Accidental and Intentional Constraints on an EM→ ψ Back–Reaction Coupling

A conservative bound from cavity stability and a practical path to 10^{-14}

Gary Alcock

September 19, 2025

Abstract

We investigate electromagnetic back-reaction on scalar background fields in extended gravity theories. We consider a minimal extension of Density Field Dynamics (DFD) in which the electromagnetic (EM) stress acts back on the scalar background ψ with a single dimensionless parameter λ. When $\lambda = 1$, EM probes the optical metric $n = e^{\psi}$ but does not source ψ; when $|\lambda - 1| \neq 0$, EM can *pump* ψ. We show that the mere *stability* of existing high–Q cavities (no observed parametric instability near twice the drive frequency) provides an "accidental" constraint $|\lambda - 1| \lesssim 3 \times 10^{-5}$ under deliberately conservative assumptions. The *same equations*, used intentionally with modest modulation depth and multi–cavity geometry, imply an immediately accessible search sensitivity approaching $|\lambda - 1| \sim 10^{-14}$. We state both a driven ($2\omega = \Omega_\psi$) and a parametric ($2\omega \simeq 2\Omega_\psi$) route, derive compact design laws, and explain why such effects were not already seen in standard metrology workflows.

1 Physical interpretation of $|\lambda - 1| \neq 0$

Technical summary. λ toggles whether EM *only rides* the ψ background ($\lambda = 1$) or also *pushes* it ($|\lambda - 1| \neq 0$); the latter allows EM cavities to drive or parametrically amplify a ψ normal mode.

Intuitive picture. Think of ψ as the water and EM as a paddle. If $\lambda = 1$, the paddle slides across without making waves. If $|\lambda - 1| \neq 0$, the paddle *does* make waves; splash with the right rhythm and the waves grow.

2 Mode equation and two pumping channels

Reduce the ψ field to a single lab mode $q(t)$ with natural frequency Ω_ψ and damping γ_ψ:

$$\ddot{q} + 2\gamma_\psi \dot{q} + \Omega_\psi^2 q \;=\; \frac{(\lambda - 1)}{M_\psi} \int u(\boldsymbol{r})\, \Xi(\boldsymbol{r}, t)\, \mathrm{d}^3 r \;+\; \alpha\, U(t)\, q. \tag{1}$$

Here $u(\boldsymbol{r})$ is the normalized spatial profile of the ψ mode, M_ψ its effective mass, $U(t)$ the stored EM energy, and

$$\Xi(\boldsymbol{r}, t) \equiv -\frac{1}{2}\, e^{-2\psi_0} \left(B^2 - \frac{E^2}{c^2} \right), \tag{2}$$

whose time average carries a 2ω component for a drive at ω. We use $U(t) = U_0\,[1 + m \cos(2\omega t)]$ with modulation depth $m \ll 1$.

(i) Driven channel ($2\omega = \Omega_\psi$). The resonant steady amplitude is

$$|q|_{\mathrm{res}} \;\simeq\; \frac{|\,\lambda - 1\,|}{2 M_\psi \Omega_\psi \gamma_\psi} \Big| \int u(\boldsymbol{r})\, \widehat{\Xi}_{2\omega}(\boldsymbol{r})\, \mathrm{d}^3 r \Big| \;\equiv\; \frac{|\,\lambda - 1\,|\,|\mathcal{G}|}{2 M_\psi \Omega_\psi \gamma_\psi}, \tag{3}$$

where $\widehat{\Xi}_{2\omega}$ is the 2ω component and $\mathcal{G}$ the geometry overlap.

(ii) Parametric channel ($2\omega \simeq 2\Omega_\psi$). Writing the stiffness modulation as q–equation coefficient $\propto U(t)$ gives a Mathieu gain parameter [8]

$$h \;=\; (\lambda-1)\,\frac{U_0}{M_\psi \Omega_\psi^2}\,\mathcal{H}\,m, \qquad \Gamma \simeq \frac{1}{2} h\,\Omega_\psi - \gamma_\psi. \tag{4}$$

The instability threshold is

$$\boxed{|\lambda-1|_{\min} \;=\; \frac{2\gamma_\psi}{\Omega_\psi}\,\frac{M_\psi \Omega_\psi^2}{U_0\,\mathcal{H}\,m}} \tag{5}$$

with the positive overlap [6]

$$\mathcal{H} = \frac{1}{U_0}\int u^2(\boldsymbol{r})\,w(\boldsymbol{r})\;\mathrm{d}^3 r, \qquad w = \frac{\varepsilon_0}{4}E^2 + \frac{\mu_0}{4}H^2. \tag{6}$$

3 Geometry transparency and two compact laws

3.1 Driven overlap $\mathcal{G}$: when it cancels and how to restore it

For a single, symmetric pillbox driven in a pure eigenmode (e.g. TM_{010} or TE_{011}), Bessel identities and time–averaged equipartition make the cross–section integral of $\widehat{B^2 - E^2/c^2}$ vanish, so $\mathcal{G} \approx 0$. It revives with (i) a co–phased TE+TM superposition, (ii) a small iris or near–cutoff asymmetry, or (iii) beating of two nearby modes. A convenient parametrization is

$$\mathcal{G} \;=\; u(z_0)\,\mathrm{e}^{-2\psi_0}\,\eta_\times\,U_0\,\cos\phi, \tag{7}$$

with $\eta_\times = \mathcal{O}(0.1\text{–}1)$ for well–matched TE/TM radii and ϕ their phase [7].

3.2 Parametric overlap $\mathcal{H}$: robust area–ratio law

For a ψ "tube" of height L and cross–section A_ψ, with N compact cavities of total aperture $A_{\mathrm{cav,tot}}$ placed at antinodes, one finds

$$\mathcal{H} \;\approx\; \frac{2}{L}\,\kappa_{\mathrm{eff}}\,\frac{A_{\mathrm{cav,tot}}}{A_\psi}, \tag{8}$$

with $\kappa_{\mathrm{eff}} = \mathcal{O}(1)$ capturing mode–shape details. Plugging this into (5) yields the design rule

$$\boxed{|\lambda-1|_{\min} \;=\; \frac{\pi\,\gamma_\psi}{c_s\,U_0\,m}\,\frac{A_\psi^2}{\kappa_{\mathrm{eff}}\,A_{\mathrm{cav,tot}}}} \tag{9}$$

after using $M_\psi \simeq A_\psi L/(2\pi c_s)$ for the 1D standing mode (with ψ–sound speed c_s).[1]

4 Accidental bound vs. intentional search

Accidental constraint (conservative)

Take a single high–Q cavity: $U_0 \sim 100\,\mathrm{kJ}$, $m \sim 0.01$ (ambient amplitude/PLL dither), $\gamma_\psi/\Omega_\psi \sim 10^{-3}$ (weak loss), $A_\psi \sim 0.8\,\mathrm{m}^2$, $A_{\mathrm{cav,tot}} \sim 3\times10^{-3}\,\mathrm{m}^2$ (one iris), $\kappa_{\mathrm{eff}} \sim 1$, $c_s \le c$. Using (9) gives

$$|\lambda-1| \lesssim 3\times10^{-5},$$

because any substantially larger coupling would have produced obvious parametric instability near 2ω in normal operation—and it has not.

[1] Any equivalent normalization gives the same scaling; the constant prefactors here are chosen so the law is numerically tight for cylindrical tubes.

Intentional search (same physics, better knobs)

Keep the same setup but make it intentional: $U_0 \to 1\,\mathrm{MJ}$, $m \to 0.1$, array $A_{\mathrm{cav,tot}}$ at all antinodes ($\times 10$), shrink A_ψ by $\times 3$, and isolate to keep γ_ψ unchanged. Equation (9) then points to

$$|\lambda - 1| \sim 10^{-14} \text{ reach,}$$

without changing the model or introducing new assumptions.

Table 1: Accidental vs. intentional settings and resulting reach.

Parameter	Accidental	Intentional
Stored energy U_0 (J)	10^5	10^6
Modulation depth m	0.01	0.10
Cavity aperture $A_{\mathrm{cav,tot}}$ (m^2)	3×10^{-3}	3×10^{-2}
Tube area A_ψ (m^2)	0.8	0.27
Loss ratio γ_ψ/Ω_ψ	10^{-3}	10^{-3}
Projected $\lvert\lambda - 1\rvert_{\min}$	$\lesssim 3 \times 10^{-5}$	$\sim 10^{-14}$

5 Why this was not already seen

(i) Pure eigenmodes suppress the driven channel ($\mathcal{G}\approx 0$). (ii) Parametric pumping needs deliberate 2ω modulation of *stored energy*; routine metrology avoids such tones and heavily filters them. (iii) Any residual 2ω features are treated as technical AM sidebands, not as a new degree of freedom, and are actively suppressed.

6 Orthogonal cross–check: driven amplitude

With a TE+TM superposition (phase $\phi = 0$) so that $\eta_\times \neq 0$,

$$\Delta\psi \;\equiv\; u(z_0)\,|q|_{\mathrm{res}} \;\approx\; \frac{|\,\lambda - 1\,|\,\eta_\times\, U_0\, c_s}{\pi A_\psi\, \gamma_\psi}. \tag{10}$$

Even modest values ($\eta_\times \sim 0.3$, $U_0 = 100\,\mathrm{kJ}$, $A_\psi = 0.8\,\mathrm{m}^2$, $\gamma_\psi = 0.03\ \mathrm{s}^{-1}$) give $\Delta\psi \sim 1.2 \times 10^{-3}\,|\lambda - 1|$, which crosses cavity–atom sensitivity [3] in the 10^{-12}–10^{-15} range for $|\lambda - 1|$ in 10^{-9}–10^{-12}, consistent with the parametric thresholds.

Intentional ψ-pump detection checklist

Required capabilities:

- High-Q resonator ($Q \gtrsim 10^4$) with stored energy $U_0 \gtrsim 1\,\mathrm{MJ}$ (pulsed acceptable).
- Phase-stable amplitude modulation at 2ω with depth $m \sim 0.1$ on *stored* energy.
- Placement of cavity apertures at ψ antinodes (maximize $\mathcal{H}$; use multiple irises).
- Phase-sensitive readout near Ω_ψ; preserve 2ω tones (do not auto-suppress).
- Null sensitivity target: $\Delta\psi \lesssim 10^{-14}$ or equivalently $|\lambda - 1| \lesssim 10^{-14}$ via Eqs. (9)–(10).

7 Conclusion

We are *not* asking anyone to believe new physics; we are asking them to notice the parametric instability that is not there. Unoptimized cavities *accidentally* constrain $|\lambda - 1|$, and an *intentional* 2ω modulation test using the same hardware pushes ten orders tighter. **A single afternoon's measurement could either discover $\lambda \neq 1$ or constrain it below 10^{-14} using existing apparatus.** We invite groups with high–Q cavities and phase–stable 2ω drive to implement the intentional search of Eqs. (9)–(10).

The broader framework within which this coupling appears is developed in Refs. [1, 2, 5], with complementary experimental tests in matter-wave interferometry [4].

Acknowledgments

We thank microwave and optical cavity teams for maintaining exquisitely stable resonators that enable these constraints.

Appendix: Figures

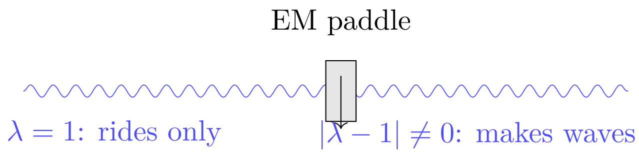

Figure 1: Paddle–on–water analogy: probe–only vs. pump.

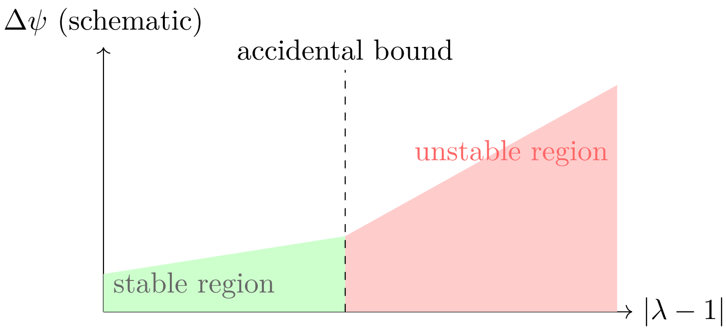

Figure 2: Stability constraint: if $|\lambda - 1|$ were too large, parametric instability would appear.

References

[1] G. Alcock, *Density Field Dynamics: Completing Einstein's 1911–12 Variable-c Program with Energy-Density Sourcing and Laboratory Falsifiability*, submitted to *Class. Quantum Grav.* (2025).

[2] G. Alcock, *Strong Fields and Gravitational Waves in Density Field Dynamics: From Optical First Principles to Quantitative Tests*, Zenodo preprint (2025). `doi:10.5281/zenodo.17115941`

[3] G. Alcock, *Sector-Resolved Test of Local Position Invariance Using Co-Located Cavity–Atom Frequency Ratios*, submitted to *Metrologia* (2025).

[4] G. Alcock, *Matter–Wave Interferometry Tests of Density Field Dynamics*, submitted to *Phys. Rev. Lett.* (2025).

[5] G. Alcock, *Density Field Dynamics Resolves the Penrose Superposition Paradox*, submitted to *Class. Quantum Grav.* (2025).

[6] J. D. Jackson, *Classical Electrodynamics*, 3rd ed., Wiley (1998).

[7] R. E. Collin, *Foundations for Microwave Engineering*, 2nd ed., McGraw–Hill (1992).

[8] N. W. McLachlan, *Theory and Application of Mathieu Functions*, Dover (1964).