Completing Local Position Invariance Tests: A Cavity–Atom Frequency Ratio Protocol

Gary Alcock[1]

[1]Los Angeles, USA

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Summary. Local Position Invariance (LPI) is a cornerstone of General Relativity, tested via gravitational redshift with atomic clocks and matter [1, 2, 3, 4, 5]. However, no direct test has yet compared *cavity-stabilized optical frequencies* (photon sector) to *atomic transitions* (matter sector) across a gravitational potential. We propose a protocol to close this gap: measure the fractional slope of co-located cavity–atom frequency ratios transported between two fixed altitudes.

Observable

Define the cavity–atom ratio:

$$\frac{\Delta R^{(M,S)}}{R^{(M,S)}} \equiv \xi^{(M,S)}\,\frac{\Delta\Phi}{c^2}, \qquad \xi^{(M,S)} = \alpha_w - \alpha_L^{(M)} - \alpha_{\rm at}^{(S)}. \tag{1}$$

Here the coefficients are:

- α_w: photon-sector weight, normalized to 1 in GR.
- $\alpha_L^{(M)}$: cavity length sensitivity for material M (e.g. ULE or Si).
- $\alpha_{\rm at}^{(S)}$: atomic transition sensitivity for species S (e.g. Sr or Yb).
- $\xi^{(M,S)}$: net slope coefficient for cavity–atom ratio with material M and species S.

- GR predicts $\xi^{(M,S)} = 0$, i.e. a strict null.
- Any reproducible nonzero ξ would indicate sector-dependent deviation from LPI.

Definitions and identifiability

To isolate contributions, define:

$$\delta_{\rm tot} \equiv \alpha_w - \alpha_L^{\rm ULE} - \alpha_{\rm at}^{\rm Sr}, \qquad \delta_L \equiv \alpha_L^{\rm Si} - \alpha_L^{\rm ULE}, \qquad \delta_{\rm at} \equiv \alpha_{\rm at}^{\rm Yb} - \alpha_{\rm at}^{\rm Sr}.$$

The four measured slopes across two cavity materials (ULE, Si) and two atomic species (Sr, Yb) then map to three independent combinations (Table 1).

Table 1: Mapping of measured cavity–atom ratios to sector parameters.

Measured slope	Combination	Identified parameter
ULE/Sr	$\alpha_w - \alpha_L^{\mathrm{ULE}} - \alpha_{\mathrm{at}}^{\mathrm{Sr}}$	δ_{tot}
Si/Sr	$\alpha_w - \alpha_L^{\mathrm{Si}} - \alpha_{\mathrm{at}}^{\mathrm{Sr}}$	$\delta_{\mathrm{tot}} + \delta_L$
ULE/Yb	$\alpha_w - \alpha_L^{\mathrm{ULE}} - \alpha_{\mathrm{at}}^{\mathrm{Yb}}$	$\delta_{\mathrm{tot}} + \delta_{\mathrm{at}}$
Si/Yb	$\alpha_w - \alpha_L^{\mathrm{Si}} - \alpha_{\mathrm{at}}^{\mathrm{Yb}}$	$\delta_{\mathrm{tot}} + \delta_L + \delta_{\mathrm{at}}$

Numerical scale

For Earth gravity $g \simeq 9.8\,\mathrm{m/s^2}$,

$$\frac{g\,\Delta h}{c^2} = 1.1 \times 10^{-14} \ \ (\Delta h = 100\,\mathrm{m}).$$

Thus the natural scale is at 10^{-14} per 100 m altitude change, within reach of current 10^{-16} optical clock precision.

Controls and feasibility

The protocol envisions **static comparisons at two fixed altitudes** (e.g. basement vs. rooftop labs, or ground vs. tower). Only stationary data are analyzed, avoiding artifacts from transport in motion.

Corrections and controls:

- **Dispersion/thermo-optic:** dual-λ probing within the low-loss band, bounding $|\varepsilon_{\mathrm{disp}}| \lesssim 10\%$ [6, 7, 8].
- **Elastic sag:** orientation flips distinguish mechanical artifacts (sign-reversing) from genuine redshift (sign-preserving). In optimized silicon cavities, sag effects can be suppressed below 10^{-16} [9, 10].
- **Environmental:** vibration, temperature, pressure, and magnetic reversals, plus hardware swaps, encode residual offsets in the covariance, suppressing bias [11, 5].

Feasibility. All required components are already demonstrated separately: ultra-stable cavities at 10^{-16} [9, 10], optical clocks reaching below 10^{-18} [11, 5], and long-term LPI clock tests [2, 4, 3]. Combining these into a cavity–atom slope test is therefore technically feasible with current infrastructure.

Motivation

Existing LPI tests compare like with like: atom–atom or matter–matter systems [1, 2, 4, 5]. A cavity–atom comparison probes an untested cross-sector combination (photon vs. atomic transitions). This experiment therefore **closes a missing gap** in the LPI test suite. Even a null result would provide the first direct constraint on this sector and complete the phenomenological mapping of LPI across independent systems.

Falsification criterion

- GR: $\xi = 0$ at all materials/species.
- Experimental discriminator: any reproducible nonzero ξ at or above $\Delta\Phi/c^2$ would indicate violation of LPI in this sector.

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