathbb{CP}^2$ orientation.** The arrow is *not* carried by the static internal Kähler structure. On $\mathbb{CP}^2$, which has *even* complex dimension $n = 2$, the flip $J \rightarrow -J$ leaves the canonical orientation $\text{vol} = \omega^n/n! \rightarrow (-1)^n \text{vol} = +\text{vol}$ *unchanged*, so the $i = \pm J$ bit is orientation-invisible (the same even-dimension fact by which CP conjugation on $\mathbb{CP}^2$ is orientation-preserving, App. L); the static $\mathbb{CP}^2$ J therefore cannot break T . The asymmetry instead arises *dynamically*. Time reversal T is *antiunitary*, so $T J_* T^{-1} = -J_*$ acts on the *dynamical*, field-level positive-frequency complex structure J_* (the global quantum i , Theorem QM.10, App. QM), not on the static internal J . The sign of J_* —equivalently the direction of $i\hbar\partial_t$ —is then fixed by the *optical arrow* $\dot{\psi}_0 > 0$ (Theorem Q.10, “Branch selection by the optical arrow,” App. Q): the primordial fast-light boundary condition $\psi_0(0^+) \rightarrow -\infty$ selects the future direction and hence $i = +J_*$, while $H \geq 0$ —argued from the stable $\psi + \chi + h^{\text{TT}}$ Hessian (Theorem QM.10)—makes the bounded-below generator $A = J_* H/\hbar$ unique. The asymmetry therefore sits in the boundary data, not in Ψ_0 , which is real. *Grounding:* the master dynamics are themselves T -symmetric; the arrow rests on a *primordial boundary condition*, the smooth beginning of the main volume’s App. Q—the same low-entropy-past input that every cosmology requires, not a consequence of time-symmetric dynamics (Rem. Q.11).

(iii) **Horizon uniformity.** The printed global state does not supply the smoothness of the CMB at $z = 1100$: Axiom PO of the main volume’s App. Q fixes the ψ fluctuation sector and the internal S^3 , not the plasma, and the print names no process that makes the plasma; the smoothness is stated, in the smooth beginning (main volume, App. Q). The pre-BBN optical engine would put the observed sky in causal contact, but it accelerates the observable scale factor, so it needs a source of negative pressure and a reheating stage, which the print does not supply (main volume, App. Q).

The exponent 57 of $E_0 = \alpha^{57} M_P^4$ is that of Theorem AH.158; the coefficients differ, and the observed dark-energy density carries the measured R (main volume, App. AU). (Theorem AH.177’s own eigenvalue step is asserted, not derived; see its status note, §AH.12 b.) $\square$

b. Four Routes to $\Omega = 60$

Theorem AH.186 (Four routes to $\Omega = 60$). *The four converging derivations (HRR Euler characteristic on bundle E over $\mathbb{CP}^2 \times S^3$, $|E_8 \text{ roots}|/4$, $|A_5|$ directly, RT MTC at level $k = 58$) all give 60.*

Proof. Direct equivariant Lefschetz–RR on the orbifold $X = \mathbb{CP}^2 \times S^3/2I$ (where $2I = \text{SL}(2, \mathbb{F}_5)$ is the binary icosahedral group double-covering A_5) gives $\chi_{\text{equiv}}(X, \mathbf{E}) = \sum_g L(g) = |A_5| = 60$ via the Lefschetz fixed-point trace. The McKay correspondence $2I \leftrightarrow \hat{E}_8$ pairs the nine irreducible representations of $2I$ with the nine nodes of the affine E_8 diagram, and E_8 has $240 = 4 \times 60$ roots. The RT modular-tensor-category at level 58 gives 60 through the Sugawara level-shift $k + h^\vee = 58 + 2$ (it has 59 simple objects). $\square$

a. *Anti-Theorem* (Five proofs of Ω are four-equivalent). The five derivations are four routes to the number 60.

c. Origin of $\mathbb{CP}^2 \times S^3$: Eight-Layer Manifold Rigidity

Proposition AH.187 ($\mathbb{CP}^2 \times S^3$ is DFD-admissible under eight rigidity axioms; uniqueness is conjectural pending exhaustive enumeration). *Among compact 7-manifolds, the pair $(M, \mathbf{E}) = (\mathbb{CP}^2 \times S^3, \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5})$ satisfies the eight DFD-admissibility axioms. Eight rigidity layers narrow the candidate space substantially, and explicit refutations of named alternatives (S^4 , $\mathbb{CP}^3$, $S^2 \times S^2$, S^7 , $T^4 \times S^3$, $K3 \times S^3$, Calabi–Yau 3-fold, G_2 holonomy, lens spaces) each fail at least one rigidity layer. Strict uniqueness across the full 7-manifold landscape is conjectural pending exhaustive enumeration; the present statement is a top-down narrowing argument, not a from-below classification.*

Plausibility argument. The eight rigidity layers each force one factor of the parameter space:

1. Compactness $\Leftrightarrow$ finite mode count.
2. Total dimension 7 $\Leftrightarrow$ correct gauge-group dimensional analysis ($\dim G_{\text{SM}} = 12$, after KK reduction).
3. Product structure $M_c \times M_g \Leftrightarrow$ Wang’s classification of compact Lie-coset products.
4. $\chi_{\text{top}}(M_c) = 3$ plus positivity and simple connectivity (main volume, App. AB) $\Rightarrow M_c = \mathbb{CP}^2$.
5. $\pi_3(M_g) = \mathbb{Z} + \text{free } SU(2) \text{ action} \Leftrightarrow M_g = S^3$.
6. Spin^c structure on $\mathbb{CP}^2 \Leftrightarrow w_2 = c_1 \pmod{2}$.
7. Self-dual Einstein metric of positive scalar curvature (Hitchin) $\Leftrightarrow$ Fubini–Study.
8. Minimal-padding bundle $\mathbf{E} = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5} \Leftrightarrow$ explicit HRR computation $\chi(\mathbf{E}) = 55 + 5 = 60$.

The eight refutations of named alternatives are individually rigorous. It is a proposition, not a theorem, because of the distinction between (a) showing that named alternatives fail, and (b) ruling out all conceivable alternatives via exhaustive product-decomposition + bundle-classification enumeration. The latter requires a separate classification result not in hand. $\square$

Scope note. This statement is not a uniqueness theorem. The “ruled out alternatives” list (S^4 , $\mathbb{CP}^3$, etc.) demonstrates non-admissibility for the named candidates; it does not constitute a from-below classification. A uniqueness theorem awaits either an exhaustive classification of compact 7-manifolds with product structure satisfying axioms 1–8, or a rigorous obstruction argument blocking all unnamed candidates. Until then, the present statement is recorded as a proposition. The main volume’s uniqueness theorem (App. AB) is conditional on seven stated axioms and does not supply that classification.

d. Master Derivation Tree of the Standard-Model Constants

Theorem AH.188 (Derivation tree of the $\text{SM} + \Lambda + H_0$ constants, given the stated inputs). *All 34 continuous parameters of the Standard Model + $\Lambda + H_0$ follow from the pair $(\Omega = 60, M_P)$ given the discrete, prefactor and continuous inputs of App. FK of the main volume (see caveat). The nine charged-fermion masses carry a 0.61% mean error (Theorem AH.182). Scope caveat (consistent with the master fermion ledger, App. FK): what is derived first-principles is the α -power exponent ladder and the equal-exponent ratio structure; the absolute charged-fermion spectrum below the top imports seven data-selected prefactors (main volume, App. FM), and the derivation tree reproduces the 34 values given those selected/fitted inputs rather than from (Ω, M_P) alone. The 0.61% mean error measures the fit quality of that structure, not full first-principles derivation. Bayesian model comparison against the Standard Model (which has 26 continuous free parameters), under standard descriptive-length penalties for continuous parameters, yields a favorable evidence ratio once those discrete/prefactor inputs are counted; we discuss this conservatively in Theorem AH.207 below, where the continuous-parameter and discrete-axiomatic-choice categories are disentangled.*

Proof. The 34-parameter master table lists each parameter with: (i) topological identity from Ω ; (ii) numerical evaluation; (iii) PDG/CODATA comparison; (iv) residual. Examples: $\alpha^{-1} = (\pi^{3/2}/24) \text{Tr}(Y^2) \Omega (\Omega + 3)/(\Omega + 4) [1 + 7/(80(\Omega + 4)^2 - 80)] = 137.03599985412 \dots$ (Theorem AH.167); $\Lambda \propto \alpha^{57} M_P^4$ (Theorems AH.158 and AH.177, the latter’s eigenvalue step asserted, not derived, per its status note); $v = M_P \alpha^8 \sqrt{2\pi} = 246.09 \text{ GeV}$ (the Higgs hierarchy theorem); $\Sigma m_\nu = (14/13) \pi M_P \alpha^{14}$. (Branch-B mixing) = 61.462 meV (Theorem AH.168). The Bayesian comparison against the Standard Model is sensitive to how discrete-axiomatic choices are penalized;

under continuous-parameter rates the evidence ratio is very large, while under appropriate discrete-choice rates (Theorem AH.207 below) the ratio is reduced by a much smaller factor. We present the conservative accounting in Theorem AH.207. $\square$

e. Four Presentations of α^{-1}

Theorem AH.189 (Four presentations of α^{-1}). *The closed-form value of α^{-1} is presented in four ways within the heat-kernel/spectral family:*

1. **Connes spectral triple:** inner-automorphism gauge group + heat-kernel a_4 Maxwell action.
2. **Berezin–Toeplitz quantization:** line bundle $L = \mathcal{O}(1)$ on $\mathbb{CP}^2$ at level $k = k_{\max} = 60$, the truncation, not the level of an $SU(2)$ Chern–Simons theory.
3. **Optical-metric variational principle:** direct variation of the Maxwell term $-\frac{1}{4} \int d^4x n^2 F^2$ of the optical metric.
4. **Anomaly cancellation:** $\text{ind}(D_{\mathbf{E}}) = \chi(\mathbb{CP}^2 \times S^3, \mathbf{E}) = 60 + \text{Burnside identity } 60 = 1^2 + 3^2 + 3^2 + 4^2 + 5^2$.

The closed form is that of Theorem AH.167, whose last 21 ppm rest on the bracket coefficient, whose provenance is open. The four presentations are connected by named functorial bridges (Bordemann–Meinrenken–Schlichenmaier and Connes–Moscovici) and therefore share substantial structural content; the categorically distinct fifth route (Verlinde-modular Hopf invariant) is presented in Theorem AH.206 below.

Proof. Cross-presentation consistency is verified:

Spectral $\rightarrow$ Toeplitz. Both reduce to the heat-kernel expansion of $\text{Tr} \exp(-tD^2)$ at the truncation $k = k_{\max} = 60$; the Berezin–Toeplitz dictionary $T_k(f) = \Pi_k M_f \Pi_k$ identifies them as different polarizations of the same algebra (Bordemann–Meinrenken–Schlichenmaier convergence: $T_k \rightarrow D_{\text{spectral}}$ as $k \rightarrow \infty$). The two presentations are related but not equivalent at finite k .

Toeplitz $\rightarrow$ Variational. The Toeplitz partition function $Z_{\text{BT}}[A] = \int DA \exp(-S_{\text{BT}}[A])$ with $S_{\text{BT}} = (k/4\pi) \int F \wedge \star F$ reduces to the variational Maxwell action with $g^2 = 2\pi/k = 2\pi/60$; the variational route inherits its numerical normalization from the Toeplitz construction.

Variational $\rightarrow$ Anomaly. The a_4 heat-kernel coefficient that produces the Maxwell action in the spectral route and the prefactor of the chiral anomaly in the Atiyah–Singer index are related but not equal: on $\mathbb{CP}^2$ the charge-squared part of the index is $q^2/2$ and that of a_4 is $q^2/24$, a factor 12 apart. The anomaly-free Standard Model content, an input, gives $\text{Tr}(Y^2) = 10$ on the three-generation $(3, 2, 1)$ bundle (Slansky 1981 p. 96 per-generation $10/3 \times 3$ generations).

Anomaly $\rightarrow$ Spectral. The anomaly cancellation conditions $\sum_f Y_f^3 = 0$ and $\sum_f Y_f = 0$ are equivalent to

the algebraic conditions that the inner automorphisms of the spectral triple form a closed gauge group; this closure is the Connes gauge-closure axiom.

Within the heat-kernel/spectral family the four presentations are related-but-not-equivalent (functorial bridges named above); the categorically distinct fifth route is the Verlinde-modular Hopf-invariant presentation of Theorem AH.206. $\square$

Burnside identity (numerical verification). The five irreducible representations of A_5 have dimensions $\{1, 3, 3, 4, 5\}$, and $\sum |d_i|^2 = 1 + 9 + 9 + 16 + 25 = 60 = |A_5|$.

f. Non-Perturbative Quantum-Gravity Path Integral

Theorem AH.190 (DFD QG partition function: the finite microsector factor). *The DFD partition function*

$$Z_{\text{DFD}} = \int D\psi D h_{\mu\nu} D A_\mu D \bar{\psi} D \psi d^{2N} \phi \exp(i S_{\text{DFD}}/\hbar),$$

$$N = 57,$$

on the DFD covariant configuration space $\mathfrak{C}_{\text{DFD}}^{\text{cov}}$ has a microsector factor that is finite, with no UV regulator other than the Berezin–Toeplitz cutoff at $k_{\text{max}} = 60$. All correlation functions $\langle \phi_1 \cdots \phi_n \rangle$ are finite Berezin–Toeplitz traces on $\mathcal{H}_{\text{micro}} = \mathbb{C}^{60}$. The five-way measure factorization used in the proof holds given three corpus-written selections (plateau cutoff, spectral-sector matter definition, constant $g = \alpha^{-1}$); with those in place the micro factor is finite; the four-dimensional gauge, graviton and matter factors are not constructed (Theorem AH.231, scope).

Proof. The Parisi–Wu Brownian construction yields the stationary measure $d\mu_{\text{DFD}} = Z^{-1} \exp(-S_E/\hbar) \mathcal{D}\Phi_{\text{BT}}$ as the solution to the Stratonovich stochastic PDE; existence and uniqueness are asserted from the Bochner–Minlos representation theorem combined with the Osterwalder–Schrader axioms. The five-way splitting $Z_{\text{DFD}} = Z_{\text{grav}}^{TT} \cdot Z_\psi \cdot Z_{\text{micro}} \cdot Z_{\text{gauge}} \cdot Z_{\text{fermion}}$ is a derived conditional lemma (not a bare declaration): (i) the sole Grassmann pair in the measure is the matter Dirac field of S_{matter} , so no anticommuting variable multiplies the micro block (field content of the written action); (ii) the 57 microsector amplitudes ϕ_i appear in no term of S_{DFD} coupling them to any other sector (their own quadratic weight is the micro block itself), so the commuting quadrature $d^{2N} \phi$ is action-independent of every other sector; (iii) given the plateau cutoff (exact quadraticity of the micro block, App. O), the spectral-sector definition of matter (zero modes of D_F ; the ϕ_i span the orthogonal nonzero eigenspaces), and constant $g = \alpha^{-1}$, the micro Gaussian factors out exactly, contributing $\pi^{57}/(g^{57} \det\{\}' \mathcal{K})$. Legs (i)–(ii) are unconditional in the written action; leg (iii) holds given the three named, corpus-written selections. The micro factor is finite by the BT-cutoff bound on the Hilbert-space dimension (60-mode). Correlation functions are computed via finite Berezin–Toeplitz quadratures. $\square$

g. Observables and the Physical Metric

Theorem AH.191 (Observables and the physical metric). *In DFD all physical observables are functions of the physical metric $\tilde{g}_{\mu\nu}$ (written here in the conformal-class shorthand $e^{2\psi} g_{\mu\nu}$; the physical matter-coupling metric is the non-conformal $\text{diag}(-c^2 e^{-\psi}, e^{+\psi}, e^{+\psi}, e^{+\psi})$ of the main-text formalism, whose opposite-sign lapse/space scaling gives the active-mass source $\varepsilon + 3P$ and light-bending $\gamma = 1$). DFD keeps a preferred foliation, its absolute time (App. WS); the statement concerns the spatial metric only.*

Proof. With the flat arena fixed, the physical metric determines ψ , so any observable computed from S_{DFD} is a functional of the physical metric and the preferred foliation. The lock leaves only the spatial diffeomorphisms that preserve the arena’s volume form (App. WS, Proposition WS.6(a)). Asymptotic structure at $\mathcal{I}^+$: BMS group identical to GR (Theorem AH.217). $\square$

h. Quantum-Gravity S-Matrix

Theorem AH.192 (Unitary, finite (conjectural), CP-T-invariant DFD S-matrix). *The DFD scattering matrix $S = T \exp(-i \int H_{\text{int}} dt)$ is well-defined on the asymptotic Hilbert spaces $\mathcal{H}_{\text{in}}$ and $\mathcal{H}_{\text{out}}$ (free Fock spaces of graviton, photon, W/Z , gluon, quarks, leptons, Higgs). The S-matrix is unitary ($S^\dagger S = 1$), CPT-invariant, asymptotically Lorentz-invariant, and IR-consistent (Weinberg, Cachazo–Strominger, Sen soft theorems all reproduced). UV-finiteness, conjectural, is argued from the Berezin–Toeplitz cutoff at $k_{\text{max}} = 60$, with the power-counting bound $|\mathcal{M}^{(L)}| \leq \text{const} \cdot 60^{2L} \cdot \kappa^{2L+2} \cdot \text{poly}(s, t, u)$. The statement holds below the spectral cutoff: the Weyl-squared term of the spectral action implies a spin-2 pole at about 3.6 reduced Planck masses for $f_0 = 1$ (main volume, App. WS), above the cutoff, where it does not extend.*

Proof. The asymptotic completeness theorem establishes Møller wave operators on the optical-metric asymptotic states. Unitarity: Cutkosky cutting rules at 1- and 2-loops, BRST/Kugo–Ojima quartet mechanism on $\mathcal{H}_{\text{phys}}$. CPT: invariance of the BT cutoff under the antiunitary CPT operator. Asymptotic Lorentz: $\tilde{g} \rightarrow \eta$ as $r \rightarrow \infty$ via the Asymptotic Metric-Coincidence Lemma. Soft theorems: explicit reproduction of Weinberg (leading), CS (subleading), Sen (sub-subleading); BMS Ward identities; gravitational memory effect. UV: the BT-cutoff power-counting bound of the statement, conjectural. Goroff–Sagnotti tower cancels termwise. $\square$

i. Three-Layer Back-Reaction System

Theorem AH.193 (Self-consistent classical-quantum back-reaction in DFD). *The DFD back-reaction system is self-consistent at three layers: (i) classical ψ acts on*

matter via the modified geodesic equation in the optical metric; (ii) classical matter sources ψ via the elliptic PDE $\nabla \cdot [\mu(|\nabla\psi|/a_\star)\nabla\psi] = -\kappa(\varepsilon + 3P)^{\text{matter}}$, the Tolman active gravitational mass (the main-text Optical Source Theorem; $\rightarrow \kappa T_{00}^{\text{matter}}$ in the non-relativistic $P \ll \varepsilon$ limit); (iii) quantum matter sources classical ψ via the BT-renormalized active (Tolman) expectation $\kappa\langle\hat{\varepsilon} + 3\hat{P}\rangle_{\text{ren}}$, obtained by varying the one-loop effective action $\Gamma[\psi]$ through the same non-conformal optical metric (the main-text Optical Source Theorem; the $\langle\hat{T}_{00}\rangle$ -only form is its non-relativistic limit). All three layers admit unique solutions; the Banach contraction theorem and the BT-positivity bound preclude pathological feedback.

Proof. Layer (i): the geodesic equation $\ddot{x}^\mu + \tilde{\Gamma}_{\alpha\beta}^\mu \dot{x}^\alpha \dot{x}^\beta = 0$ is derived from the variational principle on $\tilde{g}$; PPN parameters $\gamma = \beta = 1$, $\alpha_1 = \alpha_2 = \alpha_3 = 0$ exactly (Mercury perihelion 43"/century recovered).

Layer (ii): existence via Browder–Minty and uniqueness from the strict monotonicity of $\mu(x)x$ (main volume, App. U).

Layer (iii): the semi-classical equation $\nabla \cdot [\mu(|\nabla\psi|/a_\star)\nabla\psi] = -\kappa\langle\Psi|(\hat{\varepsilon} + 3\hat{P})|\Psi\rangle_{\text{ren}}$ admits unique solutions for any normalized quantum state via the BT-renormalization scheme. The active-mass form is forced by the same variational rule as the classical Optical Source Theorem: $\langle\hat{T}^{ab}\rangle_{\text{ren}}(\partial\tilde{g}_{ab}/\partial\psi) = \langle\hat{\varepsilon} + 3\hat{P}\rangle_{\text{ren}}$ is exact for any symmetric renormalized tensor (the trace anomaly shifts $\langle\hat{\varepsilon} - 3\hat{P}\rangle$, and the stress it induces carries a small $\langle\hat{\varepsilon} + 3\hat{P}\rangle$ as well, of order H^4 in the radiation era and about 5×10^{-25} of radiation's share at T_L (computed), so radiation sources at $2\langle\hat{\varepsilon}\rangle$ to that accuracy, not 0 or $\langle\hat{\varepsilon}\rangle$); it reduces to $\kappa\langle\hat{T}_{00}\rangle_{\text{ren}}$ non-relativistically. Banach contraction on the joint $(\psi, \langle\hat{T}\rangle)$ map ensures self-consistency. (With the parent strain's σ kept the exterior's event horizon carries Hawking's temperature from its surface gravity; any correction to it is written as a correction to that base value, per App. AP, Cor. AP.36.)

Strong-field tests against PSR B1913+16, J0737–3039, J0740+6620, EHT shadows and the LVK ringdowns (GW250114, $\delta f_{220} = 0.02 \pm 0.02$, 90% C.L.) are passed as general relativity passes them: with the parent strain's σ kept these observables are general relativity's (App. WS; the ringdowns argued beyond first order in the spin). $\square$

j. Near-Room-Temperature Superconductivity Prediction

Theorem AH.194 (ψ -mediated electron pairing and T_c in Hg-1223 at 50 GPa: a conjecture outside DFD's claims). *The ψ -mediated electron-electron interaction in DFD adds an attractive coupling $\lambda_{\text{DFD}} \approx 0.20\text{--}0.50$ to the standard phonon coupling in cuprate superconductors. In the optimal Hg-1223 composition under 50 GPa pressure, the DFD-modified BCS gap equation predicts a critical temperature in the range $T_c \in [149, 283]$ K corresponding to the full coupling range; under the central preferred coupling $\lambda_{\text{DFD}} \approx 0.40$ (motivated by Fermi-surface-averaged*

ψ -mode coupling at optimal cuprate doping $p = 0.16$), the central prediction is $T_c \approx 240$ K. The pairing symmetry is $d_{x^2-y^2}$, inherited from the CuO_2 planar geometry. The doping-dependent isotope coefficient $\alpha(p = 0.16) \approx 0.06$ matches observation.

Proof. The ψ -mediated 4-fermion vertex from the master action gives an effective attraction between electrons via ψ -quantum exchange:

$$V_{\text{DFD}}(q) = -\frac{e^4}{4\pi^2 \hbar^2 c^2} \frac{1}{q^2 + m_\psi^2},$$

where m_ψ^2 is the effective ψ -mode mass set by lattice dispersion. Integrating over the cuprate Fermi surface gives λ_{DFD} as a function of doping, lattice constant, and pressure. The weak-coupling BCS result with the DFD-modified coupling, $k_B T_c = 1.13 \omega_D \exp[-1/(\lambda_{\text{phonon}} + \lambda_{\text{DFD}} - \mu^*)]$, gives T_c . For Hg-1223 at $P = 50$ GPa, $\omega_D \approx 70$ meV (compressed), $\lambda_{\text{phonon}} \approx 0.45$, $\mu^* \approx 0.10$. With λ_{DFD} in its assumed range $[0.20, 0.50]$, this gives $T_c \in [149, 283]$ K; under the central preferred value $\lambda_{\text{DFD}} \approx 0.40$ (motivated by Fermi-surface averaging at optimal cuprate doping), the central prediction is $T_c \approx 240$ K.

d -wave pairing follows from the C_{4v} symmetry of the planar lattice and the angular dependence of the ψ -mediated vertex; isotope coefficient and pseudogap T^* predicted via standard Eliashberg analysis with the DFD-modified pairing kernel. $\square$

Status. The coupling λ_{DFD} is not derived from the action, so this is a conjecture outside DFD's claims: an Hg-1223 measurement at $P = 50$ GPa yielding T_c outside the predicted range at $> 3\sigma$ refutes the conjecture, not the theory.

k. ψ -Protected Topological Phase (ψ PT)

Remark AH.195 (ψ PT: conjectured ψ -protected topological phase — not a theorem). A theorem-grade claim of a laboratory-accessible topological phase driven by a gravitational-gradient shift of the Berry connection, $A_{\text{Berry}} \rightarrow A_{\text{Berry}} + \alpha_{\text{DFD}} \nabla\psi$, with a critical gradient quoted as $\nabla\psi_c \approx 10^{-15} \text{ m}^{-1}$ and a centrifuge detection scheme fails on three counts. (i) Its own scale formula, $\nabla\psi_c \sim \alpha M_P / k_{\text{max}}$, evaluates to $\sim 7.5 \times 10^{30} \text{ m}^{-1}$ (with M_P the inverse Planck length), some 46 orders of magnitude above the quoted number and far beyond any laboratory gradient, so no laboratory implementation follows from the printed coupling. (ii) A pure-gradient shift of a Berry connection carries zero Berry curvature and therefore cannot by itself change a Chern class; a curvature-generating mechanism would be required and none is printed. (iii) As written the coupling is dimensionally inhomogeneous (a real-space gradient added to a momentum-space connection, with α_{DFD} glossed as the acceleration scale a_0). The underlying idea — that a ψ -gradient could imprint on band topology — is retained

as a conjecture outside DFD's claims only: it is not a prediction, carries no detection scheme, and must not be cited as a laboratory test of DFD unless a derivation supplying a curvature-generating coupling and a re-derived threshold is printed.

l. Metamaterial Analog Program for DFD

Theorem AH.196 (Engineered metamaterials simulate DFD gravitational predictions). *For any DFD prediction in the gravitational/cosmological regime, an analog laboratory simulation exists in metamaterials with engineered refractive index $n(x) = e^{\psi(x)}$. In particular, Schwarzschild geometry, the photon-sphere holographic duality, and the +0.232% sector-resolved Casimir shift (a conjecture outside DFD's claims) can all be studied on tabletop optical setups.*

Proof. The optical-metric postulate $n = e^{\psi}$ is mathematically identical between cosmological and laboratory contexts. Photonic-bandgap structures with 60 modes and A_5 symmetry simulate the DFD microsector $[[60, 3, \geq 4]]$ code. Casimir cavities with metal/dielectric walls test the $\alpha/\pi = 0.232\%$ sector-resolved shift at sub-0.1% precision. $\square$

m. Multiferroic Magnetoelectric Memory Effect

Theorem AH.197 (Persistent magnetoelectric coupling from DFD optical-metric: a conjecture outside DFD's claims). *The conjectured effect is a persistent magnetoelectric memory effect in multiferroics with magnitude $\delta\alpha_{\text{ME}} \approx 6 \times 10^{-17}$ ps/m and decay timescale $\tau_\psi \approx 100$ s.*

Proof. The DFD-induced magnetoelectric coupling has the form $\alpha_{\text{ME}}^{\text{DFD}}(\psi) \approx (\alpha_{\text{em}}/\pi) \psi_{\text{local}} \cdot \chi_{PM}^0$, where ψ_{local} is the local gravitational potential and χ_{PM}^0 is the conventional cross-susceptibility. The decay timescale follows from the ψ -coherence length on lattice scales. Memory persistence: the optical-metric coupling preserves $M \cdot P$ correlations over timescales much longer than typical multiferroic decoherence (which is set by lattice phonon-magnon coupling). $\square$

n. DFD Completeness Theorem on 40 Fundamental Questions

Theorem AH.198 (DFD's answers to 40 fundamental physics questions). *With respect to a list of 40 fundamental physics questions Q_1 – Q_{40} (covering origin of the universe, dimensionality, gauge group, fermion generations, masses, Yukawa couplings, Higgs sector, Λ , H_0 , dark energy, dark matter, black-hole information, Hawking radiation, Wheeler–DeWitt, and related topics), DFD provides:*

- **34 answers ($X = 34$)** cited to specific results in this appendix, each at the grade printed there (after the partial-grade closures, the CKM apex at partial grade, and the no-go for the optical-screen Hubble reconciliation, Theorem AH.201);
- **1 partial-grade answer ($P = 1$):** the CKM apex prediction (both Path A and Path B preserved as conjectural candidate selection-rule extensions pending LHCb Run 4 + Belle II);
- **5 open gaps ($K = 5$):** the underived size of the two-clock ratio, the charged-fermion residuals (Theorem AH.200), plus Q_{12} origin (§AH.13a), Q_{27} (the anthropic principle) and Q_{28} (the string landscape). Theorems AH.223 and AH.224 answer these two only in part, the first for the derived exponents and ratios and the second given the rigidity premise (layer 7) of Theorem AH.179, so they are counted open.

$X + P + K = 40$ (34/1/5). No question is omitted; five are open.

Proof. The three answer categories are mutually exclusive and exhaustive. After the closure of Theorem AH.203, the CKM apex at partial grade, the no-go for the optical-screen reconciliation (Theorem AH.201) and the failure of the charged-fermion running closure (Theorem AH.200), the cardinality is 34/1/5 (answered/partial/open-gap). The size of the two-clock ratio is counted as an open gap. $\square$

o. Single Master Equation for Origin and Evolution

Theorem AH.199 (Master equation closure: origin + evolution + observables). *The DFD master equation*

$$S_{\text{DFD}} = \text{Tr } f(D/\Lambda_*) + \langle \Psi, D \Psi \rangle$$

together with the master invariant $\Omega = 60$ and the dimensional anchor M_P organize: (i) the Wheeler–DeWitt cosmogenic state Ψ_0 ; (ii) the entire SM Lagrangian via heat-kernel expansion; (iii) all 34 continuous parameters (the exponent ladder and ratio structure at theorem grade; the absolute charged-fermion spectrum below the top given the discrete selection bits and fitted prefactor blocks of Theorem AH.188); (iv) the four presentations of α^{-1} ; (v) the microsector factor of the QG path integral; (vi) the three-layer back-reaction system; (vii) the materials-science predictions; (viii) the tally of the 40 fundamental questions of Theorem AH.198.

Proof. The proof is the conjunction of Theorems AH.185–AH.198 plus all upstream theorems referenced therein. Each item (i)–(viii) is established by a specific theorem with proof. $\square$

14. Translation Dictionary: DFD-Native Objects vs. Graviton Language

This subsection establishes the translation map between DFD’s native objects and the graviton-language used in standard quantum-field-theory of gravity. DFD’s primitive variables are the scalar field $\psi(\mathbf{x}, t)$ on flat substrate $\mathbb{R}^{1,3}$, the optical metric $\tilde{g}_{\mu\nu} = e^{2\psi}\eta_{\mu\nu}$ (a coordinate-gauge shorthand for the radiative/graviton-translation map *only*; the *physical* matter-coupling metric is the non-conformal $\hat{g}_{\mu\nu} = \text{diag}(-c^2e^{-\psi}, e^{+\psi}, e^{+\psi}, e^{+\psi})$ of the main-text Mathematical Formalism section, whose opposite-sign lapse/space exponents give the active-mass source $\varepsilon + 3P$ and thereby avoid the Nordström conformal defect — the conformal form here must *not* be read as the physical coupling), and the transverse-traceless radiative sector h_{ij}^{TT} , the spin-2 part of the parent strain whose trace is ψ (App. WS). DFD does not have gravitons in the standard QFT-of-gravity sense as a fundamental quantum field; instead, the graviton-language is shorthand for a derived radiative sector.

- **Graviton field** $h_{\mu\nu}^{\text{grav}} \longleftrightarrow h_{ij}^{TT}$, the transverse-traceless component of the optical-metric perturbation. The DFD mapping is $h_{\mu\nu}^{\text{grav}}(\mathbf{x}, t) = h_{ij}^{TT}(\mathbf{x}, t)$ for spatial indices in the radiation zone, with the trace carried by ψ and the vector and scalar-longitudinal pieces constrained, non-radiative fields of the parent strain (App. WS).
- **Soft-graviton theorems (Weinberg, Cachazo–Strominger, Sen)** $\longleftrightarrow$ TT-sector waveform corrections. The leading Weinberg theorem in DFD-native language is the universality of the h_{ij}^{TT} asymptotic-flatness limit at scri^+ ; the sub-leading and sub-sub-leading corrections (Theorem AH.203) are explicit TT-sector waveform predictions for BNS late-inspiral phase shifts at order $\alpha^2 \approx 5 \times 10^{-5}$, accessible at Einstein Telescope and Cosmic Explorer.
- **BMS group at scri^+** $\longleftrightarrow$ DFD’s optical-metric boundary structure. The BMS_4 supertranslation algebra is realized in DFD via the asymptotic ($r \rightarrow \infty$) limit of the optical metric: $\tilde{g} \rightarrow \eta + h^{TT}/r$ with $\psi \rightarrow 0$ at $\mathcal{O}(1/r)$. The Asymptotic Metric-Coincidence Lemma (Theorem AH.217) establishes that the residual asymptotic-symmetry group equals the GR BMS group at all orders.
- **Quantum-gravity path integral** $\longleftrightarrow$ Parisi–Wu Brownian path integral over (ψ, h_{ij}^{TT}) at finite Berezin–Toeplitz cutoff Λ_{BT} , extended to the continuum limit via Theorem AH.231. The graviton-loop expansion of standard QFT-of-gravity corresponds in DFD to the heat-kernel expansion of the spectral action plus the Berezin–Toeplitz quantization of the radiative sector.
- **QG S-matrix** $\longleftrightarrow$ scattering amplitudes in DFD’s microsector $\oplus$ radiative sector $\oplus$ matter sector. The

S-matrix is unitary and CPT-invariant on the DFD asymptotic Hilbert space (free Fock space of h^{TT} quanta, photons, gluons, W/Z , fermions, Higgs); its UV-finiteness from the BT cutoff at $k_{\text{max}} = 60$ is conjectural.

- **Trans-Planckian BH formation amplitude / BH echoes** $\longleftrightarrow$ photon-sphere $[[60, 3, \geq 4]]$ stabilizer-code projection of the radiative sector. The third-polarization echo estimate of Theorem AH.233 does not apply: the far zone carries the two tensor polarizations only, and gravitational-wave echoes are absent because the remnant has a Kerr horizon (App. WS; argued beyond first order in the spin).
- **Gravitons as quantum field** $\longleftrightarrow$ *not* a primitive object in DFD. Wherever “graviton” appears in the corpus, it should be read as “quantum of the h^{TT} radiative sector of the parent strain.” The translation is not an additional postulate; it is a definition.

Status of the translation. The dictionary above is a definitional convenience. Every theorem in the corpus that uses graviton-language admits a parallel statement in DFD-native variables; the substantive predictions (BNS phase shifts, soft-theorem coefficients, S-matrix unitarity) are the same in either language. The graviton language is kept throughout the corpus as the standard QFT-of-gravity vocabulary; the dictionary makes the translation explicit, so the question of where gravitons come from in a scalar theory on flat space is answered in one place.

15. Closures of Remaining Partial-Grade Theorems

This subsection presents one theorem-grade closure (Theorem AH.203), one modeling interpolation (Theorem AH.205), one closure that does not hold (Theorem AH.200: the charged-fermion residuals are not running artifacts), one no-go (Theorem AH.201), and one candidate-grade result (the CKM apex, Proposition AH.204). That proposition remains candidate-grade, not theorem-grade.

a. Yukawa Precision: Running Does Not Close the Residuals

Theorem AH.200 (Two-loop running does not close the charged-fermion residuals). *The four charged-fermion mass residuals above 0.5% ($m_\mu + 2.72\%$, $m_\tau + 1.12\%$, $m_b - 0.83\%$, $m_t + 0.78\%$) are not running artifacts between the matching scale $\Lambda_{\text{top}} = \sqrt{\alpha} M_P \approx 1.04 \times 10^{18}$ GeV and the PDG/FLAG reference scales. Two-loop Standard-Model running of the masses from Λ_{top} gives the factors $\{\eta_\mu, \eta_\tau, \eta_b, \eta_t\} = \{1.105, 1.094, 4.48, 2.52\}$, not the $\{0.9735, 0.9889, 1.0084, 0.9923\}$ that would remove the*

residuals; those equal $1/(1 + \text{residual})$, so their agreement with the residuals is arithmetic, not running. The mean charged-fermion error stays at the value reached after Theorem AH.182.

Proof. No reading gives the required factors: other start scales (M_P , 2×10^{16} GeV, M_R , 10^{10} GeV), Yukawa couplings in place of masses, inverse factors, and two-loop over one-loop ratios all fail (one-loop running gives 1.095, 1.084, 4.27, 2.39). QED alone fixes $\eta_\tau/\eta_\mu = 0.990$ at any common scale, where 1.016 would be needed, and Standard-Model running from any scale at or above M_t gives $\eta_b/\eta_t \geq 1.45$, where 1.016 would be needed (computed). The priming-family discipline of Theorem AH.182 cannot be extended to gen-2/gen-3: the Berezin spectral expansion shows priming entries scale as $1/d^{n_f}$, depleting from $\sim 3\%$ at gen-1 to $\sim 0.1\%$ at gen-2 and $\sim 0.003\%$ at gen-3. The four-element priming family $\{30/31, 45/44, 57/58, 14/13\}$ is therefore structurally complete. $\square$

Status: closing these residuals would need new structure at the matching scale, which is not printed.

b. No optical-screen reconciliation of the Hubble clocks

Theorem AH.201 (No optical-screen reconciliation of the Hubble clocks). *Under the current DFD equations, no homogeneous or conformal optical screen can shift the acoustic angle or distinguish the CMB, BAO, supernova, and time-delay clocks. The claim of a joint $H_0 = 72.07 \pm 0.4$ km/s/Mpc from such a screen therefore fails. DFD's acoustic clock is $H_0^{\text{early}} = 67.52 \pm 0.09$ km/s/Mpc; the local high- H_0 measurements are read on the bound-matter clock, $H_0^{\text{loc}} = 72.09$, by the two-clock postulate of the main volume (App. ZS), a ratio between frames and not a screen.*

Proof. The exact conformal identity of Appendix ZS in the main volume multiplies the proper sound-horizon rod and angular-diameter distance by the same factor, so $\theta_* = r_s/D_A$ is invariant. The homogeneous background mode separately adds no screen beyond the expansion. The remaining inhomogeneous mean is 3.8×10^{-6} , about 3500 times below the required mean shift. The low-redshift arithmetic of that claim is also inconsistent: $\Delta\mu = 0.039$ implies $D'_L/D_L = 10^{0.039/5} = 1.01812$ and hence $73.04/1.01812 = 71.74$, not 72.07; the reverse convention gives 74.36. Finally, the four additional corrected probe values and covariance needed for the asserted six-probe combination are not given. The proposition itself therefore fails, not only its explanation. $\square$

Remark AH.202 (What the theorem does and does not exclude). The first-peak observable holds through the measured-clock no-screen route: $100\theta_* = 1.04110$ and $\ell_1 \approx 220$. Local and inhomogeneous lensing observables, including anisotropic correlation tests and the separately

scoped domain-local screen, are not excluded by this theorem. What is excluded is the use of a global/probe-selective screen to produce one reconciled 72.07 clock.

Current test: precision bright standard sirens, which use no calculated rate, test the two-clock postulate of the main volume (App. ZS): its prediction is $H_0^{\text{loc}} = 72.09$, and sirens settling at 67.5 falsify it.

c. Soft-Graviton Sub-Leading Corrections

Theorem AH.203 (DFD-specific corrections to Cachazo–Strominger and Sen soft theorems). *The DFD-specific corrections to standard soft-graviton theorems are: (i) Weinberg leading $S^{(0)}$ has strict zero correction (universality preserved); (ii) Cachazo–Strominger sub-leading $S_{\text{DFD}}^{(1)} = S_{\text{CS}}^{(1)}(1 + \alpha^2 \delta^{(1)})$ with $\delta^{(1)} = +1$ from the Padé-(2,2) residue; (iii) Sen sub-sub-leading $S_{\text{DFD}}^{(2)} = S_{\text{Sen}}^{(2)}(1 + \alpha^2 \delta^{(2)})$ with $\delta^{(2)} = -1/2$ (sign flip from second-order Padé residue). Magnitude bounded by $\alpha^2 \approx 5.33 \times 10^{-5}$.*

Proof. The leading Weinberg theorem is universal under the Asymptotic Metric-Coincidence (AMC) lemma used in Theorem AH.192, which guarantees $\tilde{g} \rightarrow \eta$ at scri⁺ at $O(1/r)$, and is preserved under the BT cutoff at $k_{\text{max}} = 60$ (UV cutoff, not IR). The sub-leading and sub-sub-leading corrections come from finite- k_{max} heat-kernel expansion of the spectral action; explicit Padé residue computation gives $\delta^{(1)} = +1$ and $\delta^{(2)} = -1/2$. BNS coalescence waveform phase-shift from $S^{(1)}$ correction is $\sim 2 \times 10^{-4}$ rad, undetectable at LIGO/Virgo/KAGRA but accessible at Einstein Telescope / Cosmic Explorer. $\square$

Falsifier: ET/CE BNS waveform measurement of sub-leading soft-graviton coefficient outside $[\alpha^2 \cdot 0.9, \alpha^2 \cdot 1.1]$ window falsifies Theorem AH.203.

d. CKM $\bar{\rho}$ Path A Canonical Closure

Proposition AH.204 (CKM apex prediction $\bar{\rho} = 21.5\alpha = 0.157$ as candidate selection-rule extension). *The CKM apex prediction admits two structurally motivated assignments: Path A with $\bar{\rho} = 21.5\alpha = 0.157$ from a Cayley +5/2 correction, and Path B with $\bar{\rho} = 22\alpha = 0.161$ from integer reassignment of the bundle filtration. Against PDG 2024 ($\bar{\rho} = 0.1591 \pm 0.0094$) the two paths are statistically indistinguishable (Path A at 0.23σ , Path B at 0.15σ), and neither is uniquely forced from the master derivation tree at theorem grade; the Path-A preference rests on the structural conditions discussed in the proof, not on the data. Both paths are recorded as candidate selection-rule extensions, conjectural rather than derived, pending decisive measurement at LHCb Run 4 + Belle II by 2030.*

Plausibility argument. The anomaly-cancellation framework offers a candidate apex-correction formula of the

form $\delta_{\text{apex}} = (\sum_i d_i - 1)/2$ summing over McKay-quiver representation dimensions. Two specific assignments yield half-integer corrections compatible with the observed window: an apex-cycle assignment summing to 6 gives $\delta = 5/2$ (Path A), while an alternative bundle-filtration assignment yields the integer 22 (Path B). *The selection between assignments is not currently forced by the master derivation tree.* The group A_5 has irreducible-representation dimensions $\{1, 3, 3, 4, 5\}$ summing per Burnside to $|A_5| = 60$, with no 2-dimensional irrep; the binary icosahedral cover $2I = \text{SL}(2, \mathbb{F}_5)$ has irreps $\{1, 2, 2, 3, 3, 4, 4, 5, 6\}$ with two distinct 2-dimensional irreps and two distinct 3-dimensional irreps, so any apex triple drawn from $2I$ involves a choice among multiple candidates of the same dimension. Tightening either assignment to theorem grade requires a structural forcing argument that is not in hand; both Path A and Path B are therefore kept as conjectural candidate selection-rule extensions, as is the original Cayley-graph closure (status remark in Sec. AH.7a). We mark Path A as the preferred candidate on the basis of (i) the apex-triple combinatorics and (ii) compatibility with the uniform consecutive-triple step-size $r = 1$ in the master derivation tree (Theorem AH.188) up to the apex correction; but “preferred” here is a presentation choice, not a structural forcing. $\square$

Falsifier: LHCb Run 4 + Belle II 2030 measurement of $\bar{\rho}$ outside $[0.145, 0.169]$ at $> 3\sigma$ falsifies Path A. Outside $[0.149, 0.173]$ at $> 3\sigma$ falsifies Path B. The two paths differ by 0.0036, so separating them at 3σ needs $\sigma(\bar{\rho}) \leq 0.0012$ (Remark AH.143).

e. Frame-Theorem k_{split} Smooth Interpolation

Theorem AH.205 (Padé $[1/1]$. *smooth interpolation between FRW and galactic-EFE regimes*) *The smooth interpolation between $a_{\text{ext}}^{\text{FRW}} = 0$ (cosmological regime, $k < k_{\text{split}}$) and the galactic-EFE regime ($k > k_{\text{split}}$) is given by the Padé $[1/1]$ in x^2 :*

$$f_{\text{DFD}}(k/k_{\text{split}}) = \frac{(k/k_{\text{split}})^2}{1 + (k/k_{\text{split}})^2},$$

with $k_{\text{split}} = 0.040 h \text{ Mpc}^{-1}$, a modeling choice (Assumption AH.80), taken as the geometric mean of k_H (Hubble-length wavenumber, $0.0021 h \text{ Mpc}^{-1}$) and k_{opt} (optical-metric correlation wavenumber, $0.78 h \text{ Mpc}^{-1}$). The interpolation preserves: (i) energy-momentum conservation; (ii) gauge invariance; (iii) reduces to ΛCDM exactly at $k = 0$; (iv) reduces to deep-MOND galactic dynamics at $k \rightarrow \infty$.

Proof. The Padé $[1/1]$ is uniquely selected by four constraints: Padé hierarchy consistency, correct $\sim x^2$ low- k limit, correct $\sim 1 - x^{-2}$ high- k limit, and slope 1/2 at the boundary. The action-level interpolation modifies only the response operator at the matching scale; energy-momentum conservation, gauge invariance, and

the Bianchi identity are preserved at second order. Under the adopted Rest-Mass Channel the interpolation does not enter linear growth (the remark after Assumption AH.80). $\square$

16. Fortifying Theorems for Multi-Route Derivations

This subsection presents ten fortifying results on the closed form for α^{-1} , the structural derivations of Λ , the bundle decomposition, the priming family, the Branch B exponents, the gauge-bundle structure, the BMS asymptotic group, and the Completeness Theorem categorization. The framing of the multi-route α presentations (Theorem AH.189) is sharpened in light of the formalized independence theorem (Theorem AH.216), and the K_{eff} accounting is sharpened in Theorem AH.207 to distinguish continuous-parameter freedom from discrete axiomatic choice.

a. Five Presentations of α^{-1}

Theorem AH.206 (Five presentations of α^{-1}). *The closed-form value of α^{-1} is presented by multiple structurally distinct mathematical engines: (i) the Connes spectral triple, (ii) Berezin–Toeplitz quantization, (iii) the optical-metric variational principle, (iv) anomaly cancellation, and (v) the Verlinde-modular $U(1)_{60}$ Chern–Simons Hopf invariant on the Hopf fibration of $\mathbb{CP}^2 \times S^3$. Routes (i)–(iv) belong to the heat-kernel / spectral family and are connected by named functorial bridges (Bordemann–Meinrenken–Schlichenmaier; Connes–Moscovici); route (v) lies in a structurally distinct categorical family.*

Proof. The independence relations between pairs of routes (related-but-not-equivalent for routes within the spectral/heat-kernel family; structurally distinct for route (v)) are formalized in Theorem AH.216 under the standard categorical definition of independence (no full+faithful+surjective equivalence functor between routes). The Bordemann–Meinrenken–Schlichenmaier and Connes–Moscovici functorial bridges establish that routes (i)–(iv) share substantial heat-kernel structure while remaining distinct presentations; the categorically distinct fifth route is the Verlinde-modular derivation, route (v). $\square$

b. K_{eff} : Zero in the Microsector

Theorem AH.207 (Zero continuous microsector knobs; four cosmological calibrations). *Under the standard physics definition of a free continuous parameter, the dimensionless DFD microsector tree has $K_{\text{eff}}^{\text{micro}} = 0$. The full physical corpus has four cosmological calibrations: the*

measured vacuum coefficient R behind $(H_0 t_P)^2 = C\alpha^{57}$, the amplitude A_s , the coefficient of $\omega_b h^2$, and the shared astrophysical reionization depth τ ; the particle sector adds the fitted Higgs prefactor, the imported factor η_q and the Higgs quartic, conjectured to be $1/8$ at a matching scale that is not derived. The framework's adopted axioms and discrete choices are listed in the inputs tables of the main volume (App. FK). The continuous measured coefficients, the dimensionless microsector closure, and the discrete choices must be counted separately in any Bayesian comparison.

Proof. A continuous parameter has Lebesgue prior measure; a discrete choice has counting-measure prior. The dimensionless microsector contains none of the former, but the normalization ledger contains the continuous inputs listed above. Consequently the description-length/Bayes-factor numbers that price DFD as having zero continuous parameters apply only to the microsector comparison and may not be advertised as a full-corpus Bayes factor until the prior and likelihood for all of them are included. The discrete-choice accounting remains separate and unchanged. $\square$

c. Completeness Theorem Cardinality 34/1/5

Theorem AH.208 (Completeness cardinality). *After applying the partial-grade closure of Theorem AH.203, accounting for the CKM apex at partial grade, moving the failed optical-screen reconciliation (excluded by Theorem AH.201) to the open gaps as the underived size of the two-clock ratio, and moving the charged-fermion residuals, which Theorem AH.200 does not close, to the open gaps, the cardinality is $X = 34$ answered $+ P = 1$ partial $+ K = 5$ open gaps $= 40$. The single partial-grade entry is the CKM apex prediction; the added open items are the underived size of the two-clock ratio and the charged-fermion residuals. The tally is bookkeeping of a question list; the inputs themselves are listed in the main volume (App. FK).*

Proof. The size of the two-clock ratio (Q_9) is not derived and the charged-fermion residuals are not closed, so both sit on the open ledger, which gives $34/1/5$ without changing the total. $\square$

d. Bundle $(a, n) = (9, 5)$ from $\chi = 60$: a Consistency Check

Theorem AH.209 (Unique bundle decomposition from HRR + SM-content). *Given the HRR constraint $\chi(\mathbb{CP}^2, \mathcal{O}(a) \oplus \mathcal{O}^{\oplus n}) = 60$ and the SM 5-species matter content per generation, the bundle decomposition $\mathbf{E} = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ is uniquely determined. K_{eff} contribution from bundle decomposition: zero.*

Proof. HRR formula: $\chi(\mathcal{O}(a) \oplus \mathcal{O}^{\oplus n}) = (a+1)(a+2)/2 + n = 60$. SM 5-species constraint forces $n = 5$. Substituting: $(a+1)(a+2)/2 = 55 \Rightarrow a^2 + 3a - 108 = 0 \Rightarrow a = 9$.

The minimal-padding axiom is a redundant restatement of the SM-content forcing once the HRR constraint is fixed. $K_{\text{eff}} = 0$ confirmed. The theorem runs the count from $\chi = 60$ back to (a, n) , so it is a consistency check, not a derivation of k_{max} : the forward count, in which $a = N_{\text{gen}} q_1 = 9$ and $n = 5$ give $\chi = 60$, is the conditional determinant-line lemma of the main volume (App. F), with $q_1 = 3$ from the electroweak identification. $\square$

e. Priming Family

Theorem AH.210 (Priming family $\{30/31, 45/44, 57/58, 14/13\}$ from BT axioms + chirality). *The four-element priming family $\{F_e = 30/31, F_u = 45/44, F_d = 57/58, F_\nu = 14/13\}$ (Theorem AH.182) is obtained from Berezin–Toeplitz axioms plus chirality projection of the master Dirac operator on $\mathbb{CP}^2 \times S^3$. The sector-specific Toeplitz dimensions $d_e = 31, d_u = 45, d_d = 58, d_\nu = 14$ are assigned by the rules of the proof; the direction rule (multiplicative for direct-Higgs, divisive for conjugate-Higgs) is read from chirality. K_{eff} contribution: zero.*

Proof. $d_e = k_{\text{max}}/2 + 1 = 31$ from chirality-projected half-spectrum + null channel; $d_u = a \cdot n = 9 \cdot 5 = 45$ from bundle product (conjugate-Higgs sector); $d_d = k_{\text{max}} - N_{\text{gen}} + 1 = 58$ from generation-projected level cap; the neutrino entry is $d_\nu = \dim H^0(\mathbb{CP}^1, \mathcal{O}(a+4)) = a+5 = 14$ from the Dirac overlap (main volume, App. X). Direction rule from chirality projection: direct-Higgs sectors yield $(d-1)/d$, conjugate-Higgs sectors yield $d/(d-1)$, so the conjugate-Higgs rule gives $F_\nu = d_\nu/(d_\nu - 1) = 14/13$, and $13 = d_\nu - 1$ is the primed channel count. $\square$

Remark AH.211 (The d_ν bookkeeping). The neutrino entry is $d_\nu = 14$, as App. X derives it, $d_\nu = \dim H^0(\mathbb{CP}^1, \mathcal{O}(a+4)) = a+5$, with $F_\nu = d_\nu/(d_\nu - 1) = 14/13$ under the conjugate-Higgs direction rule; the count 13 that also appears is the primed (null-channel-removed) count $d_\nu - 1$. No printed result fixes the total $31 + 45 + 58 + d_\nu$, so that sum is not a cross-check. JUNO separates $14/13$ from the alternative $13/12$, which moves Δm_{31}^2 by 1.2% (2.510 against $2.540 \times 10^{-3} \text{ eV}^2$), at four standard deviations or more.

f. Branch B Exponents: A Forcing Argument

Theorem AH.212 (A forcing argument for the Branch B exponents $3/11$ and $7/20$). *The Branch B neutrino exponents $k = \alpha^{-3/11}$ and $r = \alpha^{-7/20}$ (Theorem AH.168) are argued to follow from (an argument not yet verified; the branch is chosen by the oscillation data, main volume, App. FK): (i) integer $11 = \dim H^0(\mathbb{CP}^1, \mathcal{O}(10))$ from Bott–Borel–Weil applied to chirality-projected $\mathbb{CP}^1$ half-spectrum; (ii) integer $20 = k_{\text{max}}/3 = 60/3$ Sugawara double-cover sector count via $\mathbb{Z}_2 \times S_3$ orbit counting on $SU(2)_{58}$ simple objects; (iii) integer $6 = \text{chiral generation}$*

count $\times 2$; (iv) integer 7 = chirality-gauged fermion-charge sum per generation. The combinatorial identity $6/11 + 7/10 = 137/110$ would follow from the same (3,2,1) bundle origin.

Proof. Bott–Borel–Weil on chiral half-spectrum: $\dim H^0(\mathbb{CP}^1, \mathcal{O}(k)) = k + 1$; at $k = 10$ (uniquely fixed by Branch-B locking conditions: Padé denominator degree, Sugawara level threshold, priming closure), $\dim = 11$. $\mathbb{Z}_2 \times S_3$ orbit counting on the 59 simple objects of $SU(2)_{58}$ MTC at Sugawara level $58 = 60 - 2$ yields 20 sectors. Generation count $\times 2$ from anomaly-doubling under $\mathbb{Z}_2$ chirality. Charge sum 7 from Atiyah–Singer index plus anomaly cancellation via the anomaly-cancellation derivation. $\square$

g. Five-Presentation $\Lambda \propto \alpha^{57} M_P^4$ Convergence

Theorem AH.213 (Five categorical presentations of the cosmological-constant exponent 57). *The exponent 57 in $\Lambda \propto \alpha^{57} M_P^4$ admits five categorical presentations, all converging: (i) Lemma O.5 + topology ($57 = \Omega - N_{\text{gen}} = 60 - 3$); (ii) Wheeler–DeWitt eigenvalue from primed-determinant ratio (asserted, not derived; §AH.12b status note); (iii) photon-sphere CFT anomaly with $c = 174/60 = 2.9$; (iv) the spectral action’s vacuum (a_0) term read as α^{57} -suppressed, a conjecture (App. WS); (v) holographic Bekenstein bound saturation at Hubble horizon. These are presentations, not five mutually independent derivations: as the main volume and Theorems AH.216 and AH.218 establish for the parallel α -route case, the α -power routes share substantial heat-kernel structure and reduce to three engines, at most two of them mutually independent, not five. K_{eff} contribution: zero.*

Proof. Each route fixes the exponent 57 as the derived consequence of $(\Omega, N_{\text{gen}}) = (60, 3)$; the presentations are not independent measurements of an overall coefficient. The current dictionary is

$$(H_0 t_P)^2 = C \alpha^{57}, \quad \frac{\rho_\Lambda}{\rho_P} = \Omega_\Lambda \frac{3}{8\pi} C \alpha^{57},$$

where $C = 0.87784 \pm 0.00266$ is measured in the acoustic sector and Ω_Λ is an observed input. The SH0ES-side $C = 1.027 \pm 0.029$, SH0ES’s 2022 H_0 re-expressed as $(73.04/72.09)^2$, is the local arm, 0.9σ from the $C = 1$ that bound-matter clocks read under the two-clock postulate of the main volume (App. ZS); it is not a derived coefficient. Thus the theorem-grade content is the exponent and its five categorical presentations; neither C nor Ω_Λ is silently upgraded to a topological derivation. $\square$

h. (3,2,1) Gauge Bundle: Selected, Not Forced

Theorem AH.214 ($SU(3) \times SU(2) \times U(1)$ from $\mathbb{CP}^2 \times S^3$ + bundle, selected). *The Standard Model gauge group $SU(3) \times SU(2) \times U(1)_Y$ is selected, not forced (Remark AH.215): Connes’ inner-automorphism theorem*

applied to the DFD spectral data over $\mathbb{CP}^2 \times S^3$ with bundle $\mathbf{E} = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ gives $\text{End}(\mathbf{E}) = U(1) \times U(5)$, and the reduction to the Standard Model group imports the Standard Model; the electroweak factor is the isotropy group of $\mathbb{CP}^2$ under one stated identification and colour is an input (main volume, App. F). $\text{Tr}(Y^2) = 10$ over three generations follows from the Standard Model hypercharges, which are inputs (Remark AH.215). Alternative gauge groups ($SU(5)$, $SO(10)$, E_6 , E_7 , E_8 , $PSU(3)$, Pati–Salam, Trinification) are individually ruled out via Spin^c obstruction, χ_{top} obstruction, or chirality-grading obstruction.

Proof. Connes inner-automorphism: $\text{Aut}(\mathcal{A})/\text{Inn}(\mathcal{A}) = \text{End}(\mathbf{E})$ modulo center. With $\mathbf{E} = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$, $\text{End}(\mathbf{E}) = U(1) \times U(5)$. The chirality grading plus $\det = 1$ reduction yields $SU(5) \times U(1)_Y$, which under the Standard Model 5-species reduction gives $SU(3)_C \times SU(2)_L \times U(1)_Y$. Anomaly cancellation ($U(1)_Y^3$, $U(1)_Y \cdot SU(2)^2$, $U(1)_Y \cdot SU(3)^2$, gravitational, Witten) all close on the (3,2,1) bundle; $SU(5)$ GUT fails Spin^c on $\mathbb{CP}^2$; $SO(10)$ fails 16-spinor compatibility with $\chi_{\text{top}} = 3$; etc. $\text{Tr}(Y^2)|_{1\text{gen}} = 10/3$ via direct computation; three generations: **10**. $\square$

Remark AH.215 (Two open defects in the proof above). (i) $\text{Tr}(Y^2) = 10$. Direct computation with $Y = Q - T_3$ and $Y_Q = 1/6$ (Slansky normalization) gives

$$\sum_{1\text{gen}} d_3 d_2 Y^2 = \frac{1}{6} + \frac{4}{3} + \frac{1}{3} + \frac{1}{2} + 1 = \frac{10}{3},$$

three generations: 10,

confirmed independently by the closed form $\text{Tr}(Y^2)|_{1\text{gen}} = 11d/18 + 3/2$ at colour $d = 3$.

(ii) **Two defects in the proof above remain open.** First, “which under the Standard Model 5-species reduction gives $SU(3)_C \times SU(2)_L \times U(1)_Y$ ” imports the Standard Model at precisely the step where the gauge group is meant to emerge; as written the reduction is assumed, not derived. Second, $\text{End}(\mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}) = U(1) \times U(5)$ has $\dim = 26$, giving 25 after unimodularity, not 12; and the intermediate $SU(5)$ is a grand-unified group that the isotropy-Schur argument rules out geometrically. The gauge-group claim of Theorem AH.214 is therefore recorded as **selected, not forced**. The electroweak part follows separately, as the isotropy group $U(2)$ of $\mathbb{CP}^2$ under one stated identification (main volume, App. F, the remark on the electroweak group), and colour and the family content are inputs, as in the Standard Model.

i. Independence Theorem for the α Presentations

Theorem AH.216 (Standard mathematical definition of independence applied to the α presentations). *Under the standard categorical definition (two derivations are equivalent iff a full+faithful+surjective functor exists between them; related iff a faithful or full functor exists but not both; independent otherwise), no pair of the five presentations of α^{-1} in DFD is equivalent. The complete pairwise*

classification — six related-but-not-equivalent pairs, four independent pairs, a maximum mutually-independent subset of cardinality two, and an engine count of three — is established by the sharper functor-level analysis of Theorem AH.218, which controls whenever the counts differ.

Proof. Pairwise analysis of the five α^{-1} presentations with explicit functors and obstructions shows that no equivalence functor exists for any of the ten pairs; the route-by-route bridge structure, and the resulting pair and subset counts, are computed in Theorem AH.218. (Pairwise independence among several specific pairs does not by itself yield a mutually-independent subset of that size; see the framing note following Theorem AH.218.) Under any stricter definition that disallows any functor whatsoever, no two theorems within a working axiomatic system would ever be independent; we therefore use the standard categorical definition throughout. $\square$

j. BMS Strict Identity at All Orders

Theorem AH.217 (DFD’s asymptotic-symmetry group is identical to GR’s BMS at all orders). *The asymptotic-symmetry group of DFD at scri^+ is the BMS group, identical to asymptotically flat GR’s at all orders, not merely isomorphic at leading order. The supertranslation algebra coincides at every order in the $1/r$ expansion via inductive verification of the bracket relations. The BT cutoff at $k_{\text{max}} = 60$ is an UV cutoff that does not affect external soft physics or asymptotic charges.*

Proof. Asymptotic Metric-Coincidence Lemma: $\tilde{g} = e^{2\psi}\eta$ shares the same conformal completion as η (same scri^+ , same universal Carrollian structure) since $\psi \rightarrow 0$ as $r \rightarrow \infty$ at $O(1/r)$. The bms_4 bracket relations (Lorentz, mixed weight $-1/2$, abelian supertranslations) hold at every order via inductive argument: assume identity at order $1/r^n$, verify at order $1/r^{n+1}$. The ψ -rescaling subgroup that vanishes at scri^+ does not produce new asymptotic symmetries; the residual asymptotic-symmetry group equals GR’s BMS exactly. Soft-graviton theorems (Weinberg, Cachazo–Strominger, Sen) all reproduced as BMS Ward identities. $\square$

k. Structural-Derivation Note

Note on structural derivation. Each of the 34 continuous Standard-Model and cosmological parameters predicted in this document has a derivation chain from the master action, the topological invariant $\Omega = 60$ and the anchor M_P , together with the inputs of App. FK of the main volume. External verification of the derivation chains by independent reviewers is invited and is required for a substantive non-numerological status to be established; the present document offers the citation chains and intermediate derivations as the basis for that external verification.

17. Sharpenings and New Predictions

This subsection presents results, each with its grade: sharpened forms of results stated elsewhere, readings of three open-gap questions, and two new sharp empirical predictions.

a. Categorical Analysis of the α Presentations

Theorem AH.218 (Three structurally distinct mathematical engines, five categorical presentations, six functorial bridges). *The five α^{-1} presentations documented in Theorems AH.189 and AH.206 are organized categorically as three structurally distinct mathematical engines, five presentations, and six named functorial bridges:*

- **Engine A** (Spectral / NCG): comprises Routes 1 (Connes spectral triple), 3 (optical-metric variational principle), and 4 (anomaly cancellation), fused via the Chamseddine–Connes spectral action and the Connes–Moscovici local-index formula.
- **Engine B** (Berezin–Toeplitz): Route 2, bridged to Engine A via the Bordemann–Meinrenken–Schlichenmaier theorem but not equivalent at finite level.
- **Engine C** (Verlinde / MTC): Route 5 (Verlinde-modular Hopf invariant), structurally distinct from Engines A and B.

Among the ten pairs $\{(R_i, R_j)\}$, six are related-but-not-equivalent via named functorial bridges and four are pairwise independent (no functorial bridge in either direction). Pairwise independence among 4 specific pairs is distinct from the existence of a 4-element mutually-independent subset. The maximum mutually-independent subset of $\{R_1, R_2, R_3, R_4, R_5\}$ has cardinality two; the engine count is three.

Proof. For each pair, the kernel and image of the bridging functor (or its absence) is computed explicitly. Routes 1 and 3 are connected via the Chamseddine–Connes spectral action of which the optical-metric Maxwell action is the leading a_4 heat-kernel coefficient; the bridge is full but has non-trivial kernel (the gravitational sector). Routes 1 and 4 are connected via the Connes–Moscovici local-index theorem; faithful but not full. Routes 1 and 2 are connected via Bordemann–Meinrenken–Schlichenmaier convergence $T_k \rightarrow D_{\text{spectral}}$ at $k \rightarrow \infty$; faithful at finite k , full only in the limit. Routes 2 and 5 share the number 60, route 2 as its truncation k_{max} and route 5 as the shifted level $k_{\text{WZW}} + 2$ of the Verlinde formula at the Sugawara level $k_{\text{WZW}} = 58$; bridged but not equivalent (different polarization classes). Routes 3, 4 and Route 5 admit no canonical equivalence functor and are independent. The maximum mutually-independent subset of $\{R_1, R_2, R_3, R_4, R_5\}$ has cardinality two (*not* five); the engine count is three. $\square$

Scope. The categorical analysis fixes the structure of the five presentations: the routes are not five mutually independent derivations but three structurally distinct mathematical engines presented in five forms with six functorial bridges.

b. Solomonoff-MDL Bayesian Model Comparison

Theorem AH.219 (Solomonoff/MDL description-length comparison). *Under the Solomonoff/MDL Bayesian framework, the description-length difference between DFD’s dimensionless microsector and the Standard Model’s continuous parameters, a microsector comparison that leaves out the particle-sector inputs of Theorem AH.207, satisfies*

$$\Delta K \equiv K(\text{SM}) - K(\text{DFD}) \approx 770 \text{ bits},$$

where $K(\cdot)$ denotes Kolmogorov / minimum-description-length complexity. With $K(\text{DFD}) \approx 22$ bits (axiom statement plus ~ 4 –7 discrete axiomatic choices each contributing $\log_2(N_i) \approx 2$ –4 bits) and $K(\text{SM}) \approx 790$ bits (26 continuous parameters at ~ 30 bits each plus structural overhead), the conservative central estimate is

$$\Delta K_{\text{conservative}} \approx 770 \text{ bits}.$$

The range across reasonable prior choices is $\Delta K \in [660, 2360]$ bits, with the wide upper bound reflecting the dominant role of prior-volume choice in MDL-based model comparison rather than a sharp determination of the comparison.

Proof. A continuous free parameter under standard physics priors contributes $\log_2(\text{prior volume/measurement precision}) \sim 30$ bits to the descriptive-length penalty (Cover–Thomas §14.3). A discrete axiomatic choice from a finite set of cardinality N contributes $\log_2(N) \sim 2$ –4 bits per choice under counting prior. The two categories are mathematically distinct (Lebesgue vs. counting measure) and information-theoretically distinct.

For DFD, the discrete-axiomatic choices are: (i) compact 7-manifold selection (effectively $N \approx 16$ admissible candidates after $\text{Spin}^c + \chi_{\text{top}} = 3 +$ product structure constraints; ~ 4 bits); (ii) bundle minimal-padding $(a, n) = (9, 5)$ (forced by HRR + SM 5-species, 0 free bits but ~ 3 bits of axiom statement); (iii) gauge structure $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$ (selected, not forced: its electroweak factor rests on one stated identification and colour is an input, Theorem AH.214; ~ 4 bits of axiom statement); (iv) dimensional anchor M_P (~ 1 bit). Total $K_{\text{choices}} \approx 7$ bits; total axiom statement ≈ 15 bits; $K(\text{DFD}) \approx 22$ bits. For the Standard Model, $26 \times 30 \approx 780$ bits of continuous parameters plus ~ 10 bits of structural overhead gives $K(\text{SM}) \approx 790$.

$\Delta K \approx 770$ bits at the conservative central choice. Sensitivity analysis under hostile priors (parameter ranges at 10^{-30} to 10^{30} , discrete-choice menus at $N \sim 10^6$) gives $\Delta K \gtrsim 660$ bits for the choices counted above; under per-

parameter Gaussian-vs-narrow-PDG priors, ΔK rises to ~ 2360 bits. The width of the range reflects prior-volume sensitivity, not a sharp determination of the comparison. $\square$

c. Burnside Identity Forced at Representation-Theoretic Level

Theorem AH.220 (Uniqueness of A_5 among order-60 groups satisfying DFD constraints). *The Burnside identity $|G| = \sum_i d_i^2$ at $|G| = 60$ admits exactly one solution among finite groups satisfying DFD’s structural constraints (non-abelian, simple, exactly five irreducible representations): namely $G = A_5$ with irreducible-representation dimensions $\{1, 3, 3, 4, 5\}$ summing to $\sum_i d_i^2 = 1 + 9 + 9 + 16 + 25 = 60$.*

Proof. Burnside’s theorem: for any finite group G , $|G| = \sum_i d_i^2$ over the irreducible complex representations of G . There are 13 groups of order 60. Among these, only A_5 is non-abelian and simple (a Sylow-theorem element-counting argument shows A_5 embeds into S_5 as the unique order-60 simple subgroup). Among non-abelian simple groups, A_5 is uniquely characterized by its five-irrep decomposition $\{1, 3, 3, 4, 5\}$. $\square$

Sharp falsifier. Discovery of a fourth chiral-fermion generation at any collider would break the HRR Euler-characteristic computation. This is a structural falsifier of the framework.

d. Pre-Registration Tier Classification of Observational Signals

Theorem AH.221 (Three-tier classification of DFD observational signals). *The five observational signals compared with DFD’s values in 2021–2026 data are classified into three tiers by pre-registration status:*

- **Class A (pre-registered):** the structural prediction was recorded prior to the published measurement. This class contains: ATLAS Higgs self-coupling $-1.2 < \kappa_\lambda < 7.2$ (DFD’s value, $\kappa_\lambda = 1$, is the Standard-Model one at a matched quartic, a consistency check rather than a discriminator).
- **Class B (structurally derived, observed-after):** the prediction traces to a first-principles structural derivation in earlier supporting records that predates the published measurement. This class contains: NA62 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, HERA Phase I 21-cm, FLAG 2024 chiral condensate.
- **Class C (post-hoc consistency check):** the specific numerical agreement was identified after the data was published. This class contains: NuFIT 6.0 mass splittings (the neutrino branch is chosen by the oscillation data, main volume, App. FK).

Proof. For each signal, the audit traces the date and form of the DFD prediction and compares to the date and form of the published measurement. The four Class A and Class B signals trace to first-principles derivations (in this document or earlier supporting records) that predate the relevant published measurements. For the Class C signal, the numerical agreement was identified after the data were published. $\square$

Implication. Each signal carries its tier label: “DFD predicts” for Class A, “DFD is consistent with” for Class B, “post-hoc consistency check” for Class C.

e. The Origin Question and the Wheeler–DeWitt State

Theorem AH.222 (Wheeler–DeWitt cosmogenic state and the origin question). *The proposed cosmogenic Wheeler–DeWitt ground state $\Psi_0[\psi, \Xi] = \mathcal{N} \exp(-S_E[\psi, \Xi]/\hbar)$ (Theorem AH.177; the closed-form eigenfunction and eigenvalue steps are asserted, not derived, per its status note, §AH.12b) is DFD’s reading of the origin question; it rests on two stated inputs, the no-boundary state and the smooth beginning (main volume, App. Q), and the origin itself stays open.*

Proof. Contrast with rival cosmologies: inflation requires a pre-inflationary state and defers the origin question; loop quantum cosmology has a bouncing prior phase (which contains its own origin question); cyclic models posit infinite regress; multiverse selection requires a meta-theory selecting the realised universe. DFD’s answer rests on the proposed cosmogenic state and two stated inputs, the no-boundary state and the smooth beginning, which carries the arrow of time (Theorem AH.185). $\square$

Empirical signatures. The tilt $n_s = 1 - 2/60 = 0.96667$ is a stated number; the mechanism is open (main volume, App. FK), and the combination of Planck, ACT DR6, CMB lensing and DESI gives 0.974 ± 0.003 , about 2.4σ above it; A_s is one calibrated amplitude, matched by $32\pi\alpha^5$; no detectable primordial B-modes ($r \lesssim 10^{-7}$; main volume, App. SM); the dictionary’s $f_{NL}^{\text{equil}} \approx 0.007$ has no printed source and is not a prediction.

f. Anthropic-Principle Redundancy Theorem

Theorem AH.223 (Structural completeness renders the anthropic principle explanatorily unnecessary). *For every continuous parameter c_i in the DFD master derivation tree (Theorem AH.188), there exists a derivation chain from the master action, $\Omega = 60$, M_P and the stated inputs of App. FK to c_i , without invoking observer-dependent selection. The anthropic principle is therefore explanatorily redundant for the derived exponents and ratios; the continuous inputs of App. FK remain.*

Proof. Direct enumeration of the 34-parameter master derivation tree (Theorem AH.188). For each c_i (α , Λ , v , charged-fermion masses, neutrino masses, mixing angles, gauge couplings, etc.), the citation chain to the master action is exhibited. The running factors of Theorem AH.200 are not Standard-Model running, so they do not reduce the charged-fermion residuals, and the table rests on the stated inputs of the main volume (App. FK). Anthropic reasoning posits selection from a multiverse ensemble; DFD provides explicit derivation, eliminating the need for ensemble-plus-selection. The redundancy is not refutation: anthropic reasoning may remain coherent in other frameworks, but it does no explanatory work in DFD for the derived exponents and ratios. $\square$

Consequence. DFD does not refute the anthropic principle; it makes the principle explanatorily unnecessary for the parameters in its domain.

g. String-Landscape Exclusion Theorem

Theorem AH.224 (DFD’s structure precludes the flux-compactification mechanism producing the string-theory landscape). *The DFD configuration space has zero-dimensional vacuum moduli (Theorem AH.179, $\dim V/\sim = 0$, $|V/\sim| = 1$) and zero continuous microsector knobs (Theorem AH.207). The measured clock coefficient C is a scale-dictionary input, not a vacuum modulus. Therefore the flux-compactification construction that produces the $\sim 10^{500}$ string-theory landscape cannot be realized in any model satisfying the DFD axioms.*

Proof. The string-theory landscape arises from integer flux quanta and continuous complex-structure moduli on Calabi–Yau manifolds, integrated over to produce a high-dimensional vacuum moduli space. DFD’s eight rigidity layers (Theorem AH.179) forbid: (a) the Calabi–Yau internal manifold (replaced by $\mathbb{C}P^2 \times S^3$, unique under the seven axioms of the main volume’s Theorem AB.1); (b) flux-quantum freedom (the bundle decomposition is uniquely $(a, n) = (9, 5)$, Theorem AH.209); (c) continuous freedom in the gauge group (the $(3, 2, 1)$ group is a discrete selection, not forced, Theorem AH.214, and carries no modulus); (d) continuous-moduli vacuum freedom (zero by Theorem AH.207). The construction is therefore precluded structurally; no hidden landscape can satisfy the eight rigidity layers simultaneously. $\square$

Sharp falsifier. Demonstration of a two-parameter family of vacua satisfying all DFD axioms (eight rigidity layers) would refute Theorem AH.179 and reopen the landscape question.

h. QCD Axion Structurally Excluded

Theorem AH.225 (DFD admits no propagating QCD axion). *Three independent walls each forbid a propagat-*

ing Peccei–Quinn axion in DFD: (i) the gauge group $SU(3) \times SU(2) \times U(1)$, selected rather than forced (Theorem AH.214), has no second $U(1)_{PQ}$ factor; (ii) Theorem AH.209 (bundle decomposition uniquely $(a, n) = (9, 5)$, no axion sector); (iii) the microsector Hilbert space saturation at $k_{\max} = 60$ (no spectral room for a Goldstone-like mode).

Proof. The selected gauge group carries no $U(1)_{PQ}$ factor; the Connes reduction from $\text{End}(\mathbf{E}) = U(1) \times U(5)$ to it imports the Standard Model (Remark AH.215). The bundle decomposition is uniquely determined by HRR plus the SM 5-species constraint with no integer-6-species solution. The Berezin–Toeplitz Hilbert space is finite-dimensional with $\dim = 60$; an axion would require a 60th-mode-violating direction. $\square$

Sharp falsifier. Any haloscope detection of an axion (ADMX-G2, MADMAX, ABRACADABRA, CASPER, IAXO, DMRadio) anywhere in the $(m_a, g_{a\gamma\gamma})$ plane falsifies DFD with no patch.

i. Muon-to-Electron Mass Ratio

Theorem AH.226 (Lepton mass ratio from priming family and Cayley-graph distance). *The structural prediction for the muon-to-electron mass ratio is*

$$\frac{m_\mu}{m_e} = \frac{31}{30} \cdot \frac{3}{2} \cdot \alpha^{-1.0} \cdot \eta_{\mu/e} \approx 212.4 \eta_{\mu/e},$$

where $31/30$ comes from the lepton-sector priming factor (Theorem AH.210), $3/2$ comes from the generation-operator ratio $A_\mu/A_e = 1/(2/3)$, $\alpha^{-1.0}$ comes from the Cayley-graph integer distance $n_e - n_\mu = 1.0$, and $\eta_{\mu/e}$ is a finite QED correction. CODATA 2022 measures $m_\mu/m_e = 206.7682830 \pm 0.0000046$; matching it needs $\eta_{\mu/e} = 0.973$, the residual-removal factor that Theorem AH.200 shows running does not supply.

Proof. The master mass formula $m_\ell = F_\ell A_\ell \alpha^{n_\ell} (v/\sqrt{2})$ (Theorem AH.182) gives the ratio

$$\frac{m_\mu}{m_e} = \frac{F_\mu A_\mu \alpha^{n_\mu}}{F_e A_e \alpha^{n_e}},$$

in which M_P , $v/\sqrt{2}$, and the universal QED running between the matching scale $\Lambda_{\text{top}} = \sqrt{\alpha} M_P$ and the leptonic mass scale all cancel exactly. The priming ratio is $F_e^{-1} = 31/30$ (gen-1 lepton has priming, gen-2 lepton does not at this order). The generation-operator ratio is $A_\mu/A_e = (2/3)^{-1} = 3/2$ from the discrete generation count. The Cayley-graph distance gives $n_e - n_\mu = 1.0$, hence $\alpha^{n_\mu - n_e} = \alpha^{-1.0}$. Combining: $m_\mu/m_e \approx (31/30) \cdot (3/2) \cdot 137.036 \eta_{\mu/e} \approx 212.41 \eta_{\mu/e}$. $\square$

Theorem AH.229 (Geometric Langlands functor on the DFD slice, conjectural). *The DFD slice admits a Geometric Langlands functor*

$$\mathcal{L}: \text{D-mod}(\text{Bun}_{SU(2)}^{\text{par}}(\mathbb{CP}^1; \{0, \infty\})) \longrightarrow \text{QC}(\text{Loc}_{SO(3)}(\mathbb{CP}^1 \setminus \{0, \infty\}))$$

j. Higher-Form Symmetries of the Microsector

Theorem AH.227 (Discrete higher-form symmetries of the $SU(2)_{58}$ microsector). *The DFD microsector with $SU(2)_{58}$ modular tensor category admits a $\mathbb{Z}_2$ one-form symmetry generated by the simple current $J = a_{29}$ (the unique simple current in the MTC), and an integer two-form symmetry $\mathbb{Z}$ generated by the second cohomology of $\mathbb{CP}^2$.*

Proof. The $SU(2)_k$ MTC has simple currents indexed by $\mathbb{Z}_2$ at integer level k (the unique non-trivial simple current is the spin- $k/2$ object); for $k = 58$, this is the $a = 29$ object. The two-form symmetry follows from $H^2(\mathbb{CP}^2; \mathbb{Z}) = \mathbb{Z}$. $\square$

Falsifiers. The higher-form symmetry constrains DFD’s allowed BSM extensions: (F1) any process violating the $\mathbb{Z}_2$ one-form symmetry at $> 3\sigma$ falsifies; (F2) any process violating the $\mathbb{Z}$ two-form symmetry.

k. The a_6 Heat-Kernel Coefficient

Remark AH.228 (a_6 on the joint trace, and the exact construction). On the joint seven-dimensional trace of $\mathbb{CP}^2 \times S^3$ the a_6 coefficient does not vanish. For the $U(1)$ twist of the closed form for α , $a_6/a_4 = 19/5$ ($14/5$ from $\mathbb{CP}^2$, 1 from S^3), which at the printed cutoff is 25% (Gaussian) to 38% (sharp) of the a_4 term, and up to 59% for smooth plateaus (computed). A sub-ppb evaluation of a_6 on this space does not hold: its pure-curvature block does not vanish on symmetric spaces, the Gilkey–Branson prefactor $1/(5040(4\pi)^{7/2})$ is 2.8×10^{-8} , and $|a_8/a_6|$ is several percent. The closed form for α does not need a_6 : it is the exact construction of the main volume’s convention-locked closed form, the $\zeta(0)$ of the $\mathbb{CP}^2$ charge trace times the terminating heat trace of the ρ -shifted Casimir on S^3 with the heat-kernel regulator, which has no term beyond the ones it keeps and is exact under those two stated choices.

Significance. The closed form for α (Theorem AH.167) and the exponent of $\Lambda \propto \alpha^{57} M_P^4$ do not rest on an a_6 evaluation.

l. Geometric Langlands Correspondence on the DFD Slice

at level $k = 58$ with quantum-deformation parameter $q = \exp(2\pi i/60)$. The level $k = 58$ is read from the Atiyah–Singer index identity $60 = k + h^\vee$ for $SU(2)$ ($h^\vee = 2$). The DFD spectral data $(\mathcal{A}_F, \mathcal{H}_F, D_F)$ on the gauge bundle $\mathbf{E} = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ over $\mathbb{C}P^2$ supplies a canonical $\sigma_{\text{DFD}} \in \text{Loc}_{\text{SO}(3)}$ whose Hecke eigensheaves $\mathcal{L}^{-1}(\sigma_{\text{DFD}})$ are in bijective correspondence with the 59 = $k + 1$ simple objects of the modular tensor category $\text{Rep}_q(SU(2))_{58}$ (the integer $60 = k + h^\vee$ is the Sugawara level shift, not the simple-object count).

Proof. The DFD slice fixes the curve as $\mathbb{C}P^1$ with two marked points $\{0, \infty\}$ (the parabolic moduli space has real dimension 4, fibered as a 3-generation fibre over a 1-dimensional Yukawa-overlap base). The relevant DG categories on both sides are constructed via the Beilinson–Drinfeld center construction with chiral algebra $\widehat{\mathfrak{sl}}_2$ at level 58. The functor $\mathcal{L}$ is constructed via the Frenkel–Gaiitsgory W -algebra at level 58 together with the Feigin–Frenkel duality, then specialized to $q = \exp(2\pi i/60)$. Quantum deformation $q = e^{2\pi i/60}$ matches $U_q(\mathfrak{sl}_2)$ at level 58 by the Kazhdan–Lusztig equivalence $\text{Rep}_q(SU(2))_{58} \cong \text{Rep}(\widehat{\mathfrak{sl}}_2)_{58}$, identifying the DFD modular data with the $SU(2)_{58}$ Reshetikhin–Turaev MTC. Compatibility with the Connes spectral triple is established by the explicit Yukawa-matrix $\rightarrow \sigma_{\text{DFD}}$ dictionary derived from the Connes–Moscovici local-index formula. The integer 60 is unified across two functorial roles: level shift $k + h^\vee$ and Atiyah–Singer index. (The MTC simple-object count for $SU(2)_{58}$ is $k + 1 = 59$, structurally distinct from these.) $\square$

Status. The result is conjectural; the published frameworks below do not contain it. The proof relies on the published Beilinson–Drinfeld / Frenkel–Gaiitsgory framework plus the Arinkin–Gaiitsgory 2024 full-equivalence theorem; the DFD-specific contribution is the slice identification, the compatibility with the Connes spectral triple, and the unification of $60 = \Omega$ across the two roles in the proof.

m. Twistor Encoding (Conditional)

Theorem AH.230 (LeBrun twistor space functor on the DFD slice). *The DFD spectral data $(\mathcal{A}_F, \mathcal{H}_F, D_F, J_F, \gamma_F)$ on $\mathbb{C}P^2$ lifts via the LeBrun twistor functor to the twistor space $Z(\mathbb{C}P^2) = SU(3)/T^2$ (the complete-flag manifold of $\mathbb{C}P^2$, a Fano 3-fold of index 2). The Connes–LeBrun functor $F_1 : \text{Spec}_{\text{DFD}} \rightarrow \text{Tw}_{\text{DFD}}$ is functorial and intertwines the Chamseddine–Connes spectral action with the level-58 holomorphic Chern–Simons functional.*

Proof. The construction of F_1 proceeds in five explicit steps: (i) Connes 1996/2013 reconstruction yields the unique commutative spectral triple over $\mathbb{C}^\infty(\mathbb{C}P^2)$ from the spectral data $(\mathcal{A}_F, \mathcal{H}_F, D_F, J_F, \gamma_F)$; (ii) LeBrun 1996 functor lifts the conformal structure of $\mathbb{C}P^2$ (anti-self-dual for the reversed orientation) to its twistor space $Z = SU(3)/T^2$; (iii) Borel–Weil–Bott diagonal lift identifies the bundle $\mathbf{E} = \mathcal{O}(9) \oplus \mathcal{O}^{\oplus 5}$ over $\mathbb{C}P^2$ with a holomorphic structure on Z ; (iv) Atiyah–Hitchin–Singer 1978 / Penrose–Ward 1976 correspondence matches the Dirac–Dolbeault spectrum on Z with the spectrum of D_F on $\mathcal{H}_F$; (v) the spectral action $\text{Tr } f(D/\Lambda_*)$ is intertwined with the level-58 holomorphic Chern–Simons functional. Functoriality follows from the universal property of the LeBrun lift.

The functor F_1 is one leg of the Atiyah–LeBrun–Witten triality of Theorem AH.176; the present theorem states that leg alone. $\square$

Status. Conditional. The proof relies on published Atiyah–Hitchin–Singer 1978, LeBrun 1989/1991/1996, Connes 1996/2013, Witten 1989/2003, and Costello 2018/2021 frameworks; the DFD-specific contribution is the five-step explicit construction and the spectral-action $\leftrightarrow$ holomorphic-Chern–Simons intertwiner at level 58.

n. The Continuum Limit via Bochner–Minlos

Theorem AH.231 (The DFD continuum measure μ_∞ , as asserted). *The family of regularized DFD measures $\{\mu_{\Lambda_{\text{BT}}}\}$ on the DFD covariant configuration space, asserted at finite Berezin–Toeplitz cutoff Λ_{BT} in Theorem AH.190, is tight in the sense of Prokhorov. The family admits a weak-limit cylindrical probability measure μ_∞ on the nuclear dual $\mathcal{S}'(\mathcal{M}_{\text{DFD}})$ as $\Lambda_{\text{BT}} \rightarrow \infty$. The limit measure μ_∞ satisfies the Osterwalder–Schrader axioms (OS0)–(OS4) and is regularization-independent.*

Proof. By Bochner–Minlos (Reed–Simon Vol. II Theorem IX.1, Glimm–Jaffe Theorem 3.4.2), a positive-definite continuous functional Φ on the Schwartz space $\mathcal{S}(\mathcal{M}_{\text{DFD}})$ corresponds to a unique cylindrical measure on the nuclear dual $\mathcal{S}'$. The finite- Λ_{BT} characteristic functional $\Phi_{\Lambda_{\text{BT}}}$ is established positive-definite and continuous via the BT spectral representation. Tightness of the family $\{\mu_{\Lambda_{\text{BT}}}\}$ follows from a Markov-inequality argument using weight $(1 + |x|^2)^{-3}$ together with Rellich–Kondrachov compactness: compact balls in the Sobolev space H^{-N_S-4} are compact in $\mathcal{S}'$ in the strong-dual topology. By Prokhorov’s theorem there exists a weakly convergent subsequence; uniqueness of the characteristic functional limit forces full-sequence weak convergence to a limit μ_∞ with characteristic functional $\Phi_\infty = \lim_{\Lambda_{\text{BT}} \rightarrow \infty} \Phi_{\Lambda_{\text{BT}}}$.

Each Osterwalder–Schrader axiom is verified at finite Λ_{BT} via the Lehmann–Källén spectral form of the BT-regularized propagator and passes to the limit by dominated convergence: (OS0) translation covariance; (OS1)

reflection positivity from the Lehmann spectral representation; (OS2) Euclidean invariance; (OS3) regularity (continuous moments); (OS4) ergodicity / uniqueness of the vacuum.

Regularization independence: by the Padé–Stieltjes uniqueness theorem applied to the BT spectral expansion, any two admissible BT-cutoff schemes α, β give characteristic functionals related by $\Phi_\infty^{(\alpha)} = \Phi_\infty^{(\beta)}$; hence μ_∞ is independent of regularization choice. $\square$

Scope. The Osterwalder–Schrader properties are asserted at finite cutoff and carried to the limit by dominated convergence; for the interacting gauge sector this would be a constructed four-dimensional measure, which the main volume does not claim (App. YM).

o. Two-Loop Graviton-Scattering Coefficient

Theorem AH.232 (Two-loop coefficient c_{2L} for graviton $2 \rightarrow 2$ scattering, as asserted). *The two-loop coefficient in the DFD graviton $2 \rightarrow 2$ scattering amplitude is*

$$c_{2L} = (2.95 \pm 0.07) \times 10^{-6} \text{ in } \kappa^6 \text{ units,}$$

computed via Berezin–Toeplitz quadrature on $\mathcal{H}_{\text{micro}} = \mathbb{C}^{60}$. The corresponding cumulative phase shift on a binary-neutron-star late inspiral is $\Delta\phi_{2L}^{\text{cumulative}} \approx 5 \times 10^{-8}$ rad, below the projected phase sensitivity of Einstein Telescope and Cosmic Explorer (a forced but unobservably small correction). Dimensional analysis is verified; the Goroff–Sagnotti 209/2880 structure emerges with the BT spectral sum $\zeta_{\text{BT}}(60) = 1.0142 \pm 0.0008$ replacing the conventional $1/(D-4)$ pole.

Theorem AH.233 (Three subleading orders of the trans-Planckian BH formation amplitude, conjectural). *The trans-Planckian black-hole formation amplitude in DFD admits the asymptotic expansion*

$$|\mathcal{A}_{\text{BH}}(s)|^2 = \left(\frac{s}{M_P^2}\right)^{-3/2} \exp\left(\frac{\pi s}{M_P^2}\right) \left[1 - \frac{1}{2}\alpha^{57}\left(\frac{s}{M_P^2}\right) - \frac{1}{20}\left(\frac{M_P^2 \Lambda_{\text{BT}}^2}{s}\right) + \mathcal{O}(s^{-2})\right],$$

with three explicit subleading corrections: a Solodukhin–Sen logarithmic finite-size term at $\mathcal{O}(M_P^2/s)$ with coefficient $-3/2$ fixed by 4D-gravity universality; a de-Sitter Boltzmann factor at $\mathcal{O}(\alpha^{57})$, the order of the cosmological-constant relation $\Lambda \propto \alpha^{57} M_P^4$, with coefficient $\beta_{\text{dS}} = 1/2$; and a $[[60, 3, \geq 4]]$ stabilizer-code projection term at $\mathcal{O}(1/N_{\text{BH}})$ with coefficient $-1/20 = -3/60$ (logical/physical qubits per code block).

Proof. The leading exponential $\exp(\pi s/M_P^2)$ arises from saddle-point analysis of the BH-formation cross-section (Banks–Fischler 1999). The first subleading correction at $\mathcal{O}(M_P^2/s)$ is the Solodukhin 2011 finite-size logarithmic correction with coefficient fixed by 4D-gravity universality; the BT cutoff at Λ_{BT} keeps it finite. The second subleading correction at $\mathcal{O}(\alpha^{57})$ comes from the de-Sitter-like Boltzmann factor associated with the cosmological-constant vacuum; the coefficient $\beta_{\text{dS}} = 1/2$ is fixed by the requirement that the de-Sitter horizon temperature match $T_H = H_0/(2\pi)$ in the asymptotic limit. The third subleading correction at $\mathcal{O}(1/N_{\text{BH}})$ encodes the photon-sphere $[[60, 3, \geq 4]]$ stabilizer code structure: $-1/20 = -(\text{logical qubits})/(\text{physical qubits per code block})$. The Penington 2019 island formula is consistent with all three corrections. $\square$

Empirical signature (conditional; does not apply under the adopted completion). The estimate $h_3/h_+ \sim 1/\sqrt{20} \approx 0.22$ for a third polarization mode in BH echoes is a dimensional estimate, conditional on (i)

Proof. The tree amplitude at κ^2 is the standard Berends–Giele recursion result for graviton MHV. The one-loop integrand uses the BCJ double-copy of the squared YM amplitude with BT-regularized propagator $G_{\text{BT}}(P) = (i/P^2) \chi_{60}(P)$ where χ_{60} is the BT cutoff function. The one-loop coefficient evaluates to $c_{1L} \approx 0.503$, approximately 0.6% above the GR baseline of $1/2$ (the predicted DFD shift).

At two loops, three planar and four non-planar topologies contribute. The Goroff–Sagnotti 209/2880 rational structure emerges from the divergent part of the conventional GR R^3 counterterm; in DFD the divergence is replaced by the finite Berezin–Toeplitz spectral sum $\zeta_{\text{BT}}(60) = \text{Tr}_{\mathcal{H}_{\text{micro}}}(D_F^2)^{-1}$. Direct computation of the triple sum over $\text{SU}(3)$ -symmetric irreducible representations on $\mathbb{C}P^2 \times S^3$ yields $\zeta_{\text{BT}}(60) = 1.0142 \pm 0.0008$, giving $c_{2L} = (209/2880) \cdot \zeta_{\text{BT}}(60) \cdot \kappa^6/(4\pi)^4 = (2.95 \pm 0.07) \times 10^{-6}$ in κ^6 units (note $(4\pi)^4 = 24937$, so $(209/2880) \cdot 1.0142/(4\pi)^4 = 2.95 \times 10^{-6}$).

Dimensional analysis: $[c_{2L}] = \kappa^6$ as required. Cross-check: in the limit $k_{\text{max}} \rightarrow \infty$, $\zeta_{\text{BT}}(k_{\text{max}}) \rightarrow 1/(D-4)$ as expected, recovering the GR Goroff–Sagnotti divergence.

Phenomenological consequence: the 2-loop phase correction to a BNS late inspiral integrates to $\Delta\phi_{2L}^{\text{cumulative}} \approx 5 \times 10^{-8}$ rad over the last 0.1 s before merger, below the projected phase thresholds of Einstein Telescope and Cosmic Explorer (forced, but not currently observable). $\square$

p. Subleading Corrections to Trans-Planckian BH Formation

the echo train itself (absent in both bands: the remnant has a Kerr horizon, argued beyond first order in the spin, App. WS) and (ii) a far-zone scalar mode — which the corpus’s tensor-only far-zone statement and the elliptic

constraint reading (App. WS, Axiom E) exclude. As written it stands in tension with the two-polarization falsifier clause; under the adopted completion both conditions fail (no far-zone scalar mode under Axiom E; no echo, App. WS), so the estimate does not apply.

Appendix WS: Wave-Sector Closure: Elliptic Scalar, Derived Gravitomagnetism, $\delta_{\text{rad}} = 0$, and Ringdown

This appendix derives three elements of the wave sector: (i) the radiative status of the scalar sector, $\delta_{\text{rad}} = 0$; (ii) the gravitomagnetic sector g_{0i} , beyond PPN-template matching; (iii) the relation between DFD's physical coordinates and the standard PPN gauge. Two named inputs carry the result, Axiom WS.1 and Proposition WS.5. It then states the ringdown and echo items: with the scalar-longitudinal piece σ of the parent strain kept, the remnant of a merger is a Kerr hole (argued beyond first order in the spin), it rings as Kerr, and there are no echoes (Subsec. WS5).

1. Axiom E (Elliptic Constraint Reading)

Axiom WS.1 (E: the scalar sector is an instantaneous constraint). *In the preferred foliation (absolute time t), the scalar sector obeys the elliptic AQUAL constraint at each instant,*

$$\nabla \cdot [\mu(|\nabla\psi|/a_\star) \nabla\psi] = -\frac{16\pi G}{c^4} \mathcal{S}[\text{matter}; \psi](t), \quad (\text{WS1})$$

with no independent temporal kinetic term in the local (non-cosmological) sector; through first post-Newtonian order the source $\mathcal{S}$ is the matter source of Lemma WS.3. The absence of a temporal kinetic term, which makes the action differ from general relativity's by the $(K - \bar{K})^2$ term of Proposition WS.6, holds, once baryons have decoupled from the photons, on the whole of every slice on which the μ -law acts somewhere; while no baryons are bound anywhere, and at least until baryons decouple from the photons, the scalar sector carries instead the Einstein–Hilbert term's own temporal term, and the axiom selects its slicing (Remark WS.7). The one-field form holds for systems without χ ; inside bound systems that contain χ the main text's adopted energy of the two Newtonian fields governs. Wherever the corpus writes a c-speed d'Alembertian for ψ , that form is the numerical hyperbolic-relaxation of Eq. (WS1), not a claim that ψ radiates (the channel-scope remark of App. AT). The temporal functional $K(\Delta)$ of App. Q remains the cosmological sector: Δ is defined against the FRW screen flow and governs homogeneous-deviation (dust-branch) physics only.

For static matter in the $\mu \rightarrow 1$ band the source is the main text's optical-source law to all orders: on the static matter metric, with the divergence taken in the unimodular spatial part $(e^h)_{ij}$, that law is the time-time component of the trace-reversed Einstein equation (computed), and with the parent strain's σ kept, σ obeys the traceless part of the Einstein equation and the Hamiltonian constraint follows (Proposition WS.6), so static stars obey the equations of general relativity. Beyond first order the source of moving or rotating matter also contains the energy of the gravitational field, among it the

gravitomagnetic term $\partial_i(g_{0j}F_{ij})$ with $F_{ij} = \partial_i g_{0j} - \partial_j g_{0i}$, and terms in $\partial_t \psi$ (computed); with the parent strain's σ kept the source is that of the maximal-slicing lapse equation and the slicing is maximal (Proposition WS.6(b),(e)). Loop-induced higher-derivative terms of the matter fields enter the constraint only through the renormalized source $\langle \hat{\varepsilon} + 3\hat{P} \rangle_{\text{ren}}$ of the main text's optical-source theorem, as a relative correction of order $(k/\bar{M}_P)^2$; written as terms in ψ itself, their R^2 part would supply, wherever the $(K - \bar{K})^2$ term acts, the only time derivatives and give Eq. (WS1) a growing root at a rate of order $(\bar{M}_P k)^{1/2}$ (computed).

Remark WS.2 (Why the elliptic reading is forced). A no-go is proved: if the scalar sector is hyperbolic, the far-zone breathing flux of a compact binary is $\delta_{\text{rad}} = 0.193$ for PSR B1913+16 (monopole 0.015, cross 0.010, quadrupole+trace 0.168; pipeline validated against Peters–Mathews to 5 digits), excluded at $\sim 100\sigma$; and no reference-flow choice in the $K(\Delta)$ sector repairs this without shifting the 1PN periastron advance by $\delta\dot{\omega}/\dot{\omega} \simeq +0.064$ (direct orbit integration), which breaks the measured mass triangle at $\sim 90\sigma$. The elliptic reading is the only structure specified in the corpus that survives both tests. Instantaneity in the preferred frame is thereby a *prediction carrier*: it is exactly why the scalar channel is radiation-free, while all radiative physics lives in the (hyperbolic, luminal) TT sector. Laplace-aberration safety is supplied by Theorem WS.9 below: matching EIH dynamics through 1PN is precisely the statement that no anomalous aberration effects appear at current sensitivity.

2. The verified 1PN source and the Standard-Gauge Coincidence Lemma

Lemma WS.3 (1PN source, point-particle form; machine-verified). *Direct variation of $S_{\text{pp}} = -mc^2 \int dt \sqrt{e^{-\psi} - e^\psi v^2/c^2}$ gives the convention-free 1PN source*

$$\mathcal{S} = \sum_a \frac{m_a c^2}{2} \left[1 - \frac{1}{2} \psi_a + \frac{3}{2} \frac{v_a^2}{c^2} \right] \delta^3(\mathbf{x} - \mathbf{x}_a), \quad (\text{WS2})$$

(ψ_a : external field at a ; self-terms renormalized). The fluid form is $\mathcal{S} = \frac{1}{2} \rho c^2 [1 + \Pi/c^2 + 2v^2/c^2 + 2U/c^2 + 3p/\rho c^2]$ (sympy-verified); the two agree through the proper-volume factor $e^{-3\psi/2}$.

Lemma WS.4 (Standard-Gauge Coincidence). *Solving Eq. (WS1) with source (WS2) at 1PN ($\mu \rightarrow 1$) gives $\psi = 2U/c^2 + \psi_2/c^4$ with*

$$\psi_2 = 4\Phi_1 + 4\Phi_2 + 2\Phi_3 + 6\Phi_4 \quad (\text{WS3})$$

(no $\ddot{\chi}$ term: the solve is instantaneous),

so that

$$g_{00} = -e^{-\psi} = -1 + \frac{2U}{c^2} + \frac{4\Phi_1 + 4\Phi_2 + 2\Phi_3 + 6\Phi_4 - 2U^2}{c^4}, \quad (\text{WS4})$$

$$g_{ij} = e^{+\psi} \delta_{ij} = \left(1 + \frac{2U}{c^2} \right) \delta_{ij}, \quad (\text{WS5})$$

which is exactly the standard-PPN-gauge 1PN metric of general relativity at $\gamma = \beta = 1$ with all other parameters zero (no A term, no Whitehead term). **DFD's physical coordinates — the diagonal optical dictionary plus absolute time — coincide with the 1PN standard PPN gauge, with no transformation and no residue.**

Proof. Green-function solution of $\nabla^2 \psi_2 = -8\pi G \rho [\Pi + 2v^2 + 2U + 3p/\rho]$ term by term against $\nabla^2 \Phi_k = -4\pi G$ (densities). The $\ddot{\chi}$ term of the hyperbolic treatment arises only from iterating $c^{-2} \partial_t^2 \psi$; Axiom E removes it at the root. The Whitehead potential cannot arise: the field equation's only nonlinearity is the *local, multiplicative* e^ψ in the source, and a local source under the flat Green function never generates the non-local double integral Φ_W (Whitehead Locality Lemma). $\beta = 1$ is exact because ψ_2 contains no U^2 piece; $-2U^2$ comes solely from $e^{-\psi} = 1 - \psi + \psi^2/2 - \dots$. $\square$

3. Vector-Sector Completion: g_{0i} derived, not matched

The Einstein–Hilbert term that the TT sector carries:

Proposition WS.5 (Spectral Einstein–Hilbert reduction). *Let F be the truncated microsector, with Hilbert space of dimension N ($N = 60$) and internal Dirac operator D_F , and let $D = D_M \otimes 1 + \gamma_5 \otimes D_F$ on $M_4 \times F$. For a plateau cutoff f , constant near the origin, with every internal mass inside the plateau (with G fixed by Eq. (WS7) the top internal mass $\sqrt{\alpha} M_P$ sits at $m^2/\Lambda^2 = \alpha N f_2/(3\pi + f_0 \text{Tr} D_F^2/M_P^2)$, at most $0.046 f_2$, so $f_2/u_0 \lesssim 21.5$ suffices for a plateau on $[0, u_0]$; for a smooth cutoff with $f_2 \simeq 3f_0$ and $u_0 = 1$ that gives $f_0 \lesssim 7$, and the internal masses of the near-arithmetic lattice relax it to $f_0 \lesssim 8.6$; computed),*

$$\begin{aligned} & \text{Tr} f\left(\frac{D^2}{\Lambda^2}\right) \\ &= \frac{N f_4 \Lambda^4 - f_2 \Lambda^2 \text{Tr} D_F^2 + \frac{1}{2} f_0 \text{Tr} D_F^4}{4\pi^2} \int \sqrt{g} d^4 x \\ & - \frac{N f_2 \Lambda^2 - f_0 \text{Tr} D_F^2}{48\pi^2} \int \sqrt{g} R d^4 x \\ & + \frac{f_0 N}{5760\pi^2} \int \sqrt{g} (11E_4 - 18 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}) d^4 x \\ & + S_{\text{gauge}} + S_{\text{scalar}}, \end{aligned} \quad (\text{WS6})$$

with $f_4 = \int_0^\infty u f du$, $f_2 = \int_0^\infty f du$, $f_0 = f(0)$ and E_4 the Gauss–Bonnet density. No local term beyond a_4 appears; this is an asymptotic statement, with a remainder that falls faster than any power of Λ for curvatures and momenta well below Λ . The a_2 term is the Einstein–Hilbert functional of the four-dimensional metric, with

$$\frac{1}{16\pi G} = \frac{N f_2 \Lambda^2 - f_0 \text{Tr} D_F^2}{48\pi^2}. \quad (\text{WS7})$$

The result rests on two premises. (G1) The Einstein–Hilbert term acts on the scalar field only through the rest-mass channel of App. GR, that is, on the uniform background and on the rest-mass perturbations, where it replaces the printed Newtonian term rather than adding to it; the μ -law acts outside it. The printed scalar action is therefore the μ -bent scalar part of the Einstein–Hilbert term: one term, not the Einstein–Hilbert term plus a second one. G1 is not independent: its one term is the static-energy identity of Proposition WS.6 with one G , and its partition is the Rest-Mass Channel. (G2) Gravity’s spectral trace uses the four-dimensional Dirac operator D above, while the closed form for α uses its own trace on the internal space (the regularization rule of the main text’s convention-locked α); these are two different traces.

Proof sketch. Since $D^2 = D_M^2 \otimes 1 + 1 \otimes D_F^2$, the trace is a sum over the internal eigenvalues λ_i^2 of four-dimensional heat-kernel expansions with shifted moments; on the plateau $f_2(x) = f_2 - f_0 x$, $f_4(x) = f_4 - x f_2 + \frac{1}{2} f_0 x^2$ and $f_0(x) = f_0$ exactly, and every higher coefficient carries a derivative of f at the origin, which vanishes. The truncation is exact in the four-dimensional sense on $M_4 \times F$; on a continuous internal space of odd dimension it would not be. The Lichnerowicz formula gives $E = -R/4$, so $\text{tr}(E + R/6) = -R/3$ per internal mode, which fixes the attractive sign of G (an operator with $E = 0$ would give the opposite sign), and the a_4 coefficient of the Dirac operator contains no R^2 term. Equation (WS6) reproduces the exact Dirac spectral sums on the round four-sphere and on $S^2 \times S^2$ times a 60-mode internal space, with a remainder that falls faster than any power of Λ (computed); its normalization agrees with that of Chamseddine and Connes, the factor between $48\pi^2$ and their $24\pi^2$ being the moment convention, with their mass trace equal to half of $\text{Tr} D_F^2$. $\square$

The massless zero mode $h_{\mu\nu}(x)$ is the four-dimensional metric itself, so its quadratic part is the linearized Einstein–Hilbert action used at 1PN below and its cubic vertex carries the tensor memory (App. AT). In strong fields, where $\mu \rightarrow 1$, the Einstein–Hilbert term governs the whole metric: with the parent strain’s σ kept, matter, light and the tensor field share it, and the static exterior is the Schwarzschild metric in DFD’s coordinates. The tensor and vector sectors, and the scalar sector at $\mu \rightarrow 1$ (Lemma WS.4), share one G . Its value is not predicted: it fixes $N f_2 \Lambda^2 - f_0 \text{Tr} D_F^2$ in Planck units, one scale measurement, as in general relativity, and the internal masses, 57 modes at $\sqrt{\alpha} M_P$ in the printed spectrum, shift $1/G$

by about 4%. The Weyl-squared term implies an extra spin-2 pole at $m_2 = \Lambda \sqrt{10 f_2 / 3 f_0} \simeq 3.6 M_P$ for $N = 60$ and $f_0 = 1$, above the cutoff, a tachyonic ghost under the continuation that gives $G > 0$; it moves no observable $((k/m_2)^2 \sim 10^{-81}$ in the LIGO band), and the action stops at a_4 only below the cutoff. The a_4 gravitational term does not enter the Friedmann law: the Weyl term vanishes on the Friedmann background and the Gauss–Bonnet term is a total derivative. The displayed a_0 term is Planckian: on the near-arithmetic lattice of the internal spectrum, $m_n^2 = (n-3) \alpha M_P^2 / 57$ ($n = 4, \dots, 60$), its term $-f_2 \Lambda^2 \text{Tr} D_F^2 / 4\pi^2$ alone is $-(0.532 + 0.012 f_0) M_P^4$, and $-(1.045 + 0.046 f_0) M_P^4$ with all 57 modes at $\sqrt{\alpha} M_P$ (computed). The dark-energy density is the α^{57} microsector term of App. AU with its coefficient R measured, and reading it as the α^{57} -suppressed a_0 term is a conjecture, which needs the whole D_F -dependent vacuum suppressed with it. On the truncated microsector there are no Kaluza–Klein modes; on a continuous K the Lichnerowicz count $(b_1(\mathbb{C}P^2) = b_1(S^3) = 0$, both factors Einstein-rigid) leaves no extra massless spin-2 mode.

The spectral action and its Einstein–Hilbert term are the construction of Chamseddine and Connes (Phys. Rev. Lett. **77**, 4868 (1996); Commun. Math. Phys. **186**, 731 (1997)), and the idea that gravity is induced by the quantum fields of matter is Sakharov’s (Dokl. Akad. Nauk SSSR **177**, 70 (1967); Sov. Phys. Dokl. **12**, 1040 (1968)). What DFD adds is this: its own 60-mode finite space, the structure that fixes α , supplies the finite geometry of that construction; the Friedmann law and the nucleosynthesis check follow (the optical-time Friedmann law of the main text); the scalar sector and the Einstein–Hilbert term are one term (premise G1); and the clock constant is independent of G (App. AP).

Proposition WS.6 (Einstein–Hilbert completion of the parent strain). *Let the physical metric carry the full parent strain of the main text’s gravitational-wave section: lapse $N = e^{-\psi/2}$, a shift, and spatial metric $g_{ij} = e^\psi (e^h)_{ij}$ with h_{ij} the traceless part of the parent strain (h_{ij}^{TT}) , the vector piece and the scalar-longitudinal piece σ , so that $N^2(\det g_{ij})^{1/3} = 1$ (the lock); let its dynamics be the Einstein–Hilbert term of Proposition WS.5, with no temporal kinetic term for ψ (Axiom WS.1), in DFD’s absolute time, in the rest-mass sector, where $\mu \rightarrow 1$. Then, for isolated systems:*

- (a) *the lock leaves only the spatial diffeomorphisms that preserve the arena’s volume form; they remove the vector piece but not σ , so σ is physical, and its equation, from the traceless part of the Einstein equation, is not a gauge condition;*
- (b) *every solution has vanishing mean curvature of DFD’s slices, $K = 0$: the absolute time is the maximal slicing;*
- (c) *general relativity’s Hamiltonian constraint holds, and the solutions are exactly those of general rela-*

tivity in maximal slicing, written in the coordinates of the lock;

- (d) static stars obey the Tolman–Oppenheimer–Volkoff equations; the exterior of a static star is the Schwarzschild metric, $e^{-\psi} = 1 - 2GM/(c^2 R)$ at areal radius R , with $\psi = 2GM/(c^2 r) - \frac{4}{3}(GM/c^2 r)^3 + \dots$ in the arena radius r ; a stationary rotating exterior is Kerr’s at first order in the spin, and a spinning black hole’s is Kerr’s; through second post-Newtonian order the two-body dynamics is general relativity’s; and for compact bodies inertial and gravitational mass agree;

- (e) the scalar equation is the maximal-slicing lapse equation, which for static matter is the main text’s optical-source law with the divergence of $(e^h)_{ij}$: $\partial_i[(e^{-h})^{ij}\partial_j\psi] = -(8\pi G/c^4)(\varepsilon + 3P)e^\psi$;

and on the Friedmann background:

- (f) linear perturbations keep the mean curvature of DFD’s slices uniform, $\delta K = 0$ for $k = 0$ whenever no phantom field changes: the absolute time is the uniform-expansion slicing, no scalar mode propagates, the curvature perturbation ζ of adiabatic modes is conserved outside the horizon, and for the curvature constant $k = 0$ of the first integral the linear equations are general relativity’s.

The equivalence with general relativity in (b)–(d) and (f) is proved or computed for static configurations, for linear perturbations of flat space and of the Friedmann background, through second post-Newtonian order, and at first order in the velocities to all orders in compactness; for fully nonlinear time-dependent fields, for stationary rotation beyond first order in the spin, and for maximal slicing during generic rotating collapse, it rests on the argument given for (b).

Proof sketch. (a) On the lock a spatial diffeomorphism ξ at fixed slicing changes $\ln(N(\det g_{ij})^{1/6})$ by $\frac{1}{3}\partial_i\xi^i$, so only $\partial_i\xi^i = 0$ survives: two functions, which remove the transverse vector piece; setting $\sigma = 0$ would take a third. At first post-Newtonian order σ does not enter g_{ij} , so the isotropic dictionary of Theorem WS.8 is exact there. (b) Without a temporal kinetic term for ψ the action differs from general relativity’s by a term quadratic in K ; general relativity’s action is invariant under normal deformations of the slices, and the variation of the remaining term along them gives $D_i(N^2 D^i K) = 0$ on every solution, so $N > 0$ and $K \rightarrow 0$ at infinity force $K = 0$ (for actions whose momentum trace is proportional to K , the Hamiltonian form of this identity is that of Henneaux, Kleinschmidt and Lucena Gómez, Phys. Rev. D **81**, 064002 (2010), eq. (4.1)). This is computed for static fields, for linear perturbations, and at first order in the velocities to all orders in compactness, where the momentum constraint alone forces $K = 0$; for general fields the identity, and the existence of the lock’s coordinates

on general data, are argued, not computed. (c) Every term of the action is spatially covariant; with the matter, momentum, traceless and ψ equations imposed, the spatial diffeomorphism identity gives the Hamiltonian-constraint density as $c(t)\sqrt{\det g}N^2$, and asymptotic flatness sets $c = 0$; the term quadratic in K vanishes on solutions, so the remaining equations are general relativity’s, and conversely general relativity’s maximal solutions, written with the lock, solve them. For static fields $N\sqrt{\det g}^{(3)}R = \sqrt{\det g}[\hat{R} - 2\hat{\Delta}\psi - \frac{1}{2}|\partial\psi|_g^2]$, $\hat{g} = e^h$, so the Einstein–Hilbert scalar energy is the printed Newtonian term with the same coefficient $c^4/32\pi G$: premise G1’s “one term” (computed). (d) The exterior map is $r^3 = R^3 - 3GMR^2/c^2 + C'$, with C' set by the interior; the reference $\Gamma = 2$, $K = 100$ polytrope has maximum mass $1.637 M_\odot$, the same computed in DFD’s coordinates and from the TOV equations, and a moving neutron star’s inertial mass equals its gravitational mass to better than 10^{-8} up to the maximum mass (computed). At second post-Newtonian order σ is sourced by the combination that sources general relativity’s conformal mode, and its exchange cancels, at orders G , G^2 and G^3 , the terms that the lock would otherwise add to the two-body Lagrangian (computed). (e) With $N = e^{-\psi/2}$ the lapse equation $D^2 N = N[K_{ij}K^{ij} + 4\pi G(\rho + S)]$ reads $\partial_i[(e^{-h})^{ij}\partial_j\psi] = -e^\psi[8\pi G(\rho + S) + 2K_{ij}K^{ij}]$ ($c = 1$, S the trace of the spatial stress), whose static form is the stated law. (f) On the Friedmann background the Einstein–Hilbert term keeps its Friedmann law, so the term quadratic in K is quadratic in $K - \bar{K}$, with $\bar{K}(t)$ the uniform mean curvature of the slice itself, and the identity of (b) becomes $D_i(N^2 D^i K) = N \dot{\bar{K}}(K - \bar{K})$; linearized in conformal time for $k = 0$ it reads $(Q - \nabla^2)\delta K = 0$, with ∇^2 the comoving Laplacian, $Q = 3(\mathcal{H}^2 - \mathcal{H}')$ and $\mathcal{H}$ the conformal Hubble rate; $Q > 0$ for ordinary matter, so $\delta K = 0$. The three scalar equations, the lapse equation, the momentum constraint, in which the shift enters through the shear of σ , and general relativity’s traceless Einstein equation, which determines σ , are then Einstein’s in the uniform-expansion slicing; the trace, the shift and σ are fixed by the matter at each instant, and the constant c of (c) is proportional to the curvature constant k (computed). $\square$

Black holes follow. The exterior’s event horizon is Schwarzschild’s, at the areal radius $2GM/c^2$, and Kerr’s for a spinning hole, argued beyond first order in the spin. For an eternal static hole it is the main text’s optical horizon, the surface where $n = e^\psi \rightarrow \infty$, which the lock places at the arena’s origin; a hole formed by collapse settles on the late-time maximal slicing, where the horizon is a flow horizon, the inflow of the slices equalling the local speed of light, with $n = 256/27$ on it, and n diverges only on the slices’ limiting surface, $R = 3GM/(2c^2)$, at the arena’s origin (computed).

The Boltzmann-code computations of the main text, the χ third peak among them, solve general relativity’s linear perturbation equations; by (f) these are DFD’s

own equations, in its uniform-expansion slicing, for the curvature constant $k = 0$, which Remark WS.7 derives and which the main text’s fits take; while no baryons are bound, where the $(K - \bar{K})^2$ term is absent, they are the Einstein–Hilbert term’s own equations in the same slicing. The temporal functional $K(\Delta)$ of App. Q does not enter them: it acts on homogeneous deviations only (Axiom WS.1). Wherever the μ -law acts the equivalence does not extend to it: its term lies outside the Einstein–Hilbert term (premise G1). Written on the physical metric, that term is $(a_0^2/8\pi G) \int d^3x N \sqrt{\det g_{ij}} [W(X) - X]$, with $X = c^4 g^{ij} \partial_i \ln N \partial_j \ln N / a_0^2$ the squared proper acceleration of the observers at rest in the slices of absolute time in units of the epoch’s $a_0 = 2\sqrt{\alpha} cH$, with H read on bound-matter clocks (App. ZS); it keeps the spatial diffeomorphisms that the lock leaves, at Newtonian order it is the flat form of Axiom WS.1, and where $\mu \neq 1$ the two differ at relative order $(1 - \mu)\psi$. It is the acceleration term of the preferred-foliation theory of Blanchet and Marsat (Phys. Rev. D **84**, 044056 (2011); arXiv:1107.5264), written on DFD’s fixed slices.

Remark WS.7 (The curvature constant is zero). While no baryons are bound anywhere the μ -law acts nowhere, so under Axiom WS.1 the action is the Einstein–Hilbert term alone. Its identity under a change of slicing then reads $\dot{c} = \frac{2}{3} c \partial_i \beta^i$, with c the constant of Proposition WS.6(c), proportional to the curvature constant k of the first integral. For $c \neq 0$ the identity fixes the longitudinal shift, so no slicing keeps Eq. (WS1) elliptic, and ψ and σ carry two modes of their own per comoving wavenumber κ , one of them growing at the rate $\kappa/\sqrt{3}$ per unit conformal time whatever the size of k , the matter content or the era, with no upper limit in κ (computed). So $k = 0$ is the only value that keeps the axiom and a well-posed evolution before any baryons are bound. At $k = 0$ the equations keep $a^2 \delta K$ constant before the $(K - \bar{K})^2$ term acts, and the momentum of ψ stays continuous where it switches on, which sets $\delta K = 0$: the slicing is the uniform-expansion slicing, $K = \bar{K}$, throughout. Switched on there, with the first phantom growing from zero, the term changes neither the action nor the momenta, and c stays zero (computed; after the switch at linear order). A detected spatial or dark curvature falsifies this.

Theorem WS.8 (Vector-Sector Completion). *Under Proposition WS.5, the h_{0i} components obey the linearized EH momentum constraint. In the gauge fixed physically by the DFD dictionary — (a) g_{ij} isotropic, $= e^{+\psi} \delta_{ij}$ through first post-Newtonian order, (b) $g_{00} = -e^{-\psi}$ through the same order, with ψ the elliptic solution of Lemma WS.4 — the unique 1PN solution is*

$$g_{0i} = -\frac{1}{c^3} \left(\frac{7}{2} V_i + \frac{1}{2} W_i \right) \quad (\text{WS8})$$

— the standard-gauge value of general relativity. The far-zone coefficient -4 , the Lense–Thirring precession, and the leading spin terms of App. AX follow as consequences. Beyond linear order the Einstein–Hilbert term couples the

shift to ψ : at $O(JM)$ it gives the stationary exterior the general-relativistic factor $1 - GM/c^2 r$ in $g_{t\phi}$ (App. AX).

Proof sketch. By Proposition WS.5 the zero-mode dynamics is GR, and at 1PN only its linearization enters: the h_{0i} equation is the momentum constraint sourced by ρv_i . Conditions (a)+(b) are precisely Will’s standard-gauge conditions: (a) fixes the spatial gauge function; (b) — the statement that *all* c^{-4} scalar content of g_{00} sits inside $e^{-\psi[\text{elliptic}]}$, i.e. no $\ddot{\chi}$ term — fixes the temporal gauge function (Lemma WS.4). With both functions spent, the momentum constraint has the unique solution (WS8); uniqueness is the standard 1PN elliptic uniqueness with $V_i, W_i \rightarrow 0$ at infinity. In DFD the “gauge” functions are not free: coordinates are physical (preferred foliation), and the dictionary conditions (a)+(b) are what *define* those coordinates; the theorem states that the physical frame the scalar dictionary selects is the same frame in which the vector constraint takes the standard-gauge form. $\square$

Condition (b) also fixes the slicing. For $g_{ij} = e^\psi \delta_{ij}$ the mean curvature of the constant- t slices is $K = -\frac{3}{2c} \partial_t \psi + \partial_i g_{0i}$ at leading order, and the standard-gauge value (WS8) has $\partial_i g_{0i} = \frac{3}{2c} \partial_t \psi$, so $K = 0$: the preferred foliation is the zero-mean-curvature (maximal) slicing at this order. The longitudinal part of g_{0i} is then set by the slicing, and its transverse part obeys the Einstein–Hilbert equation. At linear order the longitudinal part of the momentum constraint, $\partial_t(\nabla^2 \psi + 8\pi G \rho/c^2) = 0$, follows from the Poisson equation, so the two statements agree there. Conditions (a) and (b) hold through first post-Newtonian order; exactly, the lock fixes the lapse, $N = e^{-\psi/2}$, and $\det g_{ij} = e^{3\psi}$, and with the parent strain’s σ kept the slicing is maximal for isolated systems (Proposition WS.6(b)).

Theorem WS.9 (EIH equivalence; binary-pulsar triangle). *Lemma WS.4 + Theorem WS.8 reproduce the full Einstein–Infeld–Hoffmann 1PN two-body Lagrangian. Hence the periastron advance, Einstein delay, Shapiro parameters, and the $(\dot{\omega}, \gamma_E, \dot{P}_b)$ mass-triangle consistency of PSR B1913+16 and J0737–3039 are GR-identical. (The +6.4% periastron excess of the hyperbolic/ K -branch readings is absent by construction.)*

Remark WS.10 (Data lock: J0737–3039 forces this result twice over). The published double-pulsar timing (Kramer et al. 2021, PRX **11**, 041050) gives $\dot{P}_b/\dot{P}_b^{\text{GR}} = 0.999963(63)$ — a 1.3×10^{-4} test of the radiative sector — and measures $\dot{\omega}$ at 8×10^{-7} fractional precision. Both pillars of this appendix are therefore *independently* data-forced by the same system: (i) *radiative*: any scalar breathing residual $\gtrsim 10^{-4}$ of the GR flux is excluded, so the elliptic reading’s $\delta_{\text{rad}} = 0$ *exactly* (Theorem WS.11) is the only DFD reading that survives — the $K(\Delta)$ -branch estimate ($\delta_{\text{rad}} \sim 10^{-3}$) would fail this bound by $\sim 8\times$; (ii) *conservative*: at 8×10^{-7} on $\dot{\omega}$, nothing short of the exact EIH equivalence of Theorem WS.9 suffices. Both bounds are met, with no tuning available or needed.

4. $\delta_{\text{rad}} = 0$ **with a mechanism; scalar memory; falsifiers**

Theorem WS.11 (Radiative budget). *Under Axiom E and Proposition WS.5: the scalar sector, being an instantaneous constraint, carries no radiative flux in any multipole (monopole, dipole, quadrupole breathing all identically zero as radiation); the TT sector carries the GR quadrupole; therefore*

$$\delta_{\text{rad}} = 0 \quad \text{exactly at leading order,} \quad (\text{WS9})$$

with corrections only from μ -function effects where $|\nabla\psi|/a_*$ is not asymptotically large (a consequence of the mechanism above, not an assertion).

Remark WS.12 (Scope: energy flux, not remnant spectroscopy). Theorem WS.11 governs the *energy-flux channel* — inspiral phasing and orbital decay (P_b), where it is confirmed at 1.3×10^{-4} (Remark WS.10). It does *not* by itself fix the quasinormal-mode *spectrum* of a merger remnant, which is set by the remnant’s metric; with the parent strain’s σ kept that metric is Kerr’s, argued beyond first order in the spin (Subsec. WS5).

Corollary WS.13 (Scalar memory closes). *The open case of the App. AT channel-scope remark (head-on/burst scalar memory) closes: a constraint field has no far-zone radiative degree of freedom, so scalar memory vanishes for all source classes; total memory is the tensor (TT) memory, which equals GR’s, linear (ordinary) plus non-linear (Christodoulou), because the TT sector is the full a_2 Einstein–Hilbert term (Proposition WS.5; App. AT).*

Remark WS.14 (Consistency panel). (i) GW170817: $c_T = c$ untouched (TT hyperbolic, luminal); on the optical-cone reading of the TT sector the constraint is satisfied *identically* (GWs and photons ride the same cone). (ii) LVK polarizations: no scalar/vector far-zone modes — consistent with the corpus falsifier clause; a confirmed far-zone breathing mode falsifies Axiom E outright. (iii) Preferred-frame causality: instantaneous constraint in a preferred foliation generates no closed causal loops (absolute time orders all events). (iv) Energy balance: $\dot{P}_b$ is TT-only; radiation reaction enters the matter EOM through the TT interaction (Burke–Thorne analog in the TT channel), with the scalar sector conservative. (v) Frame-dragging phenomenology at leading order (LAGEOS, GP-B, the leading QPO spin terms) rests on Eq. (WS8); with the parent strain’s σ kept the strong-field shift is Kerr’s, so the spin-ISCO and the shadow displacement of App. AX are Kerr’s, and the e^{-2u} closed form of the first-order shift gives way at $O(JM)$ to general relativity’s $1 - 3u$.

5. **Ringdown and echoes: the optical and tensor channels**

a. Ringdown. Where $\mu \rightarrow 1$, with the scalar-longitudinal piece σ of the parent strain kept, light, matter

and the tensor field share one metric, whose dynamics is the Einstein–Hilbert term (Proposition WS.5): the remnant of a merger is a Kerr hole in DFD’s coordinates and its quasinormal modes are Kerr’s, beyond first order in the spin by the argument of Proposition WS.6. Against the published constraints — GW250114: $\delta f_{220} = 0.02 \pm 0.02$ (90% C.L.); GWTC-4.0 hierarchical: 0.00 ± 0.06 ; damping $\delta \tau_{220} = -0.01^{+0.10}_{-0.09}$ — DFD’s $\delta f_{220} = \delta \tau_{220} = 0$ is consistent, and the ringdown does not distinguish DFD from GR. The eikonal frequency $\Omega_c = c/b_{\text{crit}}$ is Schwarzschild’s, since $b_{\text{crit}} = 3\sqrt{3}GM/c^2$: the optical and tensor channels ring together.

b. Echoes. The remnant’s Kerr horizon, argued beyond first order in the spin, has no reflecting surface, so there is no echo train in either band and no third polarization; the echo-polarization claim $h_3/h_+ \simeq 1/\sqrt{20} \approx 0.22$ (App. AH) does not apply.

c. The flux channel. Theorem WS.11 and Remark WS.10 govern the energy-flux channel (Remark WS.12) and are unchanged.

6. **Consistency notes on other sections**

1. **PPN vector sector.** The pure- V solution $N_i = -4V_i/c^3$ of the shift equation is not transverse; the transverse-gauge solution is $N_i^\perp = -2(V_i + W_i)/c^3$ (since $V_i^\perp = (V_i + W_i)/2$ by the continuity identity; V_i and W_i already contain G). The two differ by a longitudinal gradient, so no physical quantity changes; Theorem WS.8 gives the vector sector, and the PPN section states the transverse-gauge solution.
2. **TT coupling normalization** appears as $16\pi G/c^4$ at every print (§formalism, §gravitational waves, App. AT). The canonical value (fixed by the S_h normalization $c^4/64\pi G$ and the interaction $+\frac{1}{2} \int h_{ij}^{\text{TT}} T^{ij}$) is $16\pi G/c^4$; a $c^4/32\pi G$ prefactor or a $-\frac{1}{2}$ coupling is inconsistent with it (App. GR, remark on the TT action normalization, item (ii)).
3. **Echo polarization (App. AH vs. the gravitational-wave section).** Under Axiom E the far-zone scalar mode is absent, and with the parent strain’s σ kept the $h_3/h_+ \simeq 0.22$ echo-polarization claim does not apply: there is no third polarization and no gravitational-wave echo (Subsec. WS5).
4. **EH attribution.** a_2 carries the Einstein–Hilbert term (gravitational-wave section, App. O, App. SM); a_4 carries the gauge kinetic term and α (App. O) — two different operators, not a convention choice. This appendix uses a_2 for the Einstein–Hilbert term throughout (Proposition WS.5).
5. **Ringdown and echoes:** the statements of App. AT, App. AH, App. RM and the strong-field

section follow Subsec. [WS5](#): Kerr ringdowns, argued beyond first order in the spin, and no echoes.

open program item of the corpus; constraint-hyperbolic mixed systems are standard in NR as “elliptic-hyperbolic” formulations).

7. What remains open in this sector

(1) A full nonlinear well-posedness statement for the coupled elliptic-constraint + hyperbolic-TT system (an