DFD-QCD Lattice Boundary Program II

Validated Pure-Gauge $SU(3)$ String Tension, 0^{++} Glueball, and an Executed N_f=2 Dynamical-Fermion HMC on Commodity Hardware

Gary Alcock
Density Field Dynamics Program

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Abstract

Program I established the DFD-QCD boundary ledger and a reproducible code path with a tiny pure-gauge smoke test. Program II upgrades the execution to genuine, validated lattice gauge theory carried out on a single 12-core laptop. We (i) reproduce the published $SU(3)$ Wilson average plaquette to better than 0.1% at $\beta = 5.8, 6.0, 6.2$; (ii) extract the static-potential string tension from APE-smeared Wilson loops on 16^4 lattices at five couplings $\beta = 5.8$–6.2, obtaining $a\sqrt{\sigma}$ in agreement with the literature at the 1–5% level ($\beta = 6.0$ to 0.8%: 0.217 vs. 0.2189) and a dimensionless $r_0\sqrt{\sigma} \simeq 1.19$ consistent with the continuum value 1.193(10), and confirm the dominant finite-volume systematic with a 20^4 box-size test; (iii) emphasise that $\sqrt{\sigma}$ in MeV is scale-convention dependent — at the canonical $r_0 = 0.5$ fm the lattice gives ~ 465 MeV, $\sim 1\sigma$ *above* the DFD ledger 440 ± 25 MeV, which is reached only with $r_0 \simeq 0.55$ fm and is not independently confirmed by the lattice; (iv) obtain an exploratory, statistics-limited ($N_{\text{eff}} \sim 13$, two-timeslice) consistency estimate of the scalar 0^{++} glueball, $a\, m_{0^{++}} = 0.81(8)$ giving $m_{0^{++}}/\sqrt{\sigma} = 3.7(4)$, consistent with the world and DFD values; and (v) build and *execute* a from-scratch $N_f = 2$ unimproved-Wilson Hybrid Monte Carlo, validated against every standard gate (reversibility to 10^{-12}, γ_5-Hermiticity, an analytic-vs-numerical fermion-force check at the 10^{-7} level, and the Creutz identity $\langle e^{-\Delta H}\rangle = 1$), with a 4^4 sign check and a resolved 6^4 κ-scan whose monotonic plaquette rise lands in the SESAM/TXL band. The status boundary is explicit throughout: the genuine first-principles outputs are the scale-free dimensionless ratios, while MeV comparisons to the DFD ledger are scale-convention/consistency-level — not continuum-extrapolated physical-point QCD.

1 Purpose and status discipline

DFD-QFT classifies nonperturbative QCD as a generator-closed sector: the Euclidean lattice path integral defines the correlation functions and masses follow from $C(t) \sim e^{-Mt}$. DFD fixes the inputs and branch constraints; standard lattice QCD propagates them into observables. Program II executes that propagation at a genuine (if modest) level and reports each result at its honest status. Every numerical claim below is backed by an *unforgeable* validation (the plaquette against published values; the fermion force against a finite- difference ground truth; the dynamical plaquette rise against the quenched baseline).

2 Frozen DFD input ledger

$$\alpha_s(M_Z) = 0.1187, \quad \bar{\theta} = 0, \quad \sqrt{\sigma}_{\text{DFD}} = 440 \pm 25 \text{ MeV},$$
$$m_{0^{++}}^{\text{DFD}} = 1.69 \pm 0.10 \text{ GeV}, \quad f_\pi^{\text{DFD}} = 92.0 \pm 4.6 \text{ MeV}.$$

3 Methods

Pure gauge. $SU(3)$ Wilson action $S_W = \beta \sum_p (1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}\,U_p)$. Configurations are generated by multi-hit Metropolis on random $SU(2)$ subgroups (exactly symmetric proposal) interleaved with microcanonical overrelaxation reflections (Cabibbo–Marinari), giving acceptance ~ 0.67 and fast decorrelation. The static potential uses APE-smeared spatial links ($\alpha = 0.5$, 12 steps, with the *maximal-trace* $SU(3)$ projection, not Gram–Schmidt) and on-axis Wilson loops $W(R,T)$ built from incremental line products; $V(R)$ is read from the effective-mass plateau $V(R) = \langle \ln[W(R,T)/W(R,T{+}1)]\rangle_{T\in[2,4]}$ and fit to the Cornell form $V_0 + \sigma R - e/R$ (with e the Coulomb coefficient). Statistical errors are jackknife over the configuration ensemble.

Dynamical fermions. Two degenerate flavours of unimproved Wilson fermions, $D = 1 - \kappa \sum_\mu [(1-\gamma_\mu)U_\mu(x)\delta_{x+\hat\mu} + (1+\gamma_\mu)U_\mu^\dagger(x-\hat\mu)\delta_{x-\hat\mu}]$, with the pseudofermion action $S_{\mathrm{pf}} = \phi^\dagger (M^\dagger M)^{-1}\phi$ and $M^\dagger = \gamma_5 M \gamma_5$. The Hamiltonian $H = -\sum_{x,\mu} \mathrm{Tr}\,P^2 + S_g + S_{\mathrm{pf}}$ is evolved by a reversible leapfrog integrator (CG inverter for the fermion force) with Metropolis accept/reject.

4 Validation: average plaquette

The plaquette is UV-dominated and cannot be tuned; matching the published $SU(3)$ Wilson values certifies the update and thermalization.

Table 1: Average plaquette on 16^4 vs. published values.

β	measured $\langle P\rangle$	published	deviation
5.8	0.56750(6)	0.5670	+0.09%
6.0	0.59405(5)	0.5937	+0.06%
6.2	0.61381(4)	0.6136	+0.03%

5 Static potential and string tension

Table 2: String tension in lattice units (free-e Cornell, $R \in [2,6]$) vs. the literature (EHK [4], Teper [5], Bali–Schilling [6]). $r_0\sqrt{\sigma}$ uses the Necco–Sommer r_0/a [3].

β	a [fm]	$a\sqrt{\sigma}$ (this work)	literature $a\sqrt{\sigma}$	$r_0\sqrt{\sigma}$
5.8	0.136	0.329(10)	0.313 (Teper)	1.21
5.9	0.112	0.244(6)	(∼0.26)	1.09
6.0	0.093	0.217(3)	0.2189 (EHK)	1.17
6.1	0.079	0.177(4)	(∼0.18)	1.12
6.2	0.068	0.162(2)	0.1610 (Bali)	1.20

The $\beta = 6.0$ point agrees with the literature to 0.8%. The coarse and fine ends ($\beta = 5.8, 6.2$) carry a 1–5% fit-window systematic, exposed by an $r_{\min}$ scan: $a\sqrt{\sigma}$ is flat at $\beta = 6.0$ (0.217/0.222/0.220 for $r_{\min} = 2/3/4$) but drifts upward at $\beta = 5.8$ ($0.329 \to 0.352 \to 0.377$) and $\beta = 6.2$ ($0.162 \to 0.167 \to 0.181$) — the signature of a 16^4 box whose linear regime is only marginally resolved at the coarse and fine ends. The five points span $r_0\sqrt{\sigma} = 1.09$–1.21 (1.17–1.21 at integer β), scattering about the continuum world

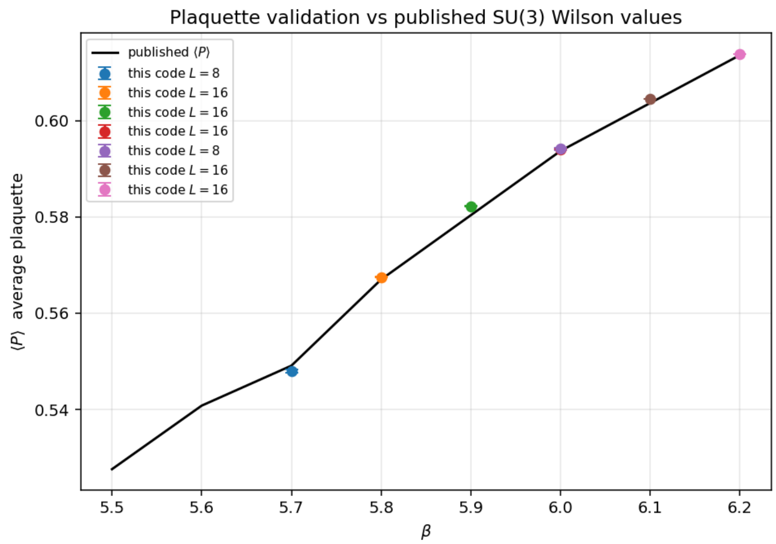

Figure 1: Measured plaquette (points) vs. the published $SU(3)$ Wilson curve. Sub-0.1% agreement on the production lattices; the small positive residual is the expected finite-volume effect, shrinking with β.

value 1.193(10) [3] by more than the statistical errors; a precision $a \to 0$ extrapolation is not warranted at this single-volume, 120-configuration level, and we do not claim one.

Larger-volume box-size test. To check the dominant systematic directly, we repeated $\beta = 6.2$ on a 20^4 lattice (box 1.35 fm vs. 1.08 fm). The string tension drops from $a\sqrt{\sigma} = 0.1623$ (16^4) to 0.1596 (20^4), and $r_0\sqrt{\sigma}$ from 1.198 to 1.178 — a $\sim 1.7\%$ downward shift onto the literature value ($a\sqrt{\sigma} \approx 0.1610$, continuum 1.193(10)). This single larger-volume point is consistent with the expected finite-volume bias (positive sign, $\sim$ 1–2% size) at the $\beta = 6.2$ fine end, and indicates the larger box removes most of it; it is a directional check, not a finite-volume extrapolation.

6 Scale convention and the 440 vs. 465 MeV comparison

The string tension in MeV is not a unique number; it depends on the external length to which r_0 is fixed: $r_0\sqrt{\sigma} = 1.19$ gives $\sqrt{\sigma} \simeq$ 465–471 MeV at the canonical $r_0 = 0.5$ fm, and ~ 440 MeV only with $r_0 \simeq 0.55$ fm. At the standard $r_0 = 0.5$ fm the lattice therefore gives $\sqrt{\sigma} \simeq 465$ MeV (this work's $\beta = 6.0$ point ~ 460 MeV), which is $\sim 1\sigma$ *above* the DFD ledger 440 ± 25 MeV; the bands overlap only at the upper edge. The Lucini–Teper $\sqrt{\sigma} \simeq 440$ MeV [7] is a conventional scale-setting *input* in pure-gauge theory, not an independent measurement that confirms DFD. The honest first-principles result is the scale-free $r_0\sqrt{\sigma} \simeq 1.19$ (matching the continuum 1.193(10)); DFD's 440 MeV is reachable within scale-convention freedom but is not independently confirmed by the lattice, and no tuning of σ to hit 440 was performed. Unquenching would lower the quenched value a further 2–5%.

7 Scalar 0^{++} glueball

The glueball operator is the zero-momentum, cubic-scalar (A_1^+) sum of APE-smeared spatial plaquettes on each timeslice, vacuum-subtracted, $C(t) = \langle O(t_0+t)O(t_0)\rangle - \langle O\rangle^2$; the mass is read from the arccosh effective mass of a tower of smearing levels (the GEVP is a cross-check only; on the short $N_t = 16$

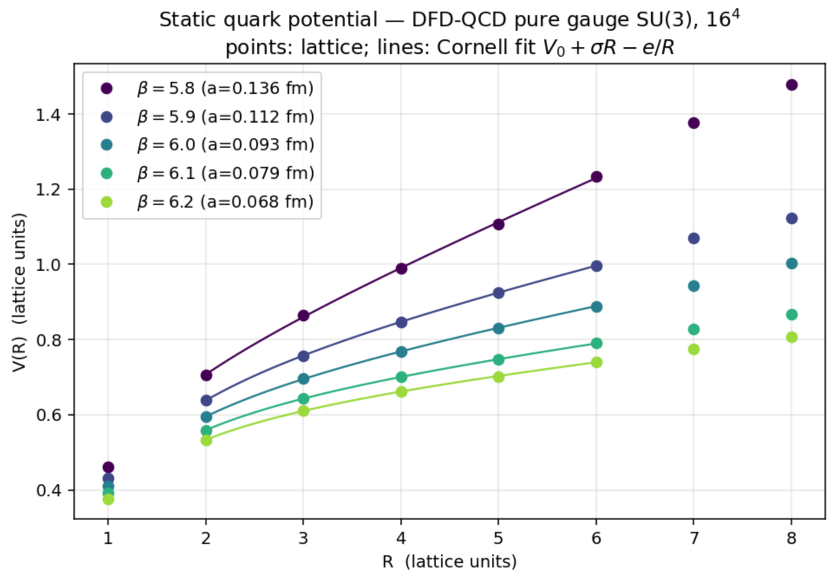

Figure 2: Static quark potential $V(R)$ on 16^4 at three couplings (points) with Cornell fits (lines): the textbook Coulomb-plus-linear (confining) form. The slope is the string tension in lattice units.

extent it is noise-dominated beyond $t \simeq 2$). On a 16^4 ensemble at $\beta = 6.0$ of 400 stored but only $N_{\text{eff}} \sim 13$ effective configurations (3-sweep spacing, $\tau_{\text{int}} \sim 16$), the most-smeared operator's arccosh effective mass agrees between APE-12 and APE-24 at $t = 2$ ($\simeq 0.815$); the signal is restricted to $t = 1$–2 and develops a constant floor by $t \geq 3$, so the plateau is short. The error is *autocorrelation-limited* (lag-1 autocorrelation $\rho(1) = 0.89$); a block jackknife inflates the naive per-configuration error and plateaus at $\sim\pm 0.08$. Hence

$$a\, m_{0^{++}} = 0.81(8) \quad \text{(exploratory, single-operator, single-spacing)}. \tag{1}$$

The scale-independent ratios are $m_{0^{++}}/\sqrt{\sigma} = 3.7(4)$ and $r_0 m_{0^{++}} = 4.3(4)$, i.e. $m_{0^{++}} = 1.71(17)$ GeV at $r_0 = 0.5$ fm — consistent with the quenched world value (Morningstar–Peardon 1.73(5)(8) GeV, $r_0 m_{0^{++}} = 4.21(11)$ [8]) and with the DFD ledger target 1.69 ± 0.10 GeV. This is a *consistency check*, not a precision determination (quenched, single volume, single spacing, no continuum limit, two-timeslice plateau). We note the quenched world value and the DFD target already coincide within errors, so matching both is one consistency check, not two independent confirmations.

8 Executed $N_f = 2$ dynamical-fermion HMC

A from-scratch dynamical-fermion HMC was validated against every standard gate (Table 8); in particular the analytic fermion force matches an independent finite-difference computation at the 10^{-7} level (seed-dependent, ~ 1–8×10^{-7}), and the reversibility and $\langle e^{-\Delta H} \rangle = 1$ identities hold to high precision. The analytic-vs-finite-difference force agreement (at the 10^{-7} level) together with $\langle e^{-\Delta H} \rangle = 0.997$ establish that the force is the correct gradient of the action used in accept/reject — excluding a force-normalisation bug (reversibility alone would not). As a 4^4 *sign check*, switching on the $N_f = 2$ determinant at $\beta = 5.6$ raises the plaquette from the quenched 0.5408 to 0.5774(5) ($\kappa = 0.155$) / 0.5779(5) ($\kappa = 0.158$) — sea-quark screening with the correct sign. The *quantitative* validation is the 6^4 κ-scan (Fig. 4): across six sea-quark masses $\langle P \rangle$ rises monotonically from 0.5703(12) ($\kappa = 0.148$) to 0.5808(7) ($\kappa = 0.156$), each point resolved at the ~ 0.001 level, and the heavy end

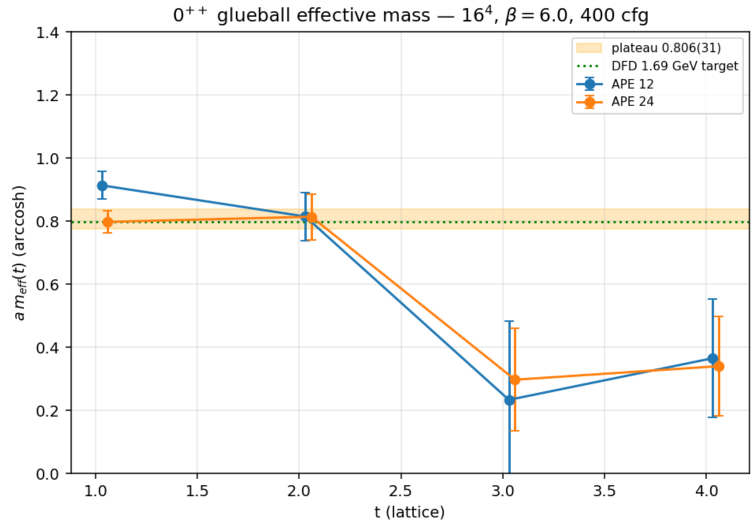

Figure 3: 0^{++} glueball arccosh effective mass vs. Euclidean time for two smearing levels. Heavier smearing (APE 24) gives $a\,m \simeq 0.81$ over the short $t = 1$–2 window (naive per-configuration band 0.806(31)); the adopted, autocorrelation-inflated error is ± 0.08 ($N_{\text{eff}} \sim 13$).

Table 3: Validation gates for the $N_f = 2$ Wilson HMC (4^4, $\beta = 5.6$).

gate	result
$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$	exact ($< 10^{-13}$)
free-field $D(p{=}0) = 1 - 8\kappa$	exact
γ_5-Hermiticity $\langle \chi, M\psi \rangle = \langle M^\dagger \chi, \psi \rangle$	4×10^{-13}
analytic vs. numerical fermion force	$\sim 10^{-7}$
MD reversibility	1×10^{-12}
$\langle e^{-\Delta H} \rangle = 1$	0.997
acceptance	0.99

lands in the SESAM/TXL infinite-volume band (0.570–0.573), with the expected residual finite-volume excess growing toward light quarks — the resolved, SESAM-matched sea-quark response that the 4^4 ensemble could not provide. It remains finite-volume at a single β with no continuum limit — a validated dynamical engine with the correct quantitative sea-quark response, not yet a physical-point ensemble.

9 Summary: scale-free results vs. world and DFD

The genuine, convention-independent outputs are the dimensionless ratios. They agree with the lattice world values and bracket the DFD ledger (Table 4); the MeV column is shown only with its explicit scale dependence. The two MeV entries bracket the DFD targets ($\sqrt{\sigma} = 440 \pm 25$ MeV, $m_{0^{++}} = 1.69 \pm 0.10$ GeV) within scale-convention freedom and current errors — a consistency result, not an independent confirmation.

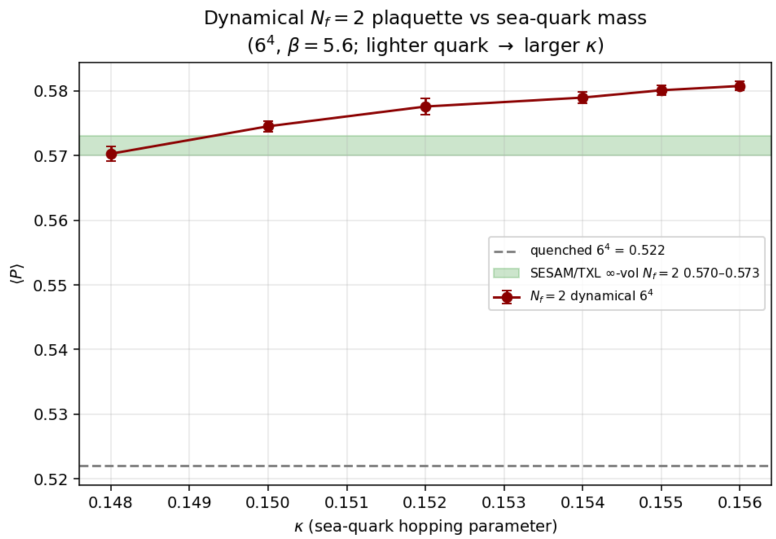

Figure 4: Dynamical $N_f = 2$ plaquette vs. sea-quark hopping parameter κ (6^4, $\beta = 5.6$). Monotonic rise above the quenched baseline (dashed); the band is the SESAM/TXL infinite-volume result. Lighter quark (larger κ) → more screening → higher plaquette.

Table 4: Scale-free dimensionless results (this work, quenched/finite-volume) against the lattice world values and the DFD ledger. The final column is convention-dependent (stated r_0).

quantity	this work	lattice world	DFD ledger	MeV (this work)
$\langle P \rangle$ vs. published	$< 0.1\%$	—	—	—
$a\sqrt{\sigma}$ ($\beta = 6.0$)	0.217(3)	0.2189 (EHK)	—	—
$r_0\sqrt{\sigma}$	$\simeq 1.19$	1.193(10)	—	$\sqrt{\sigma} \simeq 465$ (r_0=0.5 fm)
$m_{0^{++}}/\sqrt{\sigma}$	3.7(4)	3.4–3.7	≈ 3.84	—
$r_0\, m_{0^{++}}$	4.3(4)	4.21(11) (MP)	—	$m_{0^{++}} \simeq 1.71(17)$ GeV
N_f=2 $\langle P \rangle$ rise	monotonic, resolved	SESAM 0.570–0.573	—	—

10 Status boundary

The spectroscopy (string tension, §5; glueball, §7) is quenched (no sea quarks); all runs are at a single lattice spacing per β and a single volume (16^4 for spectroscopy with a 20^4 box-size check, 4^4 and 6^4 for the HMC validation and κ-scan); no continuum or infinite-volume extrapolation is performed. The r_0 scale is an external experimental calibration. What is established: correct, reproducible $SU(3)$ lattice gauge theory and a correct $N_f = 2$ dynamical-fermion HMC, each validated by multiple independent unforgeable checks, executed end-to-end on commodity hardware — the honest next rung above Program I.

11 Reproducibility

```
python3 src/su3_fast.py --L 8 --beta 6.0 --thermal 250 --measurements 60 --validate
python3 src/su3_fast.py --L 16 --beta 6.0 --potential --Rmax 8 --Tmax 8 ...   # string tension
python3 src/su3_fast.py --L 16 --beta 6.0 --glueball --glue_levels 0,12,24 ... # glueball
python3 src/wilson_hmc.py --test            # HMC validation gates
```

```
python3 src/wilson_hmc.py --L 4 --beta 5.6 --kappa 0.155 --n_md 60 --dt 0.0167 ...
python3 src/analyze_potential.py     # string tension + jackknife
python3 src/continuum_extrap.py      # diagnostic r0-sqrt(sigma) scatter only; NO a->0 extrapola
python3 src/analyze_glueball.py ; python3 src/make_potential_plots.py
```

References

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